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KVPY

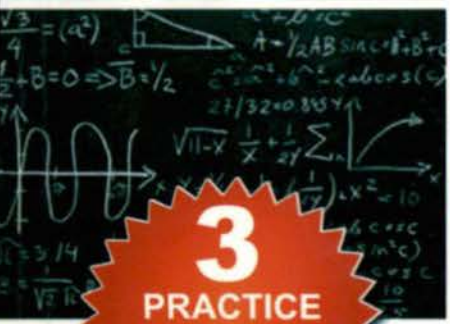
(Kishore Vaigyanik Protsahan Yojana)

STREAM - SX

Unit wise Old Examination Solved Paper

8 YEAR'S SOLVED PAPER
(2010 to 2017)

KVPY



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PHYSICS, CHEMISTRY, MATHEMATICS & BIOLOGY

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Preface

Whenever a student decides to prepare for any examination, her/his first and foremost curiosity is about the type of questions that he/she has to face.

We feel great pleasure to present before you this book. We have made an attempt to provide unit wise collection of questions asked in KVPY with answer and solutions to majority of questions. Solutions to the questions are not just sketch rather have been written in such a manner that the students will be able to understand the application of concept and can answer some other related questions too.

We firmly believe that the book in this form will definitely help a genuine, hardworking student.

We have tried our best to keep errors out of this book. Comment and criticism from readers will be highly appreciated and incorporated in the subsequent edition.

We wish to utilize the opportunity to place on record our special thanks to all team members of Content Development for their efforts to make this wonderful book.

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Physics

[Topic- wise KVPY [STREAM-SX] Questions with Solutions]

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Chapter 1

Mechanics

1. A pen of mass 'm' is lying on a piece of paper of mass M placed on a rough table. If the coefficient of friction between the pen and paper, and, the paper and table are μ_1 and μ_2, respectively, then the minimum horizontal force with which the paper has to be pulled for the pen to start slipping is given by - **[2010]**

(A) $(m + M)(\mu_1 + \mu_2)\, g$

(B) $(m\mu_1 + M\mu_2)g$

(C) $\{m\mu_1 + (m + M)\mu_2\}\, g$

(D) $m(\mu_1 + \mu_2)\, g$

2. Two masses m_1 and m_2 connected by a spring of spring constant k rest on a frictionless surface. If the masses are pulled apart and let go, the time period of oscillation is - **[2010]**

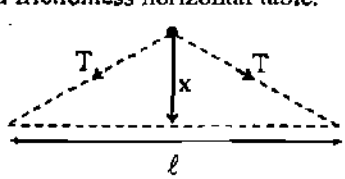
(A) $T = 2\pi\sqrt{\frac{1}{k}\left(\frac{m_1 m_2}{m_1 + m_2}\right)}$

(B) $T = 2\pi\sqrt{k\left(\frac{m_1 + m_2}{m_1 m_2}\right)}$

(C) $T = 2\pi\sqrt{\left(\frac{m_1}{k}\right)}$

(D) $T = 2\pi\sqrt{\left(\frac{m_2}{k}\right)}$

3. A bead of mass m is attached to the mid-point of a taut, weightless string of length ℓ and placed on a frictionless horizontal table.



Under a small transverse displacement x, as shown, if the tension in the string is T, then the frequency of oscillation is- **[2010]**

(A) $\frac{1}{2\pi}\sqrt{\frac{2T}{m\ell}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{4T}{m\ell}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{4T}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2T}{m}}$

4. A comet (assumed to be in an elliptical orbit around the sun) is at a distance of 0.4 AU from the sun at the perihelion. If the time period of the comet is 125 years, what is the aphelion distance ? AU : Astronomical Unit. **[2010]**

(A) 50 AU (B) 25 AU

(C) 49.6 AU (D) 24.6 AU

5. A book is resting on shelf that is undergoing vertical simple harmonic oscillations with an amplitude of 2.5 cm. What is the minimum frequency of oscillation of the shell for which the book will lose contact with the shelf ? (Assume that $g = 10\ m/s^2$) **[2010]**

(A) 20 Hz (B) 3.18 Hz

(C) 125.6 Hz (D) 10 Hz

6. Two masses m_1 and m_2 are connected by a massless spring of spring constant k and unstreched length ℓ. The masses are placed on a frictionless straight channel – which we consider our x-axis. They are initially at rest at $x = 0$ and $x = \ell$, respectively. At $t = 0$, a velocity of v_0 is suddenly imparted to the first particle. At a later time t_0, the centre of mass of the two masses is at- **[2010]**

(A) $x = \frac{m_2\ell}{m_1 + m_2}$

(B) $x = \frac{m_1\ell}{m_1 + m_2} + \frac{m_2 v_0 t}{m_1 + m_2}$

(C) $x = \frac{m_2\ell}{m_1 + m_2} + \frac{m_2 v_0 t}{m_1 + m_2}$

(D) $x = \frac{m_2\ell}{m_1 + m_2} + \frac{m_1 v_0 t}{m_1 + m_2}$

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7. A ball is dropped vertically from a height of h onto a hard surface. If the ball rebounds from the surface with a fraction r of the speed with which it strikes the latter on each impact, what is the net distance traveled by the ball up to the 10^{th} impact ? **[2010]**

(A) $2h\frac{1-r^{10}}{1-r}$ (B) $h\frac{1-r^{20}}{1-r^2}$

(C) $2h\frac{1-r^{22}}{1-r^2}-h$ (D) $2h\frac{1-r^{20}}{1-r^2}-h$

8. A certain planet completes one rotation about its axis in time T. The weight of an object placed at the equator on the planet's surface is a fraction f (f is close to unity) of its weight recorded at a latitude of 60°. The density of the planet (assumed to be a uniform perfect sphere is given by- **[2010]**

(A) $\frac{4-f}{1-f}\frac{3\pi}{4GT^2}$ (B) $\frac{4-f}{1+f}\frac{3\pi}{4GT^2}$

(C) $\frac{4-3f}{1-f}\frac{3\pi}{4GT^2}$ (D) $\frac{4-2f}{1-f}\frac{3\pi}{4GT^2}$

9. A particle of mass m undergoes oscillations about x = 0 in a potential given by $V(x) = \frac{1}{2}kx^2 - V_0\cos\left(\frac{x}{a}\right)$, where V_0, k, a are constants. If the amplitude of oscillation is much smaller than a, the time period is given by - **[2010]**

(A) $2\pi\sqrt{\frac{ma^2}{ka^2+V_0}}$ (B) $2\pi\sqrt{\frac{m}{k}}$

(C) $2\pi\sqrt{\frac{ma^2}{V_0}}$ (D) $2\pi\sqrt{\frac{ma^2}{ka^2-V_0}}$

10. A narrow but tall cabin is falling freely near the earth's surface. Inside the cabin, two small stones A and B are released from rest (relative to the cabin). Initially A is much above the centre of mass and B much below the centre of mass of the cabin. A close observation of the motion of A and B will reveal that - **[2011]**

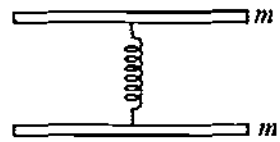
(A) both A and B continue to be exactly at rest relative to the cabin

(B) A moves slowly upward and B moves slowly downward relative to the cabin

(C) both A and B fall to the bottom of the cabin with constant acceleration due to gravity

(D) A and B move slightly towards each other vertically

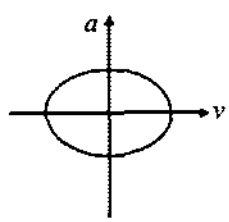
11. Two plates each of the mass m are connected by a massless spring as shown.



A weight W is put on the upper plate which compresses the spring further. When W is removed, the entire assembly jumps up. The minimum weight W needed for the assembly to jump up when the weight is removed is just more than - **[2011]**

(A) mg (B) 2 mg (C) 3 mg (D) 4 mg

12. If the speed (v) of the bob in a simple pendulum is plotted against the tangential acceleration (a), the correct graph will be represented by - **[2011]**



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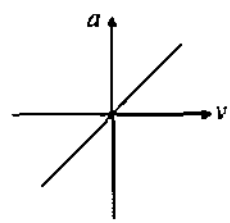

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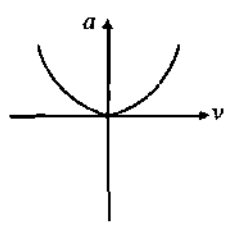

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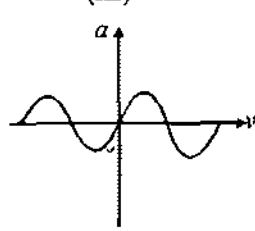

(IV)

(A) I (B) II (C) III (D) IV

13. The Quantum Hall Resistance R_H is a fundamental constant with dimensions of resistance. If h is Planck's constant and e the electron charge, then the dimension of R_H is the same as - **[2011]**

(A) e^2/h (B) h/e^2 (C) h^2/e (D) e/h^2

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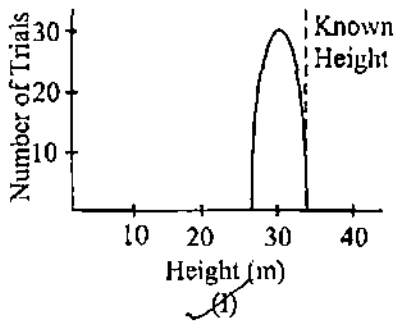
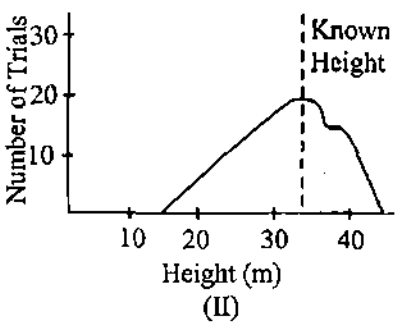
14. Four students measure the height of a tower. Each student uses a different method and each measures the height many different times. The data for each are plotted below. The measurement with highest precision is [2011]



(I)



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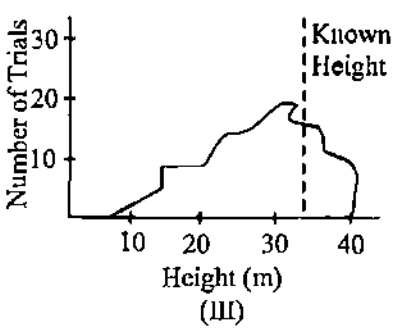

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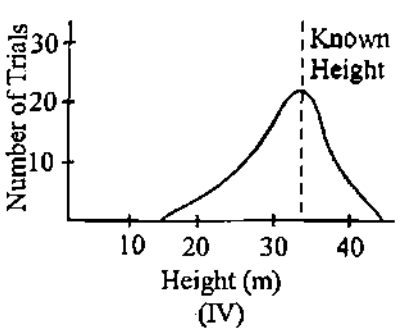
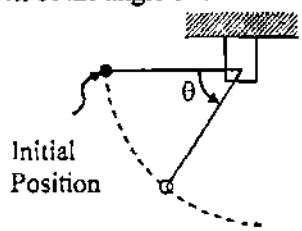


(IV)

(A) I (B) II (C) III (D) IV

15. Figure below shows a small mass connected to a string, which is attached to a vertical post. If the ball is released when the string is horizontal as shown, the magnitude of the total acceleration (including radial and tangential) of the mass as a function of the angle θ is- [2011]



(A) $g \sin\theta$ (B) $g\sqrt{3\cos^2\theta+1}$

(C) $g \cos\theta$ (D) $g\sqrt{3\sin^2\theta+1}$

16. A particle moves in a plane along an elliptic path given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. At point (0, b), the x-component of velocity is u. The y-component of acceleration at this point is - [2011]

(A) $-bu^2/a^2$ (B) $-u^2/b$

(C) $-au^2/b^2$ (D) $-u^2/a$

17. A liquid drop placed on a horizontal plane has a near spherical shape (slightly flattened due to gravity). Let R be the radius of its largest horizontal section. A small disturbance causes the drop to vibrate with frequency v about its equilibrium shape. By dimensional analysis the ratio $\frac{v}{\sqrt{\sigma/\rho R^3}}$ can be (Here σ is surface tension, ρ is density, g is acceleration due to gravity, and k is arbitrary dimensionless constant)– [2012]

(A) $k\rho g R^2/\sigma$ (B) $k\rho R^2/g\sigma$

(C) $k\rho R^3/g\sigma$ (D) $k\rho/g\sigma$

18. Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches its neighbours. Each coin is a thin disc of mass m and radius r. Note that the moment of inertia of an individual coin about an axis passing through centre and perpendicular to the plane of the coin is $mr^2/2$ [2012]

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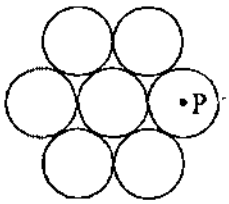
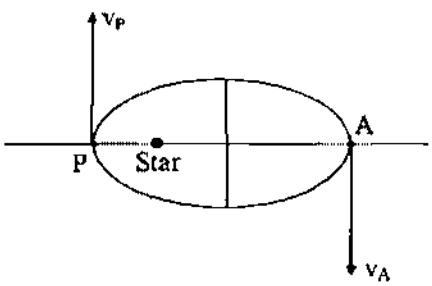


The moment of inertia of the system of seven coins about an axis that passes through the point P (the centre of the coin positioned directly to the right of the central coin) and perpendicular to the plane of the coins is–

(A) $\frac{55}{2}mr^2$ (B) $\frac{127}{2}mr^2$

(C) $\frac{111}{2}mr^2$ (D) $55\ mr^2$

19. A planet orbits in an elliptical path of eccentricity e around a massive star considered fixed at one of the foci. The point in space where it is closest to the star is denoted by P and the point where it is farthest is denoted by A. Let v_p and v_a be the respective speeds at P and A. Then – **[2012]**



(A) $\frac{v_P}{v_A}=\frac{1+e}{1-e}$ (B) $\frac{v_P}{v_A}=\frac{1+e^2}{1-e}$

(C) $\frac{v_P}{v_A}=1$ (D) $\frac{v_P}{v_A}=\frac{1+e^2}{1-e^2}$

20. A body is executing simple harmonic motion of amplitude a and period T about the equilibrium position x = 0. large numbers of snapshots are taken at random of this body in motion. The probability of the body being found in a very small interval x to x + |dx| is highest at - **[2012]**

(A) $x=\pm a$ (B) $x=\pm a/2$

(C) $x=0$ (D) $x=\pm\frac{1}{\sqrt{2}}$

21. A particle is acted upon by a force given by $F=-\alpha x^3-\beta x^4$ where α and β are positive constants. At the point x = 0, the particle is – **[2012]**

(A) In stable equilibrium

(B) In unstable equilibrium

(C) In neutral equilibrium

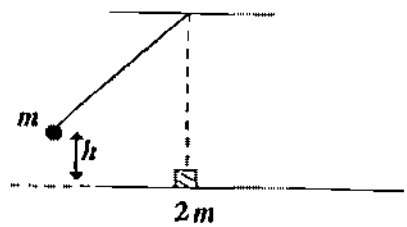
(D) Not in equilibrium

22. The potential energy of a point particle is given by the expression $V(x) = -\alpha x + \beta \sin(x/\gamma)$. A dimensionless combination of the constant α, β and γ is– **[2012]**

(A) $\alpha/\beta\gamma$ (B) $\alpha^2/\beta\gamma$

(C) $\gamma/\alpha\beta$ (D) $\alpha\gamma/\beta$

23. A ball of mass m suspended from a rigid support by an inextensible massless string is released from a height h above its lowest point. At its lowest point it collides elastically with a block of mass 2m at rest on a frictionless surface. Neglect the dimensions of the ball and the block. After the collision the ball rises to a maximum height of– **[2012]**



(A) h/3 (B) h/2

(C) h/8 (D) h/9

24. A particle released from rest is falling through a thick fluid under gravity. The fluid exerts a resistive force on the particle proportional to the square of its speed. Which one of the following graphs best depicts the variation of its speed v with time t – **[2012]**

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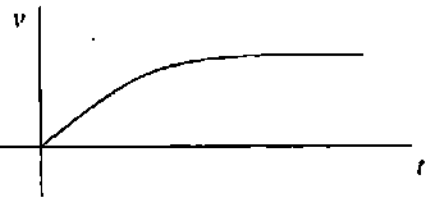
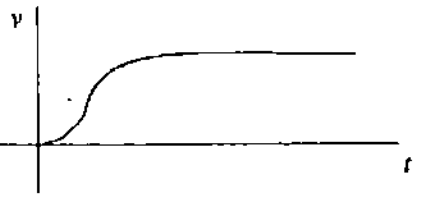
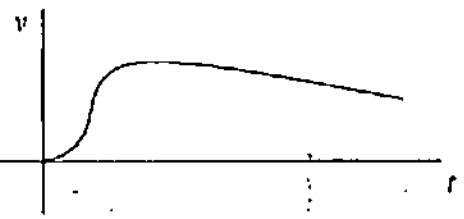
(A) 

(B) 

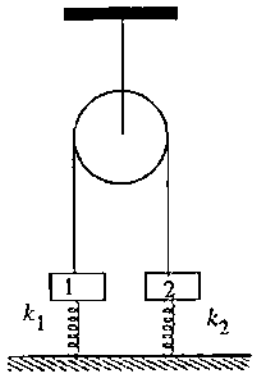
(C)

(D) 

25. A small asteroid is orbiting around the sun in a circular orbit of radius r_0 with speed V_0. A rocket is launched from the asteroid with speed $V = \alpha V_0$, where V is the speed relative to the sun. The highest value of α for which the rocket will remain bound to the solar system is (ignoring gravity due to the asteroid and effects of other planets) – **[2012]**

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) 1

26. Two blocks (1 and 2) of equal mass m are connected by an ideal string (see figure shown) over a frictionless pulley. The blocks are attached to the ground by springs having spring constants k_1 and k_2 such that $k_1 > k_2$



Initially, both springs are unstretched. The block 1 is slowly pulled down a distance x and released. Just after the release the possible values of the magnitude of the acceleration of the blocks a_1 and a_2 can be– **[2012]**

(A) Either $\left(a_1 = a_2 = \frac{(k_1+k_2)x}{2m}\right)$

or $\left(a_1 \frac{k_1 x}{m} - g \text{ and } a_2 = \frac{k_2 x}{m} + g\right)$

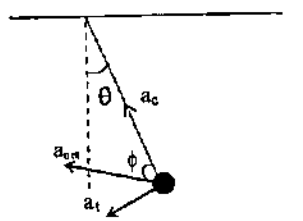
(B) $\left(a_1 = a_2 \frac{(k_1+k_2)x}{2m}\right)$ only

(C) $\left(a_1 = a_2 = \frac{(k_1-k_2)x}{2m}\right)$ only

(D) Either $\left(a_1 = a_2 = \frac{(k_1-k_2)x}{2m}\right)$

or $\left(a_1 = a_2 = \frac{(k_1 k_2)x}{(k_1+k_2)m} - g\right)$

27. A simple pendulum is released from rest at the horizontally stretched position. When the string makes an angle θ with the vertical, the angle ϕ which the acceleration vector of the bob makes with the string is given by– **[2012]**



(A) $\phi = 0$ (B) $\phi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$

(C) $\phi = \tan^{-1}(2\tan\theta)$ (D) $\phi = \pi/2$

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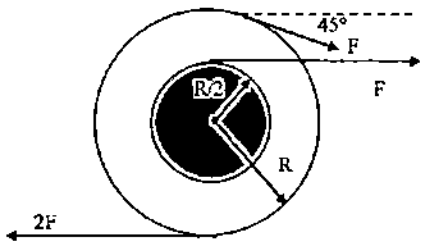
28. A wheel of radius R with an axle of radius R/2 is shown in the figure and is free to rotate about a frictionless axis through its centre and perpendicular to the page. Three forces (F, F, 2F) are exerted tangentially to the respective rims as shown in the figure. **[2013]**



The magnitude of the net torque acting on the system is nearly

(A) 3.5 FR (B) 3.2 FR
(C) 2.5 FR (D) 1.5 FR

29. Young-Laplace law states that the excess pressure inside a soap bubble of radius R is given by $\Delta P = 4\sigma/R$ where σ is the coefficient of surface tension of the soap. The Eötvös number Eo is a dimensionless number that is used to describe the shape of bubbles rising through a surrounding fluid. It is a combination of g, the acceleration due to gravity, ρ, the density of the surrounding fluid, σ and a characteristic length scale L which could be the radius of the bubble. A possible expression for Eo is **[2013]**

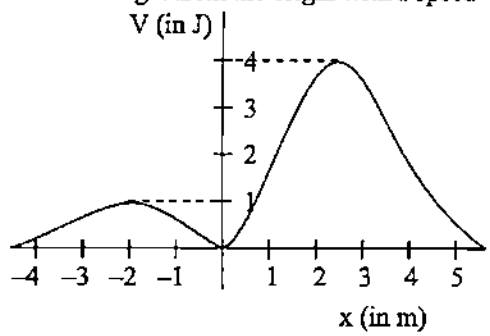
(A) $\frac{\rho g}{\sigma L^3}$ (B) $\frac{\rho L^2}{\sigma g}$ (C) $\frac{\rho g L^2}{\sigma}$ (D) $\frac{g L^2}{\sigma \rho}$

30. A plank is resting on a horizontal ground in the northern hemisphere of the Earth at a 45° latitude. Let the angular speed of the Earth be ω and its radius r_e. The magnitude of the frictional force on the plank will be : **[2013]**

(A) $mr_e\omega^2$ (B) $\frac{mr_e\omega^2}{\sqrt{2}}$

(C) $\frac{mr_e\omega^2}{2}$ (D) zero

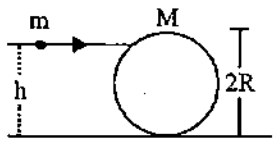
31. A point particle of mass 0.5 kg is moving along the x-axis under a force described by the potential energy V shown below. It is projected towards the right from the origin with a speed v.



What is the minimum value of v for which the particle will escape infinitely far away from the origin? **[2013]**

(A) $2\sqrt{2}$ ms^{-1}
(B) 2 ms^{-1}
(C) 4 ms^{-1}
(D) The particle will never escape

32. A bullet of mass m is fired horizontally into a large sphere of mass M and radius R resting on a smooth horizontal table



The bullet hits the sphere at a height h from the table and sticks to its surface. If the sphere starts rolling without slipping immediately on impact, then - **[2013]**

(A) $\frac{h}{R} = \frac{4m+3M}{2(m+M)}$ (B) $\frac{h}{R} = \frac{m+M}{m+2M}$

(C) $\frac{h}{R} = \frac{10m+7M}{5(m+M)}$ (D) $\frac{h}{R} = \frac{4m+3M}{m+M}$

33. A small boy is throwing a ball towards a wall 6 m in front of him. He releases the ball at a height of 1.4 m from the ground. The ball bounces from the wall at a height of 3m, rebounds from the ground and reaches the boy's hand exactly at the point of release. Assuming the two bounces (one from the wall and the other from the ground) to be perfectly elastic, how far ahead of the boy did the ball bounce from ground ? **[2013]**

(A) 1.5 m (B) 2.5 m
(C) 3.5 m (D) 4.5 m

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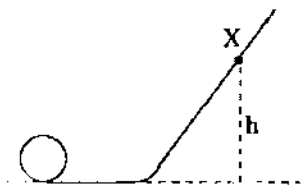
34. A ball is rolling without slipping in a spherical shallow bowl (radius R) as shown in the figure and is executing simple harmonic motion. If the radius of the ball is doubled, the period of oscillation - [2013]

(A) increases slightly
(B) is reduced by a factor of 1/2
(C) is increased by a factor of 2
(D) decreases slightly

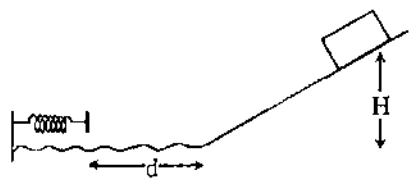
35. A solid sphere rolls without slipping, first horizontally and then up to a point X at height h on an inclined plane before rolling down, as shown -



The initial horizontal speed of the sphere is - [2013]

(A) $\sqrt{10gh/7}$ (B) $\sqrt{7gh/5}$
(C) $\sqrt{5gh/7}$ (D) $\sqrt{2gh}$

36. A block of mass m slides from rest at a height H on a frictionless inclined plane as shown in the figure. It travels a distance d across a rough horizontal surface with coefficient of kinetic friction μ, and compresses a spring of spring constant k by a distance x before coming to rest momentarily. Then the spring extends and the block travels back attainting a final height of h. Then - [2013]



(A) $h = H - 2\mu(d + x)$
(B) $h = H + 2\mu(d - x)$
(C) $h = H - 2\mu d + kx^2/mg$
(D) $h = H - 2\mu(d + x) + kx^2/2mg$

37. A uniform thin rod of length 2L and mass m lies on a horizontal table. A horizontal impulse J is given to the rod at one red. There is no friction. The total kinetic energy of the rod just after the impulse will be [2014]

(A) $\frac{J^2}{2m}$ (B) $\frac{J^2}{m}$ (C) $\frac{2J^2}{m}$ (D) $\frac{6J^2}{m}$

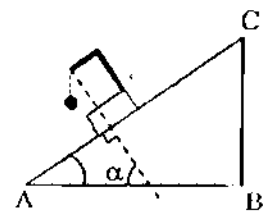
38. A solid cylinder P rolls without slipping from rest down an inclined plane attaining a speed v_P at the bottom. Another smooth solid cylinder Q of same mass and dimensions slides without friction from rest down the inclined plane attaining a speed v_Q at the bottom. The ratio of the speeds $\left(\frac{v_Q}{v_P}\right)$ is - [2014]

(A) $\sqrt{3/4}$ (B) $\sqrt{3/2}$
(C) $\sqrt{2/3}$ (D) $\sqrt{4/3}$

39. A body moves in a circular orbit of radius R under the action of a central force. Potential due to the central force is given by $V(r) = kr$ (k is a positive constant). Period of revolution of the body is proportional to- [2014]

(A) $R^{1/2}$ (B) $R^{-1/2}$ (C) $R^{3/2}$ (D) $R^{-5/2}$

40. A simple pendulum is attached to the block which slides without friction down an inclined plane (ABC) having an angle of inclination α as shown. [2014]



While the block is sliding down the pendulum oscillates in such a way that its mean position the direction of the string is-

(A) at angle α to the perpendicular to the inclined plane AC.
(B) parallel to the inclined plane AC.
(C) vertically downwards
(D) perpendicular to the inclined plane AC.

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41. Water containing air bubbles flows without turbulence through a horizontal pipe which has a region of narrow cross-section. In this region the bubbles **[2014]**

(A) move with greater speed and are smaller than in the rest of the pipe

(B) move with greater speed and are larger in size than in the rest of the pipe

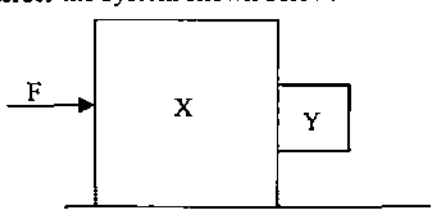
(C) move with lesser speed and are smaller than in the rest of the pipe

(D) move with lesser speed and are of the same size as in the rest of the pipe

42. A solid sphere spinning about a horizontal axis with an angular velocity ω is placed on a horizontal surface. Subsequently it rolls without slipping with an angular velocity of - **[2014]**

(A) $2\omega/5$ (B) $7\omega/5$ (C) $2\omega/7$ (D) ω

43. Consider the system shown below.



A horizontal force F is applied to a block X of mass 8 kg such that the block Y of mass 2 kg adjacent to it does not slip downwards under gravity. There is no friction between the horizontal plane and the base of the block X. The coefficient of friction between the surfaces of blocks X and Y is 0.5. Take acceleration due to gravity to be 10 ms^{-2}. The minimum value of F is **[2014]**

(A) 200 N (B) 160 N

(C) 40 N (D) 240 N

44. The maximum value attained by the tension in the string of a swinging pendulum is four times the minimum value it attains. There is no slack in the string. The angular amplitude of the pendulum is **[2014]**

(A) 90° (B) 60° (C) 45° (D) 30°

45. The dimensions of Stefan-Boltzmann constant σ can be written in terms of Planck's constant h, Boltzmann constant k_B and the speed of light c as $\sigma = h^{\alpha} k_B^{\beta} c^{\gamma}$. Here **[2014]**

(A) $\alpha = 3, \beta = 4$ and $\gamma = -3$

(B) $\alpha = 3, \beta = -4$ and $\gamma = 2$

(C) $\alpha = -3, \beta = 4$ and $\gamma = -2$

(D) $\alpha = 2, \beta = -3$ and $\gamma = -1$

46. A small box resting on one edge of the table is struck in such a way that it slides off the other edge, 1 m away, after 2 seconds. The coefficient of kinetic friction between the box and the table – **[2015]**

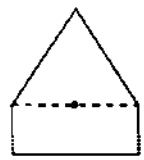
(A) must be less than 0.05

(B) must be exactly zero

(C) must be more than 0.05

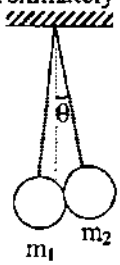
(D) must be exactly 0.05

47. Two uniform plates of the same thickness and area but of different materials, one shaped like an isosceles triangle and the other shaped like a rectangle are joined together to form a composite body as shown in the figure. If the centre of mass of the composite body is located at the midpoint of their common side, the ratio between masses of the triangle to that of the rectangle is - **[2015]**



(A) 1 : 1 (B) 4 : 3 (C) 3 : 4 (D) 2 : 1

48. wo spherical objects each of radii R and masses m_1 and m_2 are suspended using two strings of equal length L as shown in the figure (R << L). The angle, θ which mass m_2 makes with the vertical is approximately - **[2015]**



(A) $\frac{m_1 R}{(m_1 + m_2)L}$ (B) $\frac{2m_1 R}{(m_1 + m_2)L}$

(C) $\frac{2m_2 R}{(m_1 + m_2)L}$ (D) $\frac{m_2 R}{(m_1 + m_2)L}$

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49. A horizontal disk of moment of inertia 4.25 kg-m^2 with respect to its axis of symmetry is spinning counter clockwise at 15 revolutions per second about its axis, as viewed from above. A second disk of moment of inertia 1.80 kg-m^2 with respect to its axis of symmetry is spinning clockwise at 25 revolutions per second as viewed from above about the same axis and is dropped on top of the first disk. The two disks stick together and rotate as one about their axis of symmetry. The new angular velocity of the system as viewed from above is close to - **[2015]**

(A) 18 revolutions/second and clockwise
(B) 18 revolutions/second and counter clockwise
(C) 3 revolutions/second and clockwise
(D) 3 revolutions/second and counter clockwise

50. A boy is standing on top of a tower of height 85 m and throws a ball in the vertically upward direction with a certain speed. If 5.25 seconds later he hears the ball hitting the ground, then the speed with which the boy threw the ball is (take g = 10 m/s^2, speed of sound in air = 340 m/s) **[2015]**

(A) 6 m/s (B) 8 m/s (C) 10 m/s (D) 12 m/s

51. A simple pendulum of length ℓ is made to oscillate with an amplitude of 45 degrees. The acceleration due to gravity is g. Let $T_0 = 2\pi\sqrt{\ell/g}$. The time period of oscillation of this pendulum will be - **[2015]**

(A) T_0 irrespective of the amplitude
(B) slightly less than T_0
(C) slightly more than T_0
(D) dependent on whether it swings in a plane aligned with the north-south or east-west directions

52. The dimensions of the area A of a black hole can be written in terms of the universal gravitational constant G, its mass M and the speed of light c as $A = G^{\alpha}M^{\beta}c^{\gamma}$. Here - **[2015]**

(A) $\alpha = -2$, $\beta = -2$, and $\gamma = 4$
(B) $\alpha = 2$, $\beta = 2$, and $\gamma = -4$
(C) $\alpha = 3$, $\beta = 3$, and $\gamma = -2$
(D) $\alpha = -3$, $\beta = -3$, and $\gamma = 2$

53. One end of a slack wire (Young's modulus Y, length L and cross-section area A) is clamped to rigid wall and the other end to a block (mass m) which rests on a smooth horizontal plane. The block is set in motion with a speed v. What is the maximum distance the block will travel after the wire becomes taut ? **[2015]**

(A) $v\sqrt{\frac{mL}{AY}}$ (B) $v\sqrt{\frac{2mL}{AY}}$

(C) $v\sqrt{\frac{mL}{2AY}}$ (D) $L\sqrt{\frac{mv}{AY}}$

54. The moments of inertia of a non-uniform circular disc (of mass M and radius R) about four mutually perpendicular tangents AB, BC, CD, DA are I_1, I_2, I_3 and I_4, respectively (the square ABCD circumscribes the circle). The distance of the center of mass of the disc from its geometrical center is given by - **[2015]**

(A) $\frac{1}{4MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(B) $\frac{1}{12MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(C) $\frac{1}{3MR}\sqrt{(I_1 - I_2)^2 + (I_3 - I_4)^2}$

(D) $\frac{1}{2MR}\sqrt{(I_1 + I_3)^2 + (I_2 + I_4)^2}$

55. An object is propelled vertically to a maximum height of 4R from the surface of a planet of radius R and mass M. The speed of object when it returns to the surface of the planet is – **[2015]**

(A) $2\sqrt{\frac{2GM}{5R}}$ (B) $\sqrt{\frac{GM}{2R}}$

(C) $\sqrt{\frac{3GM}{2R}}$ (D) $\sqrt{\frac{GM}{5R}}$

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56. Uniform ring of radius R is moving on a horizontal surface with speed v and then climbs up a ramp of inclination 30° to a height h. There is no slipping in the entire motion. Then h is **[2016]**

(A) $v^2/2g$ (B) v^2/g (C) $3v^2/2g$ (D) $2v^2/g$

57. A particle with total mechanical energy, which is small and negative, is under the influence of a one dimensional potential $U(x) = x^4/4 - x^2/2$ J Where x is in meters. At time t = 0s, it is at x = – 0.5 m. Then at a later time it can be found **[2016]**

(A) Anywhere on the x axis
(B) Between x = – 1.0 m to x = 1.0 m
(C) Between x = – 1.0 m to x = 0.0 m
(D) Between x = 0.0 m to x = 1.0 m

58. A particle at a distance of 1 m from the origin starts moving such that $dr/d\theta = r$, where (r, θ) are polar coordinates. Then the angle between resultant velocity and tangential velocity component is **[2016]**

(A) 30 degrees
(B) 45 degrees
(C) 60 degrees
(D) Dependent on where the particle is

59. A light balloon filled with helium of density ρ_{He} is tied to a long light string of length ℓ and the string is attached to the ground. If the balloon is displaced slightly in the horizontal direction from the equilibrium and released then. **[2016]**

(A) The ballon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{air}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

(B) The ballon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{air}-\rho_{He}}{\rho_{air}}\right)\frac{\ell}{g}}$

(C) The ballon undergoes simple harmonic motion with period $2\pi\sqrt{\left(\frac{\rho_{He}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

(D) The ballon undergoes conical oscillations with period $2\pi\sqrt{\left(\frac{\rho_{air}+\rho_{He}}{\rho_{air}-\rho_{He}}\right)\frac{\ell}{g}}$

60. Consider two masses with $m_1 > m_2$ connected by a light inextensible string that passes over a pulley of radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley and the pulley turns without friction. The two masses are released from rest separated by a vertical distance 2h. When the two masses pass each other, the speed of the masses is proportional to **[2016]**

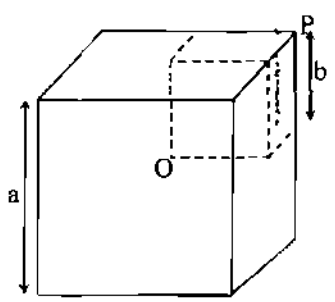
(A) $\sqrt{\frac{m_1-m_2}{m_1+m_2+\frac{I}{R^2}}}$

(B) $\sqrt{\frac{(m_1+m_2)(m_1-m_2)}{m_1+m_2+\frac{I}{R^2}}}$

(C) $\sqrt{\frac{m_1+m_2+\frac{I}{R^2}}{m_1-m_2}}$

(D) $\sqrt{\frac{\frac{I}{R^2}}{m_1+m_2}}$

61. A smaller cube with side b (depicted by dashed lines) is excised from a bigger uniform cube with side a as shown below such that both cubes have a common vertex P. Let X = a/b. If the centre of mass of the remaining solid is at the vertex O of smaller cube then X satisfies. **[2016]**



(A) $X^3 - X^2 - X - 1 = 0$
(B) $X^2 - X - 1 = 0$
(C) $X^3 + X^2 - X - 1 = 0$
(D) $X^3 - X^2 - X + 1 = 0$

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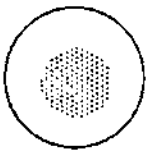
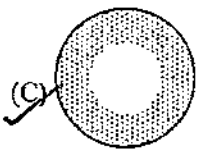
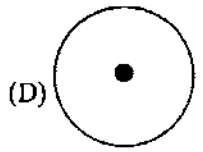
62. Consider a bowl filled with water on which some black pepper powder have been sprinkled uniformly. Now a drop of liquid soap is added at the centre of the surface of water. The picture of the surface immediately after this will look like **[5 Nov, 2017]**

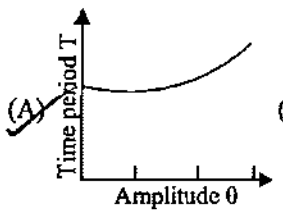
(A)

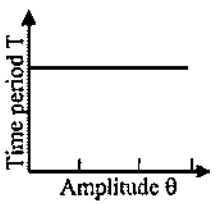
(B) 

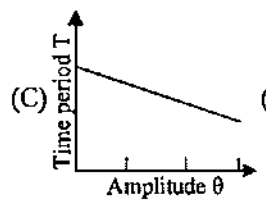
(C) 

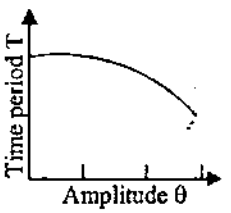
(D) 

63. Which of the following plots represents schematically the dependence of the time period of a pendulum if measured and plotted as a function of its oscillations? (Note : amplitude need not be small) **[5 Nov, 2017]**

(A) 

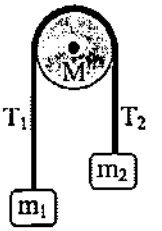
(B) 

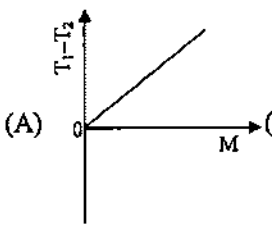
(C) 

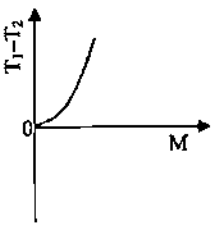
(D) 

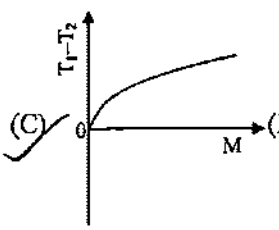
64. On a pulley of mass M hangs a rope with two masses m_1 and m_2 ($m_1 > m_2$) tied at the ends as shown in the figure. The pulley rotates without any friction, whereas the friction between the rope and the pulley is large enough to prevent any slipping. Which of the following plots best represents the difference between the tensions in the rope on the two sides of the pulley as a function of the mass of the pulley ?

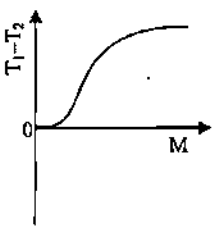
[5 Nov, 2017]



(A) 

(B) 

(C) 

(D) 

65. Two satellites S_1 and S_2 are revolving around a planet in the opposite sense in coplanar circular concentric orbits. At time t = 0, the satellites are farthest apart. The periods of revolution of S_1 and S_2 are 3 h and 24 h respectively. The radius of the orbit of S_1 is 3×10^4 km. Then the orbital speed of S_2 as observed from **[5 Nov, 2017]**

(A) the planet is $4\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1.

(B) the planet is $2\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1.

(C) S_1 is $\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1

(D) S_1 is $3\pi \times 10^4$ km h^{-1} when S_2 is closest from S_1

66. Force $\vec{F}$ applied on a body is written as $\vec{F} = (\hat{n}.\vec{F})\,\hat{n} + \vec{G}$, where $\hat{n}$ is a unit vector. The vector $\vec{G}$ is equal to **[5 Nov, 2017]**

(A) $\hat{n} \times \vec{F}$ (B) $\hat{n} \times (\hat{n} \times \vec{F})$

(C) $(\hat{n} \times \vec{F}) \times \vec{F}/|\vec{F}|$ (D) $(\hat{n} \times \vec{F}) \times \hat{n}$

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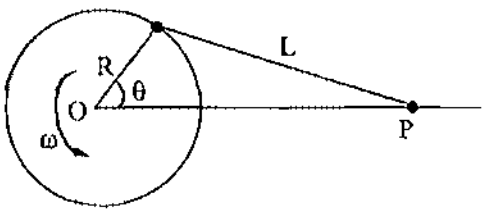
67. One end of a rod of length L=1 m is fixed to a point on the circumference of a wheel of radius $R = 1/\sqrt{3}$ m. The other end is sliding freely along a straight channel passing through the center O of the wheel as shown in the figure below. The wheel is rotating with a constant angular velocity ω about O. **[5 Nov, 2017]**



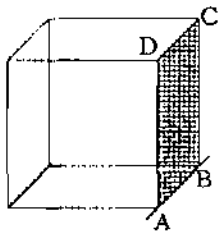
The speed of the sliding end P when θ = 60° is

(A) $\frac{2\omega}{3}$ (B) $\frac{\omega}{3}$

(C) $\frac{2\omega}{\sqrt{3}}$ (D) $\frac{\omega}{\sqrt{3}}$

68. A solid cube of wood of side 2a and mass M is resting on a horizontal surface as shown in the figure. The cube is free to rotate about a fixed axis AB. A bullet of mass m (<<M) and speed v is shot horizontally at the face opposite to ABCD at a height 4a/3 from the surface to impart the cube an angular speed ω_c. It strike the face and embeds in the cube. Then ω_c is close to (note : the moment of inertia of the cube about an axis perpendicular to the face and passing through the center of mass is $2Ma^2/3$)

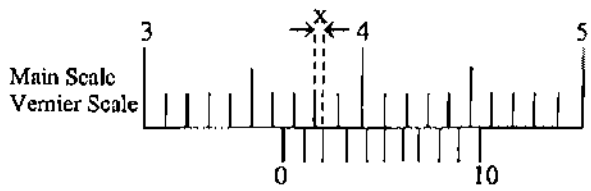
[5 Nov, 2017]



(A) Mv/ma (B) Mv/2ma

(C) mv/Ma (D) mv/2Ma

69. The figure of a centimeter scale below shows a particular position of the vernier calipers. In this position the value of x shown in the figure is (figure is not to scale) **[5 Nov, 2017]**

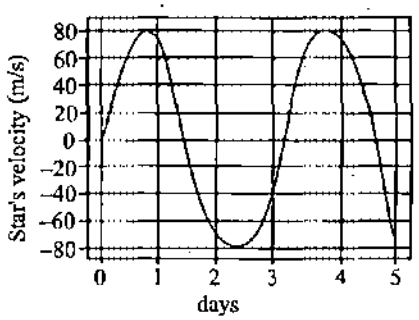


(A) 0.02 cm (B) 3.65 cm

(C) 4.15 cm (D) 0.03 cm

70. A star of mass M (equal to the solar mass) with a planet (much smaller than the star) revolves around the star in a circular orbit. The velocity of the star with respect to the center of mass of the star-planet system is shown below :

[5 Nov, 2017]



The radius of the planet's orbit is closest to
(1 A. U. = Earth-Sun distance)

(A) 0.004 A. U. (B) 0.008 A.U.

(C) 0.004 A.U. (D) 0.12 A.U.

71. The distance s travelled by a particle in time t is

$$s = ut - \frac{1}{2}gt^2$$

The initial velocity of the particle was measured to be u = 1 .11 ± 0.01 m/s and the time interval of the experiment was t = 1.01 ± 0.1 s. The acceleration was taken to be g = 9.8 ± 0.1 m/s^2. With these measurements, the student estimates the total distance travelled. How should the student report the result ? **[19 Nov, 2017]**

(A) 1.121 ± 0.1 m (B) 1.1 ± 0.1 m

(C) 1.12 ± 0.07 m (D) 1.1 ± 0.07 m

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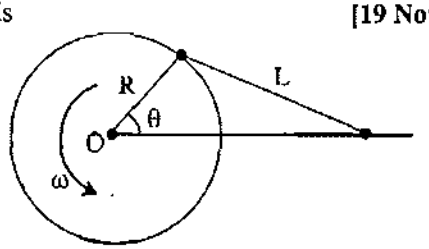
72. A massive black hole of mass m and radius R is spinning with angular velocity ω. The power P radiated by it as gravitational waves is given by $P = Gc^{-5}m^xR^y\omega^z$, where c and G are speed of light in free space, and the universal gravitational constant, respectively. Then **[19 Nov, 2017]**

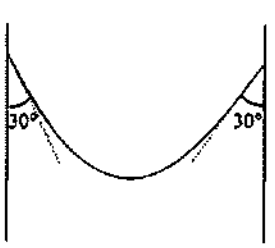
(A) $x = -1, y = 2, z = 4$
(B) $x = 1, y = 1, z = 4$
(C) $x = -1, y = 4, z = 4$
(D) $x = 2, y = 4, z = 6$

73. One end of a rod of length L is fixed to a point on the circumference of a wheel of radius R. The other end is sliding freely along a straight channel passing through the centre O of the wheel as shown in the figure below. The wheel is rotating with a constant angular velocity ω about O. Taking $T = \frac{2\pi}{\omega}$ the motion of the rod is **[19 Nov, 2017]**



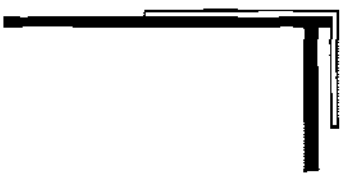
(A) simple harmonic with a period of T
(B) simple harmonic with a period of T/2
(C) not simple harmonic but periodic with a period of T
(D) not simple harmonic but periodic with a period of T/2

74. A rope of mass 5 kg is hanging between two supports as shown. The tension at the lowest point of the rope is close to (take $g = 10\ m/s^2$) **[19 Nov, 2017]**



(A) 22 N (B) 44 N
(C) 28 N (D) 14 N

75. A uniform rope of total length l is at rest on a table with fraction f of its length hanging (see figure). If the coefficient of friction between the table and the chain is μ then **[19 Nov, 2017]**

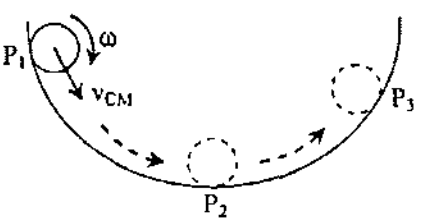


(A) $f = \mu$ (B) $f = 1/(1 + \mu)$
(C) $f = 1/(1 + 1/\mu)$ (D) $f = 1/(\mu + 1/\mu)$

76. A star of mass M and radius R is made up of gases. The average gravitational pressure compressing the star due to gravitational pull of the gases making up the star depends on R as **[19 Nov, 2017]**

(A) $\frac{1}{R^4}$ (B) $\frac{1}{R}$
(C) $\frac{1}{R^2}$ (D) $\frac{1}{R^6}$

77. A small ring is rolling without slipping on the circumference of a large bowl as shown in the figure. The ring is moving down at P_1, comes down to the lower most point P_2 and is climbing up at P_3. Let $\vec{v}_{CM}$ denote the velocity of the centre of mass of the ring. Choose the correct statement regarding the frictional force on the ring. **[19 Nov, 2017]**



(A) It is opposite to $\vec{v}_{CM}$ at the points P_1, P_2 and P_3.
(B) It is opposite to $\vec{v}_{CM}$ at P_1 and in the same direction as $\vec{v}_{CM}$ at P_3.
(C) It is in the same direction as $\vec{v}_{CM}$ at P_1 and opposite to $\vec{v}_{CM}$ at P_3.
(D) It is zero at the points P_1, P_2 and P_3.

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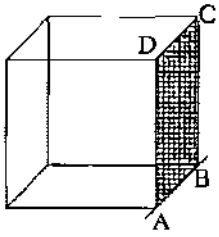
78. A solid cube of wood of side 2a and mass M is resting on a horizontal surface as shown in the figure. The cube is free to rotate about the fixed axis AB. A bullet of mass m (<<M) and speed v is shot horizontally at the face opposite to ABCD at a height 'h' above the surface to impart the cube an angular speed ω_c so that the cube just topples over. Then ω_c is (note : the moment of inertia of the cube about an axis perpendicular to the face and passing through the center of mass is $2Ma^3/3$) **[19 Nov, 2017]**



(A) $\sqrt{3gM/2ma}$ (B) $\sqrt{3g/4h}$

(C) $\sqrt{3g(\sqrt{2}-1)/2a}$ (D) $\sqrt{3g(\sqrt{2}-1)/4a}$

79. A uniform thin wooden plank AB of length L and mass M is kept on a table with its B end slightly outside the edge of the table. When an impulse J is given to the end B, the plank moves up with centre of mass rising a distance 'h' from the surface of the table. Then-

[19 Nov, 2017]

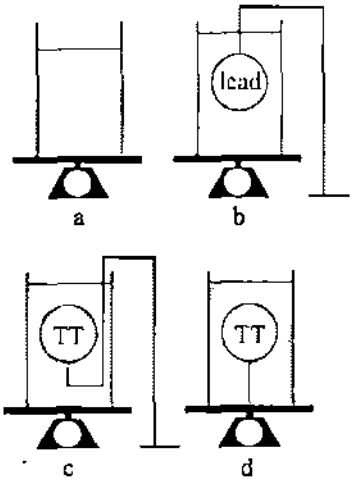
(A) $h > 9J^2/8M^2g$

(B) $h = J^2/2M^2g$

(C) $J^2/2M^2g < h < 9J^2/8M^2g$

(D) $h < J^2/2M^2g$

80. Four identical beakers contain same amount of water as shown below. Beaker 'a' contains only water. A lead ball is held submerged in the beaker 'b' by string from above. A same sized plastic ball, say a table tennis (TT) ball, is held submerged in beaker 'c' by a string attached to a stand from outside. Beaker 'd' contains same sized TT ball which is held submerged from a string attached to the bottom of the beaker. These beakers (without stand) are placed on weighing pans and register reading W_a, W_b, W_c and W_d for a, b, c and d, respectively. (Effects of the mass and volume of the stand and string are to be neglected) **[19 Nov, 2017]**



(A) $W_a = W_b = W_c = W_d$

(B) $W_b = W_c > W_d > W_a$

(C) $W_b = W_c > W_a > W_d$

(D) $W_b > W_c > W_d > W_a$

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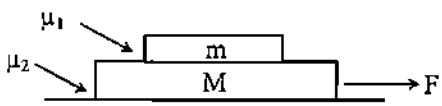
ANSWERS

1. (A)	2. (A)	3. (B)	4. (C)	5. (B)	6. (D)	7. (D)
8. (A)	9. (A)	10. (B)	11. (B)	12. (A)	13. (B)	14. (A)
15. (D)	16. (A)	17. (A)	18. (C)	19. (A)	20. (A)	21. (B)
22. (D)	23. (D)	24. (C)	25. (A)	26. (A)	27. (B)	28. (A)
29. (C)	30. (C)	31. (B)	32. (C)	33. (A)	34. (D)	35. (A)
36. (A)	37. (C)	38. (B)	39. (A)	40. (D)	41. (B)	42. (C)
43. (A)	44. (B)	45. (C)	46. (A)	47. (C)	48. (B)	49. (D)
50. (B)	51. (C)	52. (B)	53. (A)	54. (A)	55. (A)	56. (B)
57. (C)	58. (B)	59. (C)	60. (C)	61. (A)	62. (C)	63. (A)
64. (C)	65. (D)	66. (D)	67. (A)	68. (D)	69. (D)	70. (C)
71. (B)	72. (D)	73. (C)	74. (D)	75. (C)	76. (A)	77. (B)
78. (D)	79. (C)	80. (B)				

SOLUTIONS

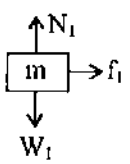
1.



For pen to start slipping maximum horizontal force on it is $f = \mu_1 mg$

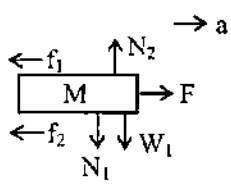
$\therefore a = \mu_1 g$ is the maximum common acceleration for both pen and paper

F.B.D. for both pen and paper



$f_1 = \mu N_1$ & $N_1 = W_1$

$\therefore f_1 = \mu_1 mg$



$\therefore N_2 = N_1 + W_1$

$F - f_1 - f_2 = Ma$

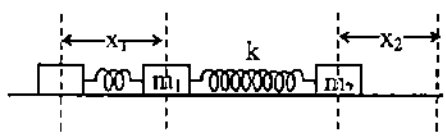
also $f_2 = \mu_2 N_2 = \mu_2 (m + M)g$

$\therefore F = f_1 + f_2 + Ma$

$F = \mu_1 mg + \mu_2 (m + M)g + M(\mu_1 g)$

$\therefore \boxed{F = (m + M)(\mu_1 + \mu_2)\, g}$

2.



Let the masses be slightly displaced by x_1 and x_2 from this equilibrium position in opposite direction so net stretch in spring is $x = x_1 + x_2$. Because of this a restoring force kx will act on each mass and therefore equation for m_1 & m_2 will be

$$m_1 \frac{d^2x_1}{dt^2} = -kx \quad \text{and} \quad m_2 \frac{d^2x_2}{dt^2} = -kx$$

but as $x = x_1 + x_2$

$$\therefore \frac{d^2x}{dt^2} = \frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2}$$

replacing values of $\frac{d^2x_1}{dt^2}$ and $\frac{d^2x_2}{dt^2}$ from acceleration equations we get

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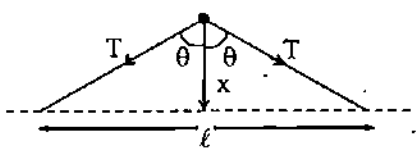
$$\frac{d^2x_2}{dt^2} = -\left(\frac{1}{m_1}+\frac{1}{m_2}\right)kx$$

also if $\frac{1}{m_1}+\frac{1}{m_2}=\frac{1}{m}$ then m is the effective mass in this case therefore

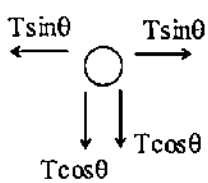
$$\frac{d^2x}{dt^2} = -\omega^2 x = \frac{-kx}{m} \text{ or } \omega^2 = \frac{k}{m} \text{ and } T = \frac{2\pi}{\omega}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}}$$

3.



Let the angle of T with the vertical be θ then F.B.D.



$\therefore 2T\cos\theta = ma$

also $\cos\theta = \frac{x}{\sqrt{x^2+\left(\frac{\ell}{2}\right)^2}}$

given $\ell >> x \quad \therefore \frac{\ell^2}{4}+x^2 \simeq \frac{\ell^2}{4}$

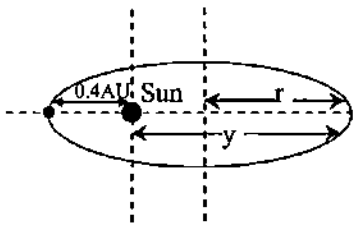
$\therefore a = \frac{-2Tx}{m\left(\frac{\ell}{2}\right)}$ (negative sign for restoring force)

or $a = -\left(\frac{4T}{m\ell}\right)x$ also this is similar to the equation of SHM i.e. $a = -\omega^2 x$

$\therefore \omega = \sqrt{\frac{4T}{m\ell}}$ & $f = \frac{\omega}{2\pi}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{4T}{m\ell}}$

4.



$\therefore r = \frac{0.4+y}{2}$

also $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$ by Kepler's law of time-periods (T_1, r_1 are taken for earth)

$$\therefore \left(\frac{1\text{ y}}{125\text{ y}}\right)^2 = \left(\frac{1\text{ AU}}{\left(\frac{0.4+y}{2}\right)\text{AU}}\right)^3$$

solving we get $\boxed{y = 49.6\text{ AU}}$

5. Book will loose contact with the shelf when $a = g$

Now $|a| = \omega^2 x \quad \therefore g = \omega^2 A$

(A → Amplitude)

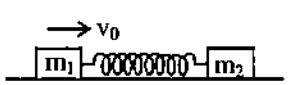
$\omega^2 = \frac{g}{A}$ also $f = \frac{\omega}{2\pi}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{g}{A}}$

replacing $g = 10\text{ m/s}^2$ and $A = 2.5\times10^{-2}\text{ m}$

We get $\boxed{f = 3.18\text{ Hz}}$

6.



$v_{COM} = \frac{m_1v_0+0}{m_1+m_2}$, $x_{iCOM} = \frac{m_1(0)+m_2(\ell)}{m_1+m_2}$

also $x_{COM} = x_{iCOM} + v_{COM}t$

$\therefore x_{COM} = \left(\frac{m_2\ell}{m_1+m_2}\right)+\frac{m_1v_0t}{m_1+m_2}$

7. Total distance =

$$\left(\frac{v_0^2}{g}+r^2\frac{v_0^2}{g}+r^4\frac{v_0^2}{g}+\text{...upto } 10^{\text{th}} \text{ terms}\right)-h$$

$$= \frac{v_0^2}{g}(1+r^2+r^4+....+10^{\text{th}}\text{ term})-h$$

also $v_0 = \sqrt{2gh}$

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$\therefore$ Total distance $= 2h\left(\frac{1-(r^2)^{10}}{1-r^2}\right) - h$

or $\boxed{\text{total distance} = \frac{2h(1-r^{20})}{(1-r^2)} - h}$

8. $v = \sqrt{\frac{GM}{r}}$ also $T = \frac{2\pi r}{v}$ or $T = 2\pi\sqrt{\frac{r^3}{GM}}$

when v is replaced by $\sqrt{\frac{GM}{r}}$

now $g_{eff} = g - \omega^2 R_e \cos^2\phi$

now $f = \frac{g - \omega^2 R_e \cos 0^\circ}{g - \omega^2 R_e \cos 60^\circ}$

solving we get $R_e = \frac{4g(f-1)}{\omega^2(f-4)}$

or $\frac{GM}{R_e^2}(f-1) = \frac{\omega^2 R_e}{4}(f-4)$

$R_e^3 = \frac{4GM}{\omega^2}\frac{(f-1)}{(f-4)}$

now $\rho = \frac{M}{\frac{4}{3}\pi R_e^3} = \frac{3}{16}\frac{\omega^2}{\pi G}\frac{(f-4)}{(f-1)}$

also $T = \frac{2\pi}{\omega}$ $\therefore$ $\boxed{\rho = \frac{3\pi(f-4)}{4T^2G(f-1)}}$

9. $V(x) = \frac{1}{2}kx^2 - V_0\cos\left(\frac{x}{a}\right)$

$E = -\frac{dV}{dx} = -\left(kx + V_0\sin\left(\frac{x}{a}\right) \times \frac{1}{a}\right)$

since $x << a$

$\therefore \sin\left(\frac{x}{a}\right) \simeq \frac{x}{a}$

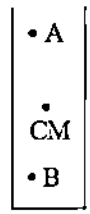
or $E = -\left(k + \frac{V_0}{a^2}\right)x$

This resembles $F = -kx$

$\therefore T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{ma^2}{ka^2 + v_0}}$

$\boxed{T = 2\pi\sqrt{\frac{m.a^2}{ka^2 + v_0}}}$

10.



$a_B > a_C > a_A$

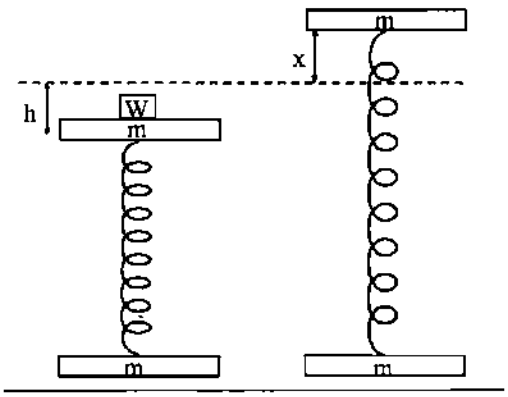
$a_B = g$

$\vec{a}_{A/CM} = \vec{a}_A - \vec{a}_{CM}$ (↑)

$\vec{a}_{B/CM} = \vec{a}_B - \vec{a}_{CM}$ (↓)

11. For lower block +ve lift, $kx \geq mg$

$\Rightarrow x \geq \frac{mg}{k}$



W/E theorem

$-mg(h+x) + \left(\frac{1}{2}kh^2 - \frac{1}{2}kx^2\right) = 0 - 0$

$\Rightarrow -mgh - \frac{m^2g^2}{k} + \frac{1}{2}kh^2 - \frac{1}{2}\frac{m^2g^2}{k} = 0$

$\frac{kh^2}{2} - mgh - \frac{3m^2g^2}{2k} = 0$

$h = \frac{mg \pm \sqrt{m^2g^2 + 3m^2g^2}}{k}$

$= \frac{mg \pm 2mg}{k} = \frac{3mg}{k}, \frac{-mg}{k}$

$\therefore h = \frac{3mg}{k}$

$W + mg = kh$

$W + mg = 3mg$

$W = 2mg$

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12. In SHM

$a = -\omega^2 x$

$v = \omega\sqrt{A^2 - x^2}$

$v^2 = \omega^2 (A^2 - x^2)$

$v^2 = \omega^2 A^2 - \omega^2 \times \frac{a^2}{\omega^2}$

$v^2 + \frac{a^2}{\omega^2} = \omega^2 A^2$

$\frac{v^2}{\omega^2 A^2} + \frac{a^2}{\omega^4 A^2} = 1$

i.e. ellipse

13. $R = \frac{V}{i} = \frac{V \times i}{i^2} = \frac{P}{i^2}$

energy $= h\nu = \frac{h}{t}$

Power $= \frac{\text{energy}}{t}$

$P = \frac{h}{t^2}$

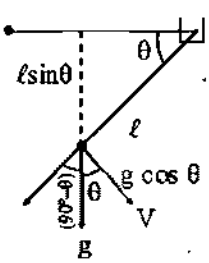
$i = \frac{e}{t}$

$\frac{P}{i^2} = \frac{h}{e^2}$

14. Precession mean

every time reading is coming nearly same.

15.



Radical acceleration

$= \frac{V^2}{R} = \frac{2g\ell\sin\theta}{\ell} = 2g\sin\theta$

tangential acceleration $= g\cos\theta$

total acceleration $= \sqrt{4g^2\sin^2\theta + g^2\cos^2\theta}$

$\Rightarrow g\sqrt{1 + 3\sin^2\theta}$

16. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$u_x = u$ at $(0, b)$

$u_y = 0$

$\frac{2x}{a^2}\frac{dx}{dt} + \frac{2y}{b^2}\frac{dy}{dt} = 0$

Again diff. w.r.t. to time

$\frac{2x}{a^2}\frac{d^2x}{dt^2} + \frac{2}{a^2}\left(\frac{dx}{dt}\right)^2 + \frac{2y}{b^2}\frac{d^2y}{dt^2} + \frac{2}{b^2}\left(\frac{dy}{dt}\right) = 0$

acceleration at (0, b) is

$a_y = \frac{-b}{a^2}u^2$

17. $\frac{v}{\sqrt{\sigma/\rho R^3}}$ is dimensionless

$k\rho g R^2/\sigma$ is also dimensionless

18. By using parallel axis theorem, $I = \frac{11}{2}mr^2$

19. Using conservation of angular momentum

$V_p r_p = V_A r_A$

$\frac{V_P}{V_A} = \frac{r_A}{r_P} = \frac{a + ae}{a - ae}$

21. $F = -\frac{\partial U}{\partial x} = -\alpha x^3 - \beta x^4$

$\left(\frac{\partial U}{\partial x}\right)_{x=0} = 0$ & $\left(\frac{\partial^2 U}{\partial x^2}\right)_{x=0} = 0$

22. $[\alpha] = MLT^{-2}$

$[\beta] = ML^2T^{-2}$

$[\gamma] = L$

23. Let velocity of each blocks are v_1 and v_2 respectively then

$mv_1 + 2mv_2 = mv$

where $v = \sqrt{2gh}$ & $v_2 - v_1 = v$

hence from above equations,

we have $v_1 = -\frac{v}{3}$

$h \propto v^2$ or $h' = \frac{h}{9}$

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24. $mg' - kv' = \frac{mdv}{dt}$

25. Rocket will remain bound to the solar system till its mechanical energy becomes zero.

i.e. $\frac{GMm}{r_0} - \frac{1}{2}mv^2 = 0$

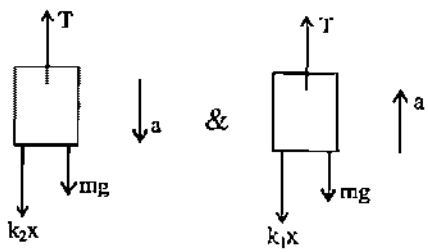
$\frac{1}{2}mv^2 = \frac{GMm}{r_0} \Rightarrow v^2 = \frac{2GM}{r_0}$

$\alpha^2 v_0^2 = \frac{2GM}{r_0}$

$\alpha^2 \times \frac{GM}{r_0} = \frac{2GM}{r_0}$

$\alpha = \sqrt{2}$

26.



$T + k_1x - mg = ma$ ....(i)

$k_2x + mg - T = ma$ ....(ii)

From equation (i) & (ii)

$T = \left(\frac{k_2 - k_1}{2}\right)x + mg$ and

$a = \left(\frac{k_1 + k_2}{2m}\right)x$ if T is non-zero.

If T is zero, then

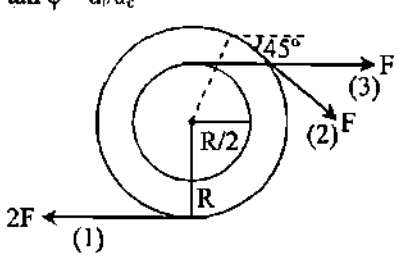
$a_1 = \frac{k_1x - mg}{m} = \frac{k_1x}{m} - g$

$a_2 = \frac{k_2x + mg}{m} = \frac{k_2x}{m} + g$

27. By energy conservation

$\tan\phi = a_t/a_c$

28.



$\tau_1 + \tau_2 + \tau_3 = \tau_{net}$

$2FR + FR + \frac{R}{2}F = \tau_{net}$

$3.5FR = \tau_{net}$

29. Based on dimensional analysis.

$\rho = [ML^{-3}]$

$g = LT^{-2}$

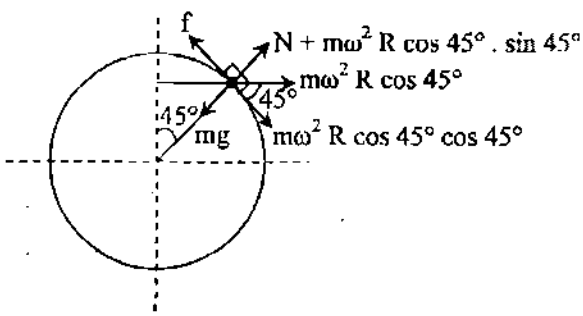
$\sigma = [MT^{-2}]$

$L = L$

$\frac{\rho g L^2}{\sigma} = \frac{ML^{-3} \times LT^{-2} \times L^2}{MT^{-2}}$

$= (M^0L^0T^0)$

30.



∵ Plank is in rest

$f = m\omega^2 R \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$f = \frac{m\omega^2 R}{2}$

31. $F = -\frac{dU}{dx}$

$(KE)_{min} = 1J$

$\frac{1}{2}mv^2 = 1 \Rightarrow \frac{1}{2} \times 0.5v^2 = 1$

$v^2 = 4 \Rightarrow v = 2m/s$

with such speed it able to cross $x = -2$ and then it will go to ∞.

32. By principal of conservation of momentum

$mv_0 = (M + m)V$ ... (1)

by principal of conservation of angular momentum

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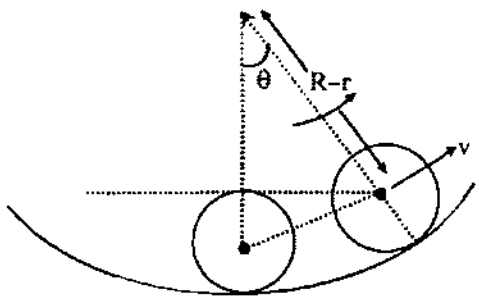
$$\omega\left(\frac{2}{5}MR^2 + mR^2\right) = mv_0(h - R) \ldots (2)$$

$v_0 = R\omega$

by (1) and (2)

$$\frac{h}{R} = \frac{7M + 10m}{5(M + m)}$$

34.



Total energy remain constant

$$TE = \frac{1}{2}Mv^2 + \frac{1}{2} \times \frac{2}{5} Mr^2 \times \omega^2 + Mg[R - r]$$

$$[1 - \cos\theta] = \frac{dTE}{dt}$$

$$M \times v\frac{dv}{dt} + \frac{Mr^2}{5} \times \frac{1}{r^2} \times 2v\frac{dv}{dt} + Mg[R - r]$$

$$(+\sin\theta)\frac{d\theta}{dt} = 0$$

$$\Rightarrow v\frac{dv}{dt} + \frac{2}{5}v\frac{dv}{dt} + g[R - r]\sin\theta \times \frac{v}{R - r} = 0$$

$$\frac{7}{5}a + g\sin\theta = 0$$

$$a = -\frac{5}{7}g\theta$$

$$\alpha(R - r) = -\frac{5}{7}g\theta$$

$$\therefore \alpha = -\frac{5g\theta}{7(R - r)}$$

$$\omega = \sqrt{\frac{5g}{7(R - r)}}$$

$$T = 2\pi\sqrt{\frac{7(R - r)}{5g}}$$

As r increase T decrease slightly.

35. Using energy conservation

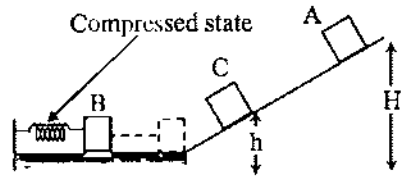
$$\frac{1}{2} \times Mv^2 + \frac{1}{2} \times \frac{2}{5}MR^2 \times \left(\frac{v}{R}\right)^2 = Mgh$$

$$\frac{v^2}{2} + \frac{v^2}{5} = gh$$

$$\frac{7v^2}{10} = gh$$

$$\Rightarrow v = \sqrt{\frac{10gh}{7}}$$

36. Using work energy theorem conservation at A & B



$$mgH - \mu mg(d + x) - \frac{1}{2}kx^2 = \Delta KE = 0 \quad \ldots (1)$$

using work energy theorem at B & C

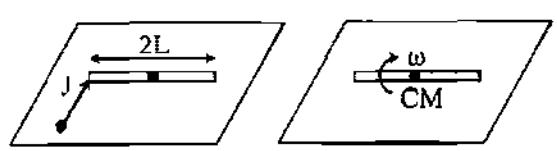
$$\frac{1}{2}kx^2 - \mu mg(d + x) - mgh = 0 \quad \ldots (2)$$

from (1) & (2)

$mgH - 2\mu mg(d + x) = mgh$

$h = H - 2\mu(d + x)$

37.



$J = mv$ .... (1)

where v is the velocity of centre of mass.

After impulse rod get angular velocity ω

Angular impulse = Iω

$$J \times L = \frac{m(2L)^2}{12} \times \omega \quad \ldots(2)$$

$$J = \frac{mL\omega}{3}$$

$$\boxed{\omega = \frac{3J}{mL}}$$

from equation (1); $v = \frac{J}{m}$

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Kinetic energy = KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow \frac{1}{2}m\frac{J^2}{m^2} + \frac{1}{2}\times\frac{m\times 4L^2}{12}\times\frac{9J^2}{m^2L^2}$

$\Rightarrow \frac{J^2}{2m} + \frac{36J^2}{24m}$

$\Rightarrow \frac{48J^2}{24m} \Rightarrow \frac{2J^2}{m}$

38. If perfect rolling (solid cylinder P)
According to energy conservation law

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}I\left(\frac{v_P}{R}\right)^2$$

Here,
$I \rightarrow$ moment of inertia, $R \rightarrow$ Radius

$$I = \frac{mR^2}{2}$$

$$\omega = \frac{v_P}{R}$$

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}\frac{mR^2}{2}\frac{v_P^2}{R^2}$$

$$mgh = \frac{1}{2}mv_P^2\left[1+\frac{1}{2}\right] = \frac{1}{2}mv_P^2\times\frac{3}{2}$$

$$mgh = \frac{3}{4}mv_P^2$$

$$v_P^2 = \frac{4}{3}gh \qquad \ldots.(1)$$

If sliding without friction
(solid cylinder Q)
According to energy conservation law

$$mgh = \frac{1}{2}mv_Q^2$$

$$\Rightarrow \quad v_Q^2 = 2gh \qquad \ldots.(2)$$

from equation (1) and (2)

$$\frac{v_Q^2}{v_P^2} = \frac{2gh}{\left(\frac{4}{3}gh\right)} = \frac{3}{2} \Rightarrow \frac{v_Q}{v_P} = \sqrt{\frac{3}{2}}$$

39. $F = -\frac{dU}{dr} = \frac{-d}{dr}[qV]$ $\quad q \rightarrow$ constant

$$F = -q\left[\frac{dV}{dr}\right]$$

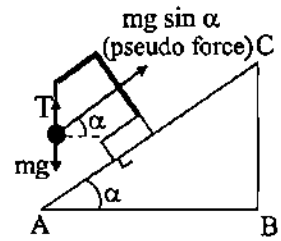
$$F = -qk \leftarrow \left(\begin{matrix} v = kr \\ \frac{dV}{dr} = k \end{matrix}\right)$$

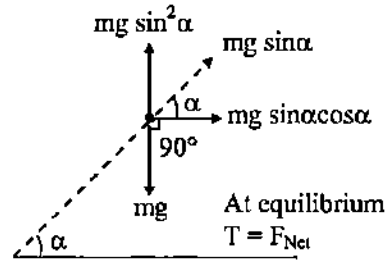
$$m\omega^2R = -qk \Rightarrow m\left(\frac{2\pi}{T}\right)^2 R = -qk$$

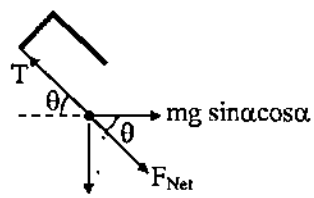
$$\frac{m(4\pi^2)R}{T^2} = -k$$

$$\Rightarrow T^2 \propto R \Rightarrow T \propto R^{1/2}$$

40. Block slides downward along the inclined plane with acceleration g sin α.







$mg[1 - \sin^2\alpha] \Rightarrow mg\cos^2\alpha$

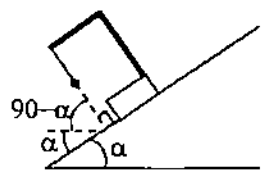
$$\tan\theta = \frac{mg\cos^2\alpha}{mg\sin\alpha\cos\alpha}$$

$\tan\theta = \cot\alpha$

$\tan\theta = \tan(90-\alpha)$

$\theta = (90-\alpha)$

string is perpendicular to inclined plane.



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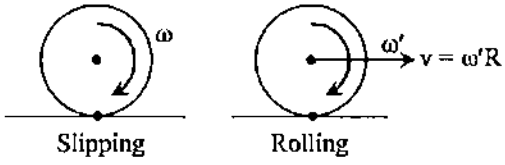
41. According to Bernoulli theorem
In the region of narrow cross section of pipe, KE of fluid will be greater and pressure energy will be lesser.
$\Rightarrow$ less pressure results into larger in size of air bubble and greater KE results its greater speed.

42. Initial sphere is slipping and finally it start rolling. During its motion τ about point of contact is zero.
$\therefore$ Angular momentum of sphere about point of contact remain conserved.

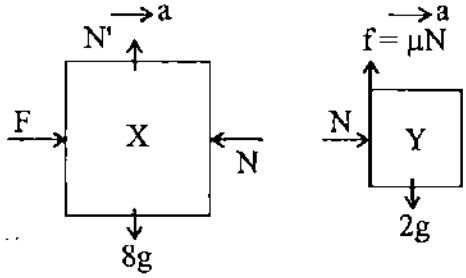


$I\omega = (I + MR^2)\,\omega'$

$$\frac{2}{5}MR^2\omega = \left(\frac{2}{5}MR^2 + MR^2\right)\omega'$$

$$\omega' = \frac{2\omega}{7}$$

43. According to free body diagram



$F - N = 8a$ ....(1)
$N = 2a$ ....(2)
$f = 2g = 20$ ....(3)
$\Rightarrow \mu N = 20$

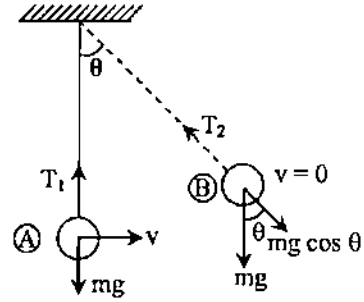
$$\Rightarrow N = \frac{20}{\mu} = \frac{20}{0.5} = 40 \quad \text{....(4)}$$

$\Rightarrow N = 2a$

$$\Rightarrow a = \frac{N}{2} = \frac{40}{2} = 20\ m/s^2 \quad \text{....(5)}$$

$F = N + 8a = 10\,a$ [from equation (1)]
$F = 10 \times 20 = 200$ newton

44.



Centripetal force at point A :

$$T_1 - mg = \frac{mv^2}{\ell} \quad \text{....(1)}$$

At point B :

$$T_2 = mg\cos\theta \quad \text{....(2)}$$

According to question

$$T_1 = 4T_2 \quad \text{....(3)}$$

$$\Rightarrow \quad mg + \frac{mv^2}{\ell} = 4\,mg\cos\theta$$

[from equation (1) & (2)]

$$\Rightarrow \quad mg(4\cos\theta - 1) = \frac{mv^2}{\ell} \quad \text{....(4)}$$

According to conservation of energy between point A and B

$$\text{Also } \frac{1}{2}mv^2 + 0 = 0 + mg\ell(1 - \cos\theta)$$

$$mv^2 = 2\,mg\ell(1 - \cos\theta)$$

$$\frac{mv^2}{\ell} = 2\,mg(1 - \cos\theta) \quad \text{....(5)}$$

From equation (4) & (5)

$$mg(4\cos\theta - 1) = 2\,mg(1 - \cos\theta)$$

$$\Rightarrow \quad 4\cos\theta - 1 = 2 - 2\cos\theta$$

$$\Rightarrow \quad 6\cos\theta = 3$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

45. $\sigma = h^\alpha k_B^\beta c^\gamma$
$\sigma \rightarrow$ Steffan boltzmann constant
$h \rightarrow$ Planck's constant
$k_B \rightarrow$ Boltzmann constant
$c \rightarrow$ speed of sight

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According to stefan's law

$$\frac{Q}{At} = \sigma T^4$$

$$\sigma = \frac{Q}{At} \times \frac{1}{T^4}$$

$$[\sigma] = \frac{[M^1L^2T^{-2}]}{[L^2][T][K^4]} = [M^1T^{-3}K^{-4}]$$

& $E = h\nu \Rightarrow h = \frac{E}{\nu} \Rightarrow [h] = \frac{[M^1L^2T^{-2}]}{[T^{-1}]}$

$[h] = [M^1L^2T^{-1}]$

$[c] = [LT^{-1}]$

$$E = \frac{3}{2}k_BT \Rightarrow [k_B] = \frac{[M^1L^2T^{-2}]}{[K]}$$

$[k_B] = [M^1L^2T^{-2}K^{-1}]$

According to homogeneity principle of dimension

$[M^1T^{-3}K^{-4}] = [M^1L^2T^{-1}]^\alpha [M^1L^2T^{-2}K^{-1}]^\beta [L^1T^{-1}]^\gamma$

on comparing powers00000000 of M, L, T, K on both sides

$\Rightarrow$ $\alpha + \beta = 1$ ....(1)

$2\alpha + 2\beta + \gamma = 0$ ....(2)

$-\alpha - 2\beta - \gamma = -3$ ....(3)

$-\beta = -4$ ....(4)

$\Rightarrow$ $\beta = 4$

$\alpha = -3$

Putting values is equation (2)

$2(-3) + 2(4) + \gamma = 0$

$\gamma = -2$

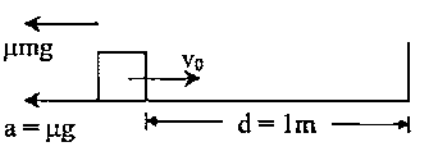
Ans :

$\alpha = -3$

$\beta = 4$

$\gamma = -2$

46.



Velocity after time t

$v = v_0 - \mu gt$

$v \geq 0 \Rightarrow v_0 \geq \mu gt$ ....(i)

Displacement in time 't'

$$\Delta X = v_0t - \frac{1}{2}\mu gt^2$$

$$1 = 2v_0 - \frac{1}{2}\mu g(2)^2$$

$2v_0 = (1 + 2\mu g)$

$$v_0 = \left(\frac{1}{2} + \mu g\right) \quad \text{....(ii)}$$

$\frac{1}{2} + \mu g > \mu gt$ (t = 2 sec)

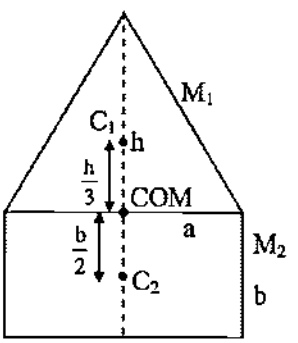
$\frac{1}{2} + \mu g > 2\mu g$

$\mu g < \frac{1}{2}$

$\Rightarrow \mu < \frac{1}{2g}$

$\Rightarrow \mu < 0.05$

47.



Equal are

$\frac{1}{2}ah = ab$

$h = 2b$

$M_1\frac{h}{3} = M_2\frac{b}{2}$ [centre of mass of combination at the mid-point of their common edge]

$$\frac{M_1}{M_2} = \frac{3}{2}\frac{b}{h}$$

$$\frac{M_1}{M_2} = \frac{3}{2}\left[\frac{1}{2}\right]$$

$$\frac{M_1}{M_2} = \frac{3}{4}$$

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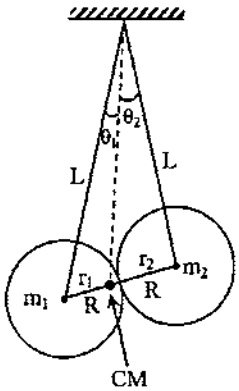
48.



Using concept of COM

$m_1r_1 = m_2r_2$

$r_1 + r_2 = 2R$

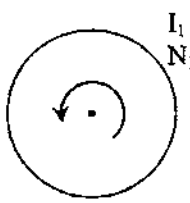
$\left(\frac{m_2}{m_1}+1\right) r_2 = 2R$

$r_2 = \frac{2m_1R}{m_1+m_2}$

$L \sin\theta_2 = r_2$ [R << L]

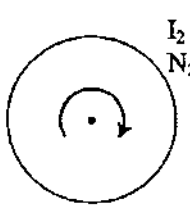
$\theta_2 = \frac{2m_1R}{(m_1+m_2)L}$

49.



$I_1 = 4.25$ kg-m^2

$N_1 = 15$rps



$I_2 = 1.8$ kg-m^2

$N_2 = 25$rps

Their axis of rotation is common.

Angular momentum conservation

$I_1\omega_1 - \omega_2I_2 = (I_1 + I_2)\,\omega$

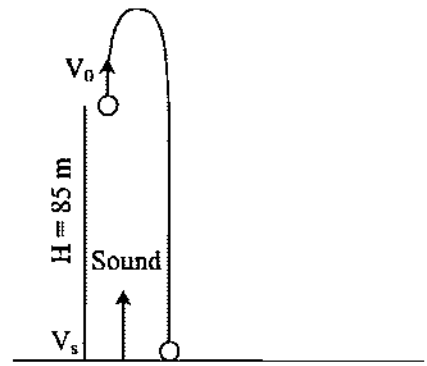
$2\pi\,(4.25)\,N_1 - 2\pi\,(1.8)\,N_2 = (4.25 + 1.80)\,N\,(2\pi)$

$(4.25 \times 15 - 1.8 \times 25) = (6.05)\,N$

$63.75 - 45 = 6.05\,N$

$N = 3$ rev/s.

50.



Time taken to reach sound after hit $t_s = \frac{H}{V_s}$

$t_s = \left(\frac{85}{340}\right)$ sec ; $t_s = 0.25$ sec

For ball time of flight T_f

$T_f + t_s = 5.25$ sec

$T_f = 5.25 - 0.25$

$T_f = 5$ sec

For ball $V_0t - \frac{1}{2}gt^2 = -H$

$\Rightarrow V_0(5) - \frac{1}{2}10(5)^2 = -85$

$5V_0 = 40$

$V_0 = 8$ m/s

51. $T = 2\pi\sqrt{\frac{\ell}{g}}\left(1+\frac{\theta^2}{16}\right)$ (This is valid when θ is not small)

$T_0 = 2\pi\sqrt{\frac{\ell}{g}}$ (for small θ)

$T = T_0\left(1+\frac{\theta^2}{16}\right)$

$T > T_0$

52. $A = G^\alpha M^\beta C^\gamma$

$[M^0L^2T^0] = [M^{-1}L^3T^{-2}]^\alpha\,[M]^\beta[LT^{-1}]^\gamma$

$-\alpha + \beta = 0 \Rightarrow \alpha = \beta$

$3\alpha + \gamma = 2$

$-2\alpha - \gamma = 0$

on solving

$\alpha = 2, \beta = 2, \gamma = -4$

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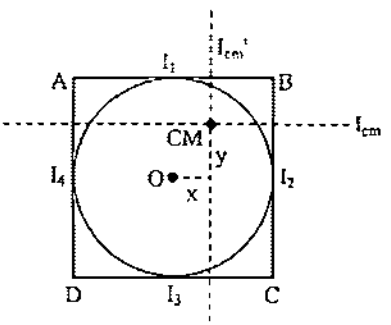
53. Initially wire is slack so it do not have any deformation energy. When block is given some velocity it move due to kinetic energy, one wire get taut. Internal force get develop in wire and KE start decreases and deformation energy of wire increase. Till block come at rest using energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times (\text{strain})^2 \times A \times L$$

$$\frac{1}{2}mv^2 = \frac{1}{2}Y \times \left(\frac{x}{L}\right)^2 \times A \times L$$

$$x = v\sqrt{\frac{mL}{AY}}$$

54.



$I_{cm} + m(R + y)^2 = I_3$ ...(1)

$I_{cm} + m(R - y)^2 = I_1$ ...(2)

from (1) & (2)

$I_1 - I_3 = m[(R - y)^2 - (R + y)^2]$

$I_1 - I_3 = m(2R)(-2y)$ ...(3)

$I_{cm}' + m(R + x)^2 = I_4$ ...(4)

$I_{cm}' + m(R - x)^2 = I_2$ ...(5)

from (4) & (5)

$(I_2 - I_4) = m[(R - x)^2 - (R + x)^2]$

$I_2 - I_4 = m[(2R)(-2x)]$ ...(6)

$(3)^2 + (6)^2$

$\Rightarrow (I_1 - I_3)^2 + (I_2 - I_4)^2 = (m^2 \times 4R^2 \times 4(x^2 + y^2)$

distance of CM from

$$O = \sqrt{x^2 + y^2} = \frac{1}{4mR\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}}$$

55. Using energy conservation

$$-\frac{GMM}{R} + \frac{1}{2}mV^2 = -\frac{GMM}{5R}$$

$$V = 2\sqrt{\frac{2GM}{5R}}$$

V is the velocity by which object is projected. When object return to earth its speed will be V.

56. $$\frac{1}{2}mV^2 + \frac{1}{2}mR^2\left(\frac{V}{R}\right)^2 = mgh$$

{Using conservation of energy)

$$m\left(\frac{V^2}{2} + \frac{V^2}{2}\right) = mgh$$

$$h = \frac{V^2}{g}$$

57. at $t = 0$, $x = 0.5$

$$u = \frac{x^4}{4} - \frac{x^2}{2}$$

$$\Rightarrow \frac{1}{4} \times \frac{1}{16} - \frac{1}{4} \times \frac{1}{2} \Rightarrow \left|\frac{1}{4}\right|$$

$$\frac{du}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x$$

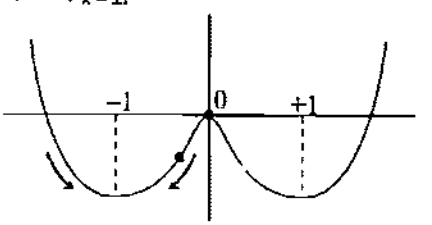
$$\frac{du}{dx} = x(x^2 - 1)$$

$\frac{du}{dx} = 0$ at point of maxima & minima

$x = 0$; $x = \pm 1$

$\left(\frac{d^2u}{dx^2}\right)_{x=0}$ = – 1 point of maxima

$\left(\frac{d^2u}{dx^2}\right)_{x=\pm 1}$ = 2 point of minima



particle will found between (–1, 0)

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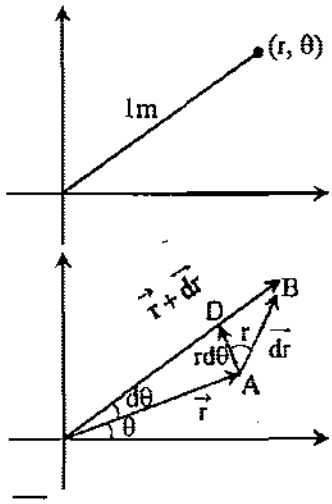
58.



$\overline{AB}$ = Direction of resultant velocity

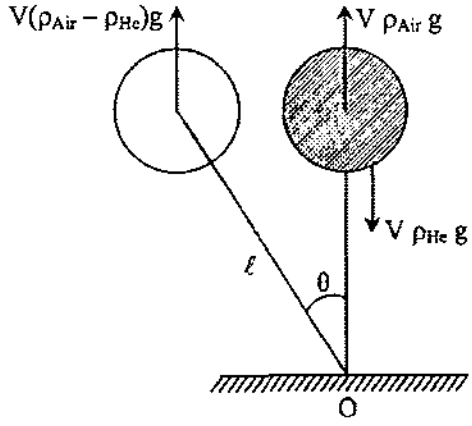
$\overline{AD}$ = Direction of tangential velocity

$$\forall \tan\alpha = \frac{dr}{rd\theta} = \frac{r}{r}$$

$\tan\alpha = 1$

$\alpha = 45°$

59.



$\tau_\theta = V(\rho_{Air} - \rho_{He})\, g\, \ell \sin\theta$

For small angular displacement (θ)

$\tau_\theta = V(\rho_{Air} - \rho_{He})\, g\, \ell\, \theta$

$I\alpha = V[\rho_{Air} - \rho_{He}]\, g\, \ell\, \theta$

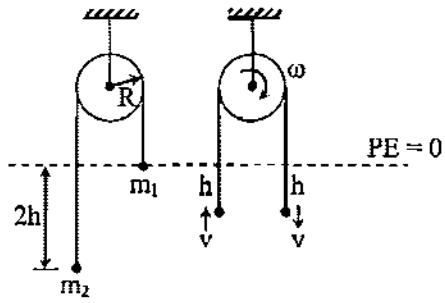
$\rho_{He} V \ell^2 \alpha = V[\rho_{Air} - \rho_{He}]\, \ell\, \theta\, g$

$$\alpha = \left[\frac{\rho_{Air} - \rho_{He}}{\rho_{He}}\right]\frac{g}{\ell}\theta$$

$$\omega = \sqrt{\left(\frac{\rho_{Air} - \rho_{He}}{\rho_{He}}\right)\frac{g}{\ell}}$$

$$= 2\pi\sqrt{\frac{\ell}{g}\frac{\rho_{He}}{(\rho_{Air} - \rho_{He})}}$$

60.



The total mechanical energy of system = conserved

Hence, $KE_i + PE_i = KE_f + PE_f$

$$0 - m_2 g \times 2h = \frac{1}{2}m_2 v^2 + \frac{1}{2}m_1 v^2 + \frac{1}{2}I\omega^2 - m_1 gh - m_2 gh$$

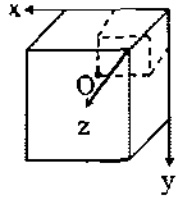
Also $\omega = \frac{v}{R}$

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$$

$$(m_1 - m_2)gh = \frac{v^2}{2}\left[m_1 + m_2 + \frac{I}{R^2}\right]$$

$$v \propto \sqrt{\frac{m_1 - m_2}{m_1 + m_2 + \frac{I}{R^2}}}$$

61. Centre of mass of remaining cube x coordinate = b



$$X_{CM} = \frac{\rho a^3 \times \frac{a}{2} - \rho b^3 \times \frac{b}{2}}{\rho a^3 - \rho b^3}$$

We will consider removed mass as a negative mass

$$b = \frac{\frac{\rho a^4}{2} - \frac{\rho b^4}{2}}{\rho a^3 - \rho b^3}$$

$$a^3 b - b^4 = \frac{a^4}{2} - \frac{b^4}{2}$$

$2a^3 b - 2b^4 = a^4 - b^4$

put $a = bx \Rightarrow 2b^4 x^3 b - 2b^4 = b^4 x^4 - b^4$

$2x^3 - 1 = = x^4$

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$2x^3 - 2 + 1 = x^4$

$2[x^3 - 1] = (x^2 - 1)(x^2 + 1)$

$2[x - 1][x^2 + 1 + x] = [x - 1][x + 1][x^2 + 1]$

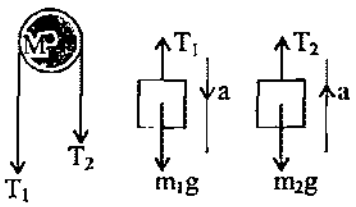
$2x^2 + 2 + 2x - x^3 + x + x^2 + 1$

$x^3 - x^2 - x - 1 = 0$

62. due to soap bubble surface tension is reduced therefore in that area. Black paper powder will sink.

63. Time period will increase as the amplitude is increases.

64.



$mg - T_1 = m_1 a$ ...(1)

$T_2 - m_2 g = m_2 a$ ...(2)

$(T_1 - T_2) \times R = I\alpha$ $[\because a = r\alpha]$

$= \frac{Ia}{R}$

(1) + (2)

$(m_1 - m_2)g + (T_2 - T_1) = (m_1 + m_2)a$

$(T_1 - T_2) = \frac{I}{R^2}\left[\frac{(m_1 - m_2)g + (T_2 - T_1)}{m_1 + m_2}\right]$

$\Rightarrow (T_1 - T_2)\left[1 + \frac{I}{k^2(m_1 + m_2)}\right]$

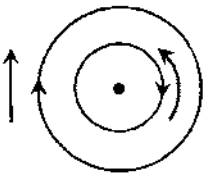
$= \frac{I}{k^2}\frac{(m_1 - m_2)}{m_1 + m_2}g$

$\Rightarrow T_1 - T_2 = \left[\frac{I(m_1 - m_2)g}{I + (m_1 + m_2)R^2}\right]$

$= \frac{\frac{1}{2}MR^2(m_1 - m_2)g}{\left(\frac{1}{2}MR^2 + (m_1 + m_2)R^2\right)}$

$T_1 - T_2 = \frac{M(m_1 - m_2)g}{M + 2(m_1 + m_2)}$

65. $T^2 \propto r^3$



$\frac{9}{24^2} = \left(\frac{3\times10^4}{r^3}\right)^3$

$\frac{3\times3}{24\times24} = \left(\frac{3\times10^4}{r}\right)^3$

$\frac{1}{4} = \frac{3\times10^4}{r}$

$r = 12 \times 10^4$

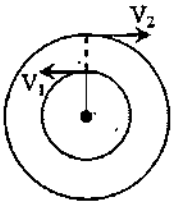
orbital speed of S_2 seen from planet = $\omega_2 r$

$= \frac{2\pi}{24} \times 12 \times 10^4$

$= \pi \times 10^4$ km h^{-1}

$V_1 = \omega_1 r_1 = \frac{2\pi}{3} \times 3 \times 10^4$

$\Rightarrow 2\pi \times 10^4$ km h^{-1}



$\left(\frac{2\pi}{3} + \frac{2\pi}{24}\right)t = \pi$

$\frac{9t}{24} = \frac{1}{2}$

$t = \frac{12}{9}$ hr

Angle rotate by both satellite

$\theta_1 = \frac{2\pi}{3} \times \frac{12}{9} \Rightarrow \frac{8\pi}{9}$

$\theta_2 = \frac{2\pi}{24} \times \frac{12}{9} \Rightarrow \frac{\pi}{9}$

velocity of S_2 seen from $S_1 = V_1 + V_2$

$= 3\pi \times 10^4$ km h^{-1}

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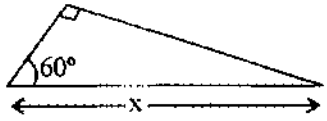
66. $(\hat{n}\times\vec{F})\times\hat{n} = -[\hat{n}\times(\hat{n}\times\vec{F})]$

$= -[\hat{n}(\hat{n}.\vec{F}) - \vec{F}(\hat{n}.\hat{n})]$

$= \vec{F} - \hat{n}(\hat{n}.\vec{F})]$

$= \vec{G}$

67.



$\tan 60° = \frac{L}{R} = \frac{1}{1/\sqrt{3}} = \sqrt{3} = \tan\theta$

$\therefore \quad x = \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{R^2 + x^2 - L^2}{2Rx}$

$\Rightarrow R^2 + x^2 - L^2 = 2Rx\cos\theta$

$\Rightarrow 2x\frac{dx}{dt} = 2R\left[x(-\sin\theta) + \cos\theta\frac{dx}{dt}\right]$

$\frac{dx}{dt}[x - R\cos\theta] = -Rx\sin\theta\frac{d\theta}{dt}$

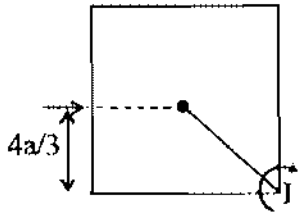
$\therefore \quad -\frac{dx}{dt} = v \;\&\; \frac{d\theta}{dt} = \omega$

$\Rightarrow v = \frac{Rx\sin\theta\omega}{x - R\cos\theta}$

$= \frac{\frac{1}{\sqrt{3}}\frac{2}{\sqrt{3}}\frac{\sqrt{3}}{2}\cdot\omega}{\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\cdot\frac{1}{2}} = \frac{\frac{\omega}{\sqrt{3}}}{\frac{1}{\sqrt{3}}\frac{3}{2}} = \frac{2\omega}{3}$

$v = \frac{2\omega}{3}$

68.



$I = I_{cm} + M(a\sqrt{2})^2$

$= \frac{8}{3}Ma^2$

$\therefore \quad mvr = I\omega$

$\Rightarrow \omega = \frac{mv\left(\frac{4a}{3}\right)}{\frac{8}{3}Ma^2}$

$= \frac{mv}{2Ma}$

69. $x = 0.3\text{ cm} - 3 \times$ scale division of vernier calipers

$= 0.3\text{ cm} - 3 \times \frac{9}{100}$

$= \frac{30 - 27}{100}$

$= \frac{3}{100}$

$= 0.03\text{ cm}$

70. $T^2 = \frac{4\pi^2}{GM}a^2$

$\frac{4\pi^2}{GM} = 1$

T = in year

a = radius in A.U.

$\therefore T = 3\text{ days} = \frac{3}{365}\text{ year}$

$\therefore a = \left(\frac{3}{365}\right)^{2/3}$

$a = 0.04$ A.U.

71. This question is related to significant numbers as in the question it is asked how student REPORT the result.

On analysis the values of u, t and g the reported result must have three significant number.

Hence correct answer is [B]

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72. $P = ML^2T^{-3}$, $c = LT^{-1}$

$\omega = T^{-1}$, $R = L$, $m = M$

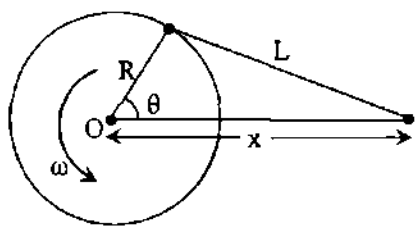
$G = M^{-1}L^3T^{-2}$

$[ML^2T^{-3}] = [M^{-1}L^3T^{-2}][LT^{-1}]^{-5} M^x L^y T^{-z}$

solve we get

$x = 2, y = 4, z = 6$

73.



$$\cos\theta = \frac{R^2 + x^2 - L^2}{2Rx}$$

$\Rightarrow x^2 = 2Rx \operatorname{cosec}\theta + L^2 - R^2$

displacement of S.H. M. is in the form of

$x = A \sin \omega t + c$

Therefore it is not S. H. M.

It is S. H. M. only which.

It $L = R$.

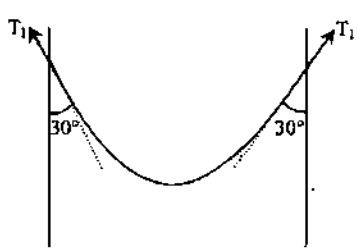
$\Rightarrow x^2 = 2Rx \cos\theta$

$\Rightarrow x = 2R \cos\theta$

$x = 2R \cos \omega t$ [S.H. M.]

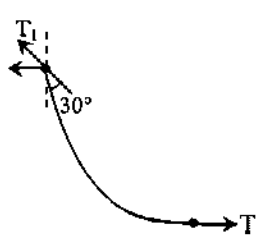
But period of this motion is T

74.



$\therefore 2T_1 \cos 30° = mg = 5 \times 10 = 50.$

$2T_1 \cos 30° = 50$

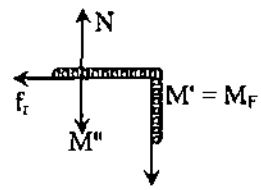


$T = T_1 \sin 30°$

$$= \frac{50}{\frac{2\sqrt{3}}{2}} \cdot \frac{1}{2} = \frac{25}{\sqrt{3}} = \frac{25\sqrt{3}}{3}$$

$T = 14.41$ N

75.



$f_r = \mu N = \mu M'' g = \mu (M - M_f)g$

at equilibrium.

$f_r = M'g$

$\mu M(1 - f)g = M_F g$

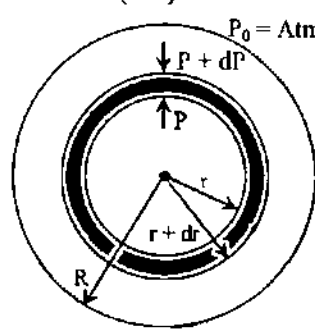
$\mu(1 - f) = f$

$\mu = (\mu + 1) f$

$$f = \frac{1}{\left(1 + \frac{1}{\mu}\right)}$$

76. Consider a spherical shell of radius r and radial thickness dr. P & P + dP are pressure at its inner and outer surface.

Let g_r = gravitational acceleration at distance r (< R)



For equilibrium of this shell -

$(P + dP)(4\pi r^2) + (4\pi r^2 dr)\rho g_r = (P)(4\pi r^2)$

$$\{\rho = \frac{3M}{4\pi R^3} = \text{Density of sphere}\}$$

$\Rightarrow dP = -\rho g_r dr$

$\because g_r = \frac{4}{3}\pi G\rho r$

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$\Rightarrow dP = -\frac{4}{3}\pi G\rho^2 r\, dr$

{(–) ve sign indicates that pressure is decreasing with radius}

$\Rightarrow \int_{P}^{P_0} dP = \int_{r}^{R} -\frac{4}{3}\pi G\rho^2 r\, dr$

$\Rightarrow P_0 - P = -\frac{4}{3}\pi G\rho^2\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$

$\Rightarrow P = P_0 + \frac{4}{3}\pi G\rho^2\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$

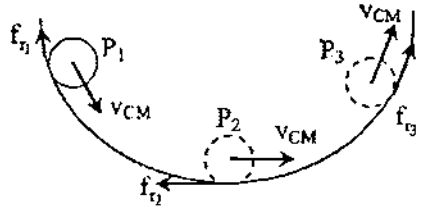
$P = P_0 + \frac{3GM^2}{8\pi R^4}\left(1 - \frac{r^2}{R^2}\right)$

Hence $P \propto \frac{1}{R^4}$

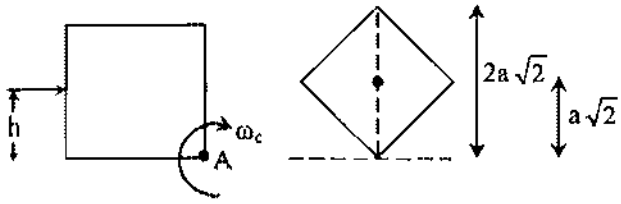
$\Rightarrow$ Average pressure will also be proportional to $\frac{1}{R^4}$

$\Rightarrow$ correct answer is [A]

77. fr_1 will increase ω.
fr_3 will decrease ω.



78.



conservation of energy

$\frac{1}{2} I_A \omega_c^2 = Mg(a\sqrt{2} - a).$

$I_A = I_{cm} + Ma^2 = \frac{2}{3}Ma^2 + M(a\sqrt{2})^2$

$I_A = \frac{8}{9}Ma^2$

$\frac{1}{2}\frac{8}{3}Ma^2\omega_c^2 = Mga(\sqrt{2} - 1)$

$\omega_c = \sqrt{3g(\sqrt{2} - 1)/4a}$

79.

Case-I

Considering angular momentum w.r.t. end A

ω = Angular velocity just after impulse then

$(\omega)\left(\frac{ML^2}{3}\right) = J(L)$

$\omega = \frac{3J}{ML}$

$\Rightarrow$ velocity of CM

$V_{cm_1} = \frac{\omega L}{2} = \frac{3J}{2M}$ ...(1)

Case-II

Apply conservation o moment, if

V_{cm_2} = Velocity of CM just after impulse

then $M V_{cm_2} = J$

$V_{cm_2} = \frac{J}{M}$ ...(2)

comparing (1) & (2)

$V_{cm_2} < V_{cm_1}$

$\Rightarrow$ velocity of CM just after impulse would be between above two extreme values

with (1), $Mgh_{max.} < \frac{1}{2}(M)\left(\frac{3J}{2M}\right)^2$

$\Rightarrow h_{max} < \frac{9J^2}{8M^2g}$ ...(3)

with (2), $Mgh_{max} > \frac{1}{2}M\left(\frac{J}{M}\right)^2$

$h_{max} > \frac{J^2}{2M^2g}$ ...(4)

Use (3) & (4)

Hence $\frac{J^2}{2M^2g} < h_{max} < \frac{9J^2}{8M^2g}$

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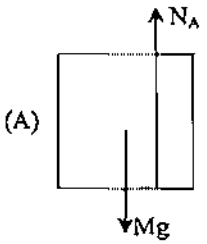
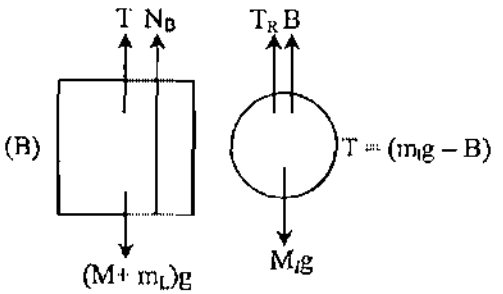
80.

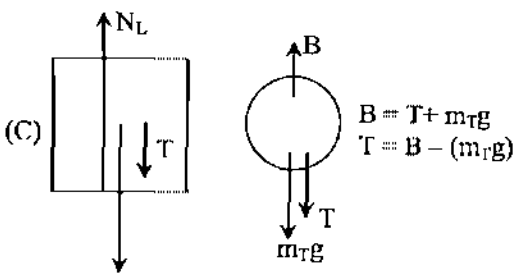


$N_A = Mg$



$N_B = (M + M_L)g - T$

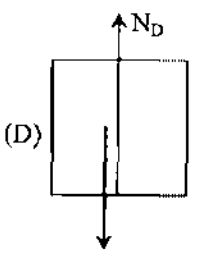
$N_B = Mg + B$



$N_C = T + (M + m_T)\, g$

Therefore $N_L = B - m_T g + (M + m_T)g$

$N_L = B + Mg$



$N_D = (M + m_T)\, g.$

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Chapter 2

Heat & Wave

1. A van der Waal's gas obeys the equation of state $\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$. Its internal energy is given by $U = CT - \frac{n^2a}{V}$. The equation of a quasistatic adiabat for this gas is given by - **[2010]**

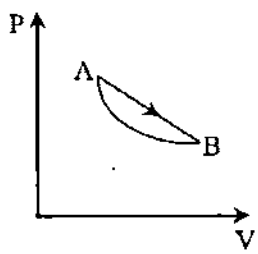
(A) $T^{C/nR}V$ = constant

(B) $T^{(C+nR)/nR}V$ = constant

(C) $T^{C/nR}(V-nb)$ = constant

(D) $P^{(C+nR)/nR}(V-nb)$ = constant

2. An ideal gas is made to undergo a cycle depicted by the PV diagram alongside. The curved line from A to B is an adiabat **[2010]**



Then-

(A) The efficiency of this cycle is given by unity as no heat is released during the cycle

(B) Heat is absorbed in the upper part of the straight line path and released in the lower part

(C) If T_1 and T_2 are the maximum and minimum temperatures reached during the cycle, then the efficiency is given by $1-\frac{T_2}{T_1}$

(D) The cycle can only be carried out in the reverse of the direction shown in figure

3. A bus driving along at 39.6 kmph is approaching a person who is standing at the bus stop, while honking repeatedly at an interval of 30 seconds. If the speed of the sound is 330 m/s, at what interval will the person hear the horn ? **[2010]**

(A) 31 seconds

(B) 29 seconds

(C) 30 seconds

(D) the interval will depend on the distance of the bus from the passenger

4. Velocity of sound measured at a given temperature in oxygen and hydrogen is in the ratio - **[2010]**

(A) 1 : 4 (B) 4 : 1 (C) 1 : 1 (D) 32 : 1

5. A piece of hot copper at 100°C is plunged into a pond at 30°C. The copper cools down to 30°C, while the pond, being huge, stays at its initial temperature. Then- **[2010]**

(A) copper loses some entropy, the pond stays at the same entropy

(B) copper loses some entropy, and the pond gains exactly the same amount of entropy

(C) copper loses entropy, and the pond gains more than this amount of entropy

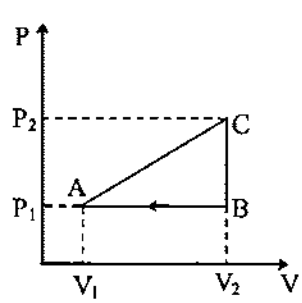
(D) both copper and the pond gain in entropy

6. An ideal gas with heat capacity at constant volume C_V undergoes a quasistatic process described by PV^α in a P-V diagram, where α is a constant. The heat capacity of the gas during this process is given by- **[2010]**

(A) C_V (B) $C_V + nR$

(C) $C_V + \frac{nR}{1-\alpha}$ (D) $C_V + \frac{nR}{1-\alpha^2}$

7. An ideal gas with constant heat capacity $C_V = \frac{3}{2}nR$ is made to carry out a cycle that is depicted by a triangle in the figure given below. **[2010]**



The following statement is true about the cycle-

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(A) The efficiency is given $1 - \frac{P_1V_1}{P_2V_2}$

(B) The efficiency is given by $1 - \frac{1}{2}\frac{P_1V_1}{P_2V_2}$

(C) Net heat absorbed in the cycle is $(P_2 - P_1)(V_2 - V_1)$

(D) Heat absorbed in part AC is given by

$$2(P_2V_2 - P_1V_1) + \frac{1}{2}(P_1V_2 - P_2V_1)$$

8. A container with rigid walls is covered with perfectly insulating material. The container is divided into two parts by a partition. One part contains a gas while the other is fully evacuated (vacuum). The partition is suddenly removed. The gas rushes to fill the entire volume and comes to equilibrium after a little time. If the gas is <u>not</u> ideal, **[2011]**

(A) the initial internal energy of the gas equals its final internal energy

(B) the initial temperature of the gas equals its final temperature

(C) the initial pressure of the gas equals its final pressure

(D) the initial entropy of the gas equals its final entropy

9. Two bulbs of identical volumes connected by a small capillary are initially filled with an ideal gas at temperature T. Bulb 2 is heated to maintain a temperature 2T while bulb 1 remains at temperature T. Assume throughout that the heat conduction by the capillary is negligible. Then the ratio of final mass of the gas in bulb 2 to the initial mass of the gas in the same bulb is close to - **[2011]**

(A) 1/2 (B) 2/3 (C) 1/3 (D) 1

10. Two rods, one made of copper and the other steel of the same length and cross sectional area are joined together. (The thermal conductivity of copper is 385 $J.s^{-1}.m^{-1}.K^{-1}$ and steel is 50 $J.s^{-1}.m^{-1}.K^{-1}$.) If the copper end is held at 100°C and the steel end is held at 0°C, what is the junction temperature (assuming no other heat losses) ? **[2011]**

(A) 12°C (B) 50°C (C) 73°C (D) 88°C

11. Jet aircrafts fly at altitudes above 30,000 ft where the air is very cold at – 40°C and the pressure is 0.28 atm. The cabin is maintained at 1 atm pressure by means of a compressor which exchanges air from outside adiabatically. In order to have a comfortable cabin temperature of 25°C, we will require in addition - **[2011]**

(A) a heater to warm the air injected into the cabin

(B) an air-conditioner to cool the air injected into the cabin

(C) neither a heater nor an air-conditioner ; the compressor is sufficient

(D) alternatively heating and cooling in the two halves of the compressor cycle

12. A speaker emits a sound wave of frequency f_0. When it moves towards a stationary observer with speed u, the observer measures a frequency f_1. If the speaker is stationary, and the observer moves towards it with speed u, the measured frequency is f_2. Then - **[2011]**

(A) $f_1 = f_2 < f_0$ (B) $f_1 > f_2$

(C) $f_1 < f_2$ (D) $f_1 = f_2 > f_0$

13. At 23°C, a pipe open at both ends resonates at a frequency of 450 hertz. At what frequency does the same pipe resonate on a hot day when the speed of sound is 4 percent higher than it would be at 23°C ? **[2011]**

(A) 446 Hz (B) 454 Hz

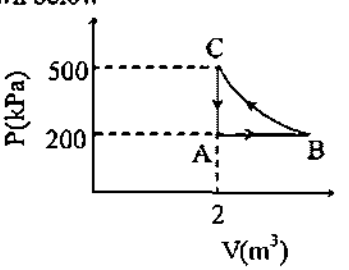
(C) 468 Hz (D) 459 Hz

14. One mole of an ideal gas at initial temperature T, undergoes a quasi-static process during which the volume V is doubled. During the process the internal energy U obeys the equation $U = aV^3$, where a is a constant. The work done during this process is- **[2011]**

(A) 3RT / 2 (B) 5RT / 2

(C) 5RT / 3 (D) 7RT / 3

15. A constant amount of an ideal gas undergoes the cyclic process ABCA in the PV diagram shown below



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The path BC is an isothermal. The work done by the gas during one complete cycle, beginning and ending at A, is nearly- [2011]

(A) 600 kJ (B) 300 kJ
(C) –300 kJ (D) –600 kJ

16. An ideal monatomic gas expands to twice its volume. If the process is isothermal, the magnitude of work done by the gas is W_i. If the process is adiabatic, the magnitude of work done by the gas is W_a. Which of the following is true ? [2012]

(A) $W_i = W_a > 0$ (B) $W_i > W_a = 0$
(C) $W_i > W_a > 0$ (D) $W_i > W_a > 0$

17. The ratio of the speed of sound to the average speed of an air molecule at 300 K and 1 atmospheric pressure is close to – [2012]

(A) 1 (B) $\sqrt{1/300}$
(C) $\sqrt{300}$ (D) 300

18. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is held at a temperature of 100°C while the other one is kept 0° C. If the two are brought into contact, then, assuming no heat loss to the environment, the final temperature that they will reach is – [2012]

(A) 50° C (B) Less than 50° C
(C) More than 50° C (D) 0° C

19. A cylindrical steel rod of length 0.10 m and thermal conductivity 50 $W.m^{-1} K^{-1}$ is welded end to end to copper rod of thermal conductivity 400 $W.m^{-1}. K^{-1}$ and of the same area of cross section but 0.20 m long. The free end of the steel rod is maintained at 100° C and that of the copper and at 0° C. Assuming that the rods are perfectly insulated from the surrounding, the temperature at the junction of the two rods – [2012]

(A) 20° C (B) 30° C (C) 40° C (D) 50° C

20. The total energy of a black body radiation source is collected for five minutes and used to heat water. The temperature of the water increases from 10.0° C to 11.0°C. The absolute temperature of the black body is doubled and its surface area halved and the experiment repeated for the same time. Which of the following statements would be most nearly correct ? [2012]

(A) The temperature of the water would increase from 10.0° C to a final temperature of 12° C
(B) The temperature of the water would increase from 10.0° C to a final temperature of 18° C
(C) The temperature of the water would increase from 10.0° C to a final temperature of 14°C
(D) The temperature of the water would increases from 10.0° C to a final temperature of 11° C

21. A standing wave in a pipe with a length L = 1.2 m is described by

$y(x,t) = y_0 \sin[(2\pi/L)x] \sin[(2\pi/L)x + \pi/4]$

Based on above information, which one of the following statements is incorrect. [2012]

(Speed of sound in air is 300 $m\,s^{-1}$)–

(A) The pipe is closed at both ends
(B) The wavelength of the wave could be 1.2 m
(C) There could be a node at x = 0 and antinode at x = L/2
(D) The frequency of the fundamental mode of vibrations is 137.5 Hz

22 The equation of state of n moles of a non-ideal gas can be approximated by the equations

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where a and b are constants characteristic of the gas. Which of the following can represent the equation of a quasistatic adiabat for this gas (Assume that C_V, the molar heat capacity at constant volume, is independent of temperature) ? [2013]

(A) $T(V - nb)^{R/C_V} = \text{constant}$

(B) $T(V - nb)^{C_V/R} = \text{constant}$

(C) $\left(T + \frac{ab}{V^2 R}\right)(V - nb)^{R/C_V} = \text{constant}$

(D) $\left(T + \frac{n^2 ab}{V^2 R}\right)(V - nb)^{C_V/R} = \text{constant}$

23. An engine moving away from a vertical cliff blows a horn at a frequency f. Its speed is 0.5% of the speed of sound in air. The frequency of the reflected sound received at the engine is [2013]

(A) 0.990 f (B) 0.995 f
(C) 1.005 f (D) 1.010 f

24. The bulk modulus of a gas is defined as $B = -VdP/dV$. For an adiabatic process the variation of B is proportional to P^n. For an ideal gas, n is [2013]

(A) zero (B) 1 (C) 5/3 (D) 2

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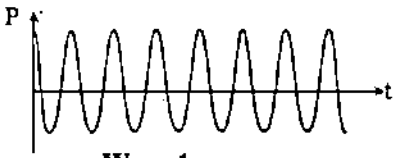
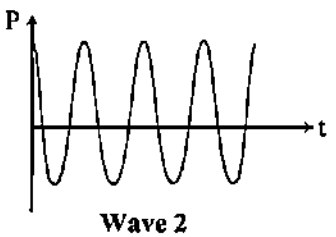
25. The average distance between molecules of an ideal gas at STP is approximately of the order of **[2013]**

(A) 1 nm (B) 100 nm
(C) 100 cm (D) 1 μm

26. The figure below shows pressure variation in two different sound waves in air with time at a given position. Both the figures are drawn to the same scale. **[2013]**



Wave 1

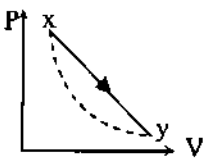


Wave 2

Which of the following statements is true ?

(A) Wave 1 has lower frequency and smaller amplitude compared to wave 2
(B) Wave 1 has higher frequency and greater amplitude compared to wave 2
(C) Wave 1 has shorter wavelength and greater amplitude compared to wave 2
(D) Wave 1 has shorter wavelength and smaller amplitude compared to wave 2

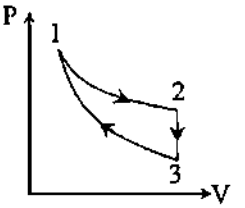
27. In the P-V diagram below the dashed curved line is an adiabat.



For a process that is described by a straight line joining two points X and Y on the adiabat (solid line in the diagram) heat is : (hint : Consider the variations in temperature from X to Y along the straight line) **[2013]**

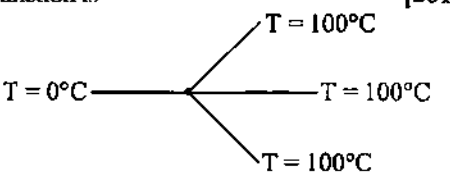
(A) absorbed throughout from X to Y
(B) released throughout from X to Y
(C) absorbed from X up to an intermediate point Z (not shown in the figure) and then released from Z to Y
(D) released from X up to an intermediate point Z (not shown in the figure) and then absorbed from Z to Y

28. The three processes in a thermodynamic cycle shown in the figure are : Process 1→2 is isothermal ; Process 2→3 is isochoric (volume remains constant); Process 3→1 is adiabatic. The total work done by the ideal gas in this cycle is 10 J. The internal energy decreases by 20 J in the isochoric process. The work done by the gas in the adiabatic process is –20 J. The heat added to the system in the isothermal process is – **[2013]**



(A) 0 J (B) 10 J (C) 20 J (D) 30 J

29. A metallic prong consists of 4 rods made of the same material, cross-sections and same lengths as shown. The three forked ends are kept at 100°C and the handle end is at 0°C. The temperature of the junction is – **[2013]**



(A) 25°C (B) 50°C (C) 60°C (D) 75°C

30. A solid expands upon heating because – **[2014]**

(A) the potential energy of interaction between atoms in the solid is asymmetric about the equilibrium positions of atoms.
(B) the frequency of vibration of the atoms increases.
(C) the heating generates a thermal gradient between opposite sides.
(D) a fluid called the caloric flows into the interatomic spacing of the solid during heating thereby expanding it.

31. Consider two thermometers T_1 and T_2 of equal length which can be used to measure temperature over the range θ_1 to θ_2. T_1 contains mercury as the thermometric liquid while T_2 contains bromine. The volumes of the two liquids are the same at the temperature θ_1. The volumetric coefficients of expansion of mercury and bromine are $18 \times 10^{-5} K^{-1}$ and $108 \times 10^{-5} K^{-1}$, respectively . The increase in length of each liquid is the same for the same increase in temperature. If the diameters of the capillary tubes of the two thermometers are d_1 and d_2 respectively, then the ratio $d_1 : d_2$ would be closest to **[2014]**

(A) 6.0 (B) 2.5 (C) 0.6 (D) 0.4

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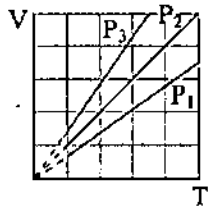
32. An ideal gas follows a process described by $PV^2 = C$ from (P_1, V_1, T_1) to (P_2, V_2, T_2) (C is a constant). Then **[2014]**
(A) if $P_1 > P_2$ then $T_2 > T_1$
(B) if $V_2 > V_1$ then $T_2 < T_1$
(C) if $V_2 > V_1$ then $T_2 > T_1$
(D) if $P_1 > P_2$ then $V_1 > V_2$

33. A whistle emitting a loud sound of frequency 540 Hz is whirled in a horizontal circle of radius 2 m and at a constant angular speed of 15 rad/s. The speed of sound is 330 m/s. The ratio of the highest to the lowest frequency heard by a listener standing at rest at a large distance from the center of the circle is **[2014]**
(A) 1.0 (B) 1.1 (C) 1.2 (D) 1.4

34. One mole of a monoatomic ideal gas is expanded by a process described by $PV^3 = C$ where C is a constant. The heat capacity of the gas during the process is given by (R is the gas constant). **[2014]**
(A) 2 R (B) $\frac{5}{2}$R (C) $\frac{3}{2}$R (D) R

35. A closed bottle containing water at 30°C is open on the surface of the moon. Then – **[2015]**
(A) the water will boil
(B) the water will come as a spherical ball
(C) the water will freeze
(D) the water will decompose into hydrogen and oxygen

36. The state of an ideal gas was changed isobarically. The graph depicts three such isobaric lines. Which of the following is true about the pressures of the gas ? **[2015]**

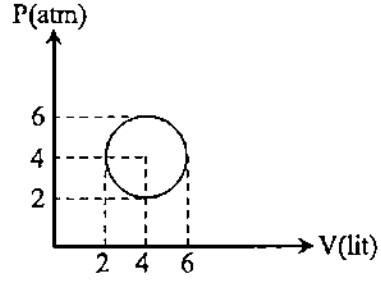


(A) $P_1 = P_2 = P_3$ (B) $P_1 > P_2 > P_3$
(C) $P_1 < P_2 < P_3$ (D) $P_1 / P_2 = P_3 / P_1$

37. The intensity of sound during the festival season increased by 100 times. This could imply a decibel level rise from - **[2015]**
(A) 20 to 120 dB (B) 70 to 72 dB
(C) 100 to 10000 dB (D) 80 to 100 dB

38. A horizontal steel railroad track has a length of 100 m when the temperature is 25°C. The track is constrained from expanding or bending. The stress on the strack on a hot summer day, when the temperature is 40°C, is (Note : the linear coefficient of thermal expansion for steel is 1.1×10^{-5}/°C and the Young's modulus of steel is 2×10^{11} Pa) **[2015]**
(A) 6.6×10^7 Pa (B) 8.8×10^7 Pa
(C) 3.3×10^7 Pa (D) 5.5×10^7 Pa

39. An ideal gas undergoes a circular cycle centered at 4 atm, 4 lit as shown in the diagram. The maximum temperature attained in this process is closed to - **[2015]**



(A) 30/R (B) 36/R (C) 24/R (D) 16/R

40. A gas at initial temperature T undergoes sudden expansion from volume V to 2V. Then - **[2016]**
(A) The process is adiabatic
(B) The process is isothermal
(C) The work done in this process is nRT $\ell n_e(2)$ where n is the number of moles of the gas.
(D) The entropy in the process does not change

41. A container is divided into two equal part I and II by a partition with a small hole of diameter d. The two partitions are filled with same ideal gas, but held at temperature $T_I = 150$ K and $T_{II} = 300$ K by connecting to heat reservoirs. Let λ_I and λ_{II} be the mean free paths of the gas particles in the two parts such that $d \gg \lambda_I$ and $d \gg \lambda_{II}$. Then λ_I/λ_{II} is close to - **[2016]**
(A) 0.25 (B) 0.5 (C) 0.7 (D) 1.0

42. A nurse measures the blood pressure of a seated patient to be 190 mm of Hg - **[2016]**
(A) The blood pressure at the patient's feet is less than 190 mm of Hg
(B) The actual pressure is about 0.25 times the atmospheric pressure
(C) The blood pressure at the patient's neck is more than 190 mm of Hg
(D) The actual pressure is about 1.25 times the atmospheric pressure

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43. Thermal expansion of a solid is due to the **[2016]**

(A) symmetric characteristic of the inter atomic potential energy curve of the solid

(B) asymmetric characteristic of the inter atomic potential energy curve of the solid

(C) double well nature of the inter-atomic potential energy curve of the solid

(D) Rotational motion of the atoms of the solid

44. A nuclear fuel rod generates energy at a rate of 5×10^8 Watt/m^3. It is in the shape of a cylinder of radius 4.0 mm and length 0.20 m . A coolant of specific heat 4×10^3 J/(kg-K) flows past it at a rate of 0.2 kg/s. The temperature rise in this coolant is approximately - **[2016]**

(A) 2°C (B) 6 °C

(C) 12 °C (D) 30 °C

45. A hearing test is conducted on an aged person. It is found that her threshold of hearing is 20 decibels at 1 kHz and it rises linearly with frequency to 60 decibels at 9 kHz. The minimum intensity of sound that the person can hear at 5 kHz is- **[2016]**

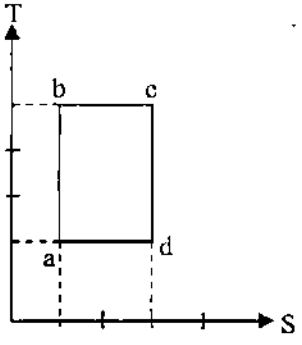
(A) 10 times than that at 1 kHz

(B) 100 times than that at 1 kHz

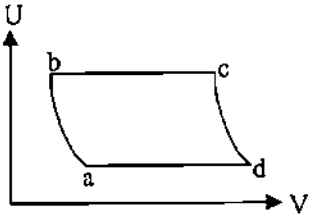
(C) 0.5 times than that at 9 kHz

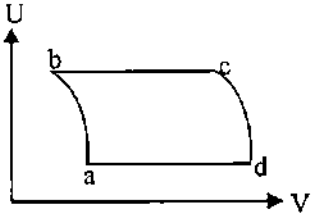
(D) 0.05 times than that at 9 kHz

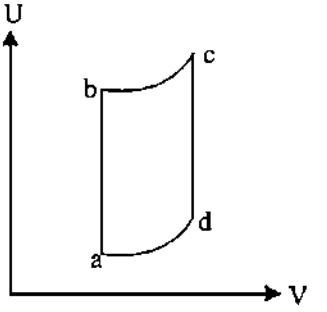
46. An ideal gas is taken reversibly around the cycle a-b-c-d-a as shown on the T (temperature) – S (entrophy) diagram **[2016]**

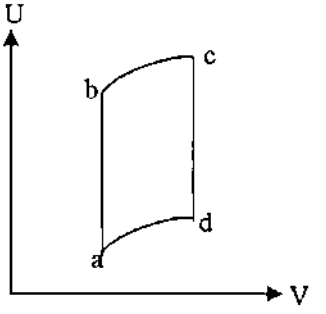


The most appropriate representation of above cycle on a U (internal energy) – V (volume) diagrame is

(A)


(B)


(C)


(D)


47. The heat capacity of one mole an ideal is found to be $CV = 3R\,(1 + aRT)/2$ where a is a constant. The equation obeyed by this gas during a reversible adiabatic expansion is – **[2016]**

(A) $TV^{3/2}e^{aRT} = \text{constant}$

(B) $TV^{3/2}e^{3aRT/2} = \text{constant}$

(C) $TV^{3/2} = \text{constant}$

(D) $TV^{3/2}e^{2aRT/3} = \text{constant}$

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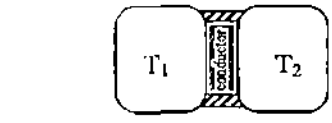
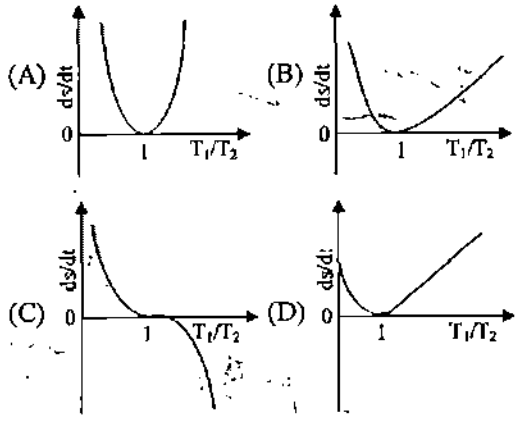
48. A transverse wave of frequency 500 Hz and speed 100 m/s is traveling in the positive x direction on a long string. At time t = 0 s the displacements at x = 0.0 m and at x = 0.25 m are 0.0 m and 0.02 m, respectively. The displacement at x = 0.2 m at $t = 5 \times 10^{-4}$ s is **[5 Nov, 2017]**

(A) –0.04 m (B) –0.02 m
(C) 0.04 m (D) 0.02 m

49. A thin piece of thermal conductor of constant thermal conductivity insulated on the lateral sides connects two reservoirs which are maintained at temperatures T_1 and T_2 as shown. Assuming that the system is in steady state, which of the following plots best represents the dependence of the rate of change of entropy of the ratio of temperatures T_1/T_2 **[5 Nov, 2017]**





50. Two bottles A and B have radii R_A and R_B and heights h_A and h_B respectively with $R_B = 2R_A$ and $h_B = 2h_A$. These are filled with hot water at 60°C. Consider that heat loss for the bottles takes place only from side surfaces. If the time the water to cool down to 50°C is t_A and t_B for the bottles A and B, respectively, then t_A and t_B are best related as **[5 Nov, 2017]**

(A) $t_A = t_B$
(B) $t_B = 2t_A$
(C) $t_B = 4t_A$
(D) $t_A = t_A/2$

51. The number of gas molecules striking per second per square meter of the top surface of a table placed in a room at 20°C and 1 atmospheric pressure is of the order of ($k_B = 1.4 \times 10^{-23}$ J/K, and the average mass of an air molecules is 5×10^{-27} kg) **[5 Nov, 2017]**

(A) 10^{27} (B) 10^{23}
(C) 10^{25} (D) 10^{29}

52. One mole of an ideal monatomic gas undergoes the following four reversible processes :

Step1 : It is first compressed adiabatically from volume V_1 to $1m^3$.
Step 2 : then expanded isothermally to volume $10\ m^3$.
Step 3 : then expanded adiabatically to volume V_3.
Step 4 : then compressed isothermally to volume V_1.

If the efficiency of the above cycle is 3/4 then V_1 is, **[5 Nov, 2017]**

(A) $2m^3$ (B) $4m^3$
(C) $6\ m^3$ (D) $8\ m^3$

53. A gas obeying the equation of state PV = RT undergoes a hypothetical reversible process described by the equation, $PV^{5/3} \exp\left(-\frac{PV}{E_0}\right) = c_1$ where c_1 and E_0 are dimensioned constants. Then, for this process, the thermal compressibility at high temperature **[5 Nov, 2017]**

(A) approaches a constant value.
(B) is proportional to T.
(C) is proportional to $T^{1/2}$
(D) is proportional to T^2.

54. Consider the following statements for air molecules in an air tight container.

(I) the average speed of molecules is larger than root mean square speed
(II) mean free path of molecules is larger than the mean distance between molecules
(III) mean free path of molecules increases with temperature
(IV) the rms speed of nitrogen molecule is smaller than oxygen molecule

The true statements are : **[19 Nov, 2017]**

(A) only II (B) II & III
(C) II & IV (D) I, II & IV

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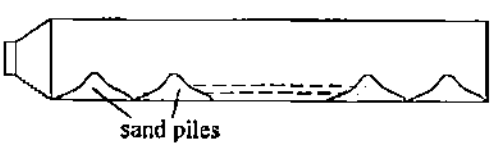
55. A light beam travelling along the x axis with planar wavefront is incident on a medium of thickness t. In the region, where light is falling the refractive index can be taken to be varying such that $\frac{dn}{dy} > 0$. The light beam on the other side of the medium will emerge **[19 Nov, 2017]**

(A) parallel to the x-axis
(B) bending downward
(C) bending upward
(D) split into two or more beams

56. A bomb explodes at time t = 0 in a uniform, isotropic medium of density ρ and releases energy E, generating a spherical blast wave. The radius R of this blast wave varies with time t as : **[19 Nov, 2017]**

(A) t (B) $t^{2/5}$
(C) $t^{1/4}$ (D) $t^{3/2}$

57. A closed pipe of length 300 cm contains some sand. A speaker is connected at one of its ends. The frequency of the speaker at which the sand will arrange itself in 20 equidistant piles is close to (velocity of sound is 300 m/s) **[19 Nov, 2017]**



(A) 10 kHz (B) 5 kHz
(C) 1 kHz (D) 100 kHz

58. A planet of radius R_p is revolving around a star of radius R^*, which is at temperature T^*. The distance between the star and the planet is d. If the planet's temperature is $f\ T^*$, then f is proportional to **[19 Nov, 2017]**

(A) $\sqrt{R^*}/d$ (B) R^*/d
(C) $R^* R_p/d^2$ (D) $(R^*/d)^4$

59. One mole of an ideal monatomic gas undergoes the following four reversible processes :

Step 1 – it is first compressed adiabatically from volume 8.0 m^3 to 1.0 m^3.
Step 2 – then expanded isothermally at temperature T_1 to volume 10.0 m^3.
Step 3 – then expanded adiabatically to volume 80.0 m^3.
Step 4 – then compressed isothermally at temperature T_2 to volume 8.0 m^3.

Then T_1/T_2 is **[19 Nov, 2017]**

(A) 2 (B) 4
(C) 6 (D) 8

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ANSWERS

1. (C)	2. (B)	3. (B)	4. (A)	5. (C)	6. (C)	7. (B)
8. (A)	9. (B)	10. (D)	11. (B)	12. (B)	13. (C)	14. (D)
15. (D)	16. (C)	17. (A)	18. (B)	19. (A)	20. (B)	21. (B,D)
22. (A)	23. (A)	24. (B)	25. (A)	26. (D)	27. (C)	28. (D)
29. (D)	30. (B)	31. (D)	32. (B)	33. (C)	34. (D)	35. (A)
36. (B)	37. (D)	38. (C)	39. (B)	40. (A)	41. (C)	42. (D)
43. (B)	44. (B)	45. (B)	46. (A)	47. (A)	48. (D)	49. (B)
50. (B)	51. (A)	52. (D)	53. (A)	54. (A)	55. (C)	56. (B)
57. (C)	58. (A)	59. (B)				

SOLUTIONS

1. For adiabatic process

$dQ = 0$ and $-dU = dW$

$\Rightarrow -nC_V \Delta T = P\Delta V$ or $-nC_V dT = PdV$

when change is very small

now given $U = CT - \frac{n^2a}{V}$

$\therefore\ dU = CdT + \frac{n^2a}{V^2}dV$

put this value of dU in $-dU = dW$

$\therefore -\left(CdT + \frac{n^2a}{V^2}dV\right) = PdV \quad ....(1)$

also $P = \left(\frac{nRT}{V-nb}\right) - \frac{n^2a}{V^2}$ replace it in (1)

$-\left(CdT + \frac{n^2a}{V^2}dV\right) = \left(\left(\frac{nRT}{V-nb}\right) - \frac{n^2a}{V^2}\right)dV$

$\therefore -CdT = \left(\frac{nRT}{V-nb}\right)dV$

$\therefore -\frac{C}{nR}\frac{dT}{T} = \frac{dV}{V-nb}$

Integrating we get

$$-\ell n\, T^{C/nR} = \ell n\,(V-nb) + k$$

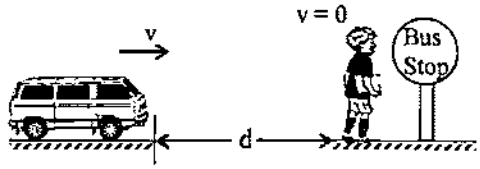
(k → constant of integration)

$\therefore\ \ell n\,(T^{C/nR})(V-nb) = -k$

or $(T^{C/nR})(V-nb) = \text{constant}$

2. From the analysis of P-V diagram we can easily say that B is the correct option.

3.



$v = 39.6$ km/hr $= 11$ m/s,

$t_1 = \frac{d}{330}$ and $t_2 = \frac{d-11\times30}{330}$

Now $\Delta t = t_1 - t_2 = 1$

now $t_1 = 30$ sec $\therefore\ t_2 = 29$ sec.

4. $v = \sqrt{\frac{\gamma RT}{M}} \qquad \therefore\ v \propto \frac{1}{\sqrt{M}}$

$\frac{V_0}{V_H} = \sqrt{\frac{M_H}{M_0}} = \sqrt{\frac{2}{32}} = \sqrt{\frac{1}{16}}$

$\frac{V_0}{V_H} = \frac{1}{4}$

5. Using theory of entropy it is evident that answer is (C).

6. Direct formula is to be used

$$C = C_V + \frac{nR}{1-\alpha}$$

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7. $C_V = \frac{3}{2}R,\ C_P = C_V + R = \frac{5R}{2}$

$f = 3$

$W = \frac{1}{2}(V_2 - V_1)(P_2 - P_1)$

For BA $Q = nC_P\Delta T = n\left(\frac{5}{2}R\right)\Delta T$

$= \frac{5}{2}(P_1V_1 - P_2V_2)$

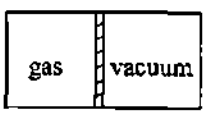
For AC $Q_{AC} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + nC_V\Delta T$

now $nC_V\Delta T = \frac{3}{2}(P_2V_2 - P_1V_1)$

now

$$\eta = \frac{W}{Q} = \frac{\frac{1}{2}(V_2 - V_1)(P_2 - P_1)}{\frac{1}{2}\times(P_1 + P_2)(V_2 - V_1) + \frac{3}{2}(P_2V_2 - P_1V_1)}$$

using formula for heat we can calculate heat absorbed in AC.

8. 

expansion is against

vacuum $\therefore \Delta W = 0$

Insulated container $\therefore\ \Delta Q = 0$

First law of thermodynamics

$\Delta Q = \Delta W + \Delta U$

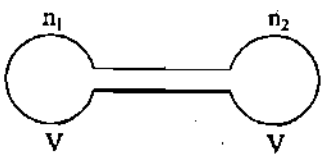
$0 = 0 + \Delta U$

$0 = 0 + \Delta U$

$\Delta U = 0$

9. Mole conservation

$n_1 + n_2 = n$



Initial no. of moles $= n_1 = n_2 = \frac{n}{2}$

finally when temp of 1 vessel is T & another is 2T

$n_1 = \frac{PV}{RT}$

$n_2 = \frac{PV}{R2T} \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$

$n_1 + n_2 = n$

$n_1 = \frac{2n}{3};\ n_2 = \frac{n}{3}$

mass of gas $\propto n_1$

$\therefore \frac{M_2}{M_1} = \frac{\frac{n}{3}}{\frac{n}{2}} = \frac{2}{3}$

10. $\frac{100 - T}{R_1} = \frac{T - 0}{R_2}$

$\frac{100 - T}{T} = \frac{R_1}{R_2}$

$R = \frac{L}{KA}$

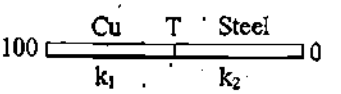
$\frac{R_1}{R_2} = \frac{k_2}{k_1}$

$\frac{100 - T}{T} = \frac{50}{385} = \frac{10}{77}$

$7700 - 77\,T = 10\,T$

$7700 = 87\,T$

$T = \frac{7700}{87} = 88°C$



11. $PV^\gamma = C$

$P^{1-\gamma}T^\gamma = C$

$(0.28)^{1-\gamma} \times (233)^\gamma = 1^{1-\gamma} \times T^\gamma$

$\gamma = \frac{7}{5}$

$(0.28)^{1-7/5} \times (233)^{7/5} = 1^{1-7/5} \times T^{7/5}$

$T^{7/5} = 233^{7/5} \times (0.28)^{-2/5}$

$T = 233\,(0.28)^{-2/7}$

$T = \frac{233}{(0.28)^{2/7}}$

T is coming

more than 298 K or 25°C

$\therefore$ T is more than 25°C

so to cool it an extra ac is required.

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12. S. → u .O

$$f_1 = \frac{f_0[v]}{v-u}$$

S. v ← .O

$$f_2 = f_0\frac{[v+u]}{v}$$

$$f_2 - f_1 = f_0\left(\frac{u+v}{v} - \frac{v}{v-u}\right)$$

$$f_2 - f_1 = \frac{-u^2 f_0}{(v)(v-u)} = -ve$$

$\therefore f_1 > f_2$

13. $f\lambda = v$

$f \propto v$

$$\frac{f_1}{f_2} = \frac{v_1}{v_2}$$

$$\frac{450}{f_2} = \frac{v_1}{1.04v_1}$$

$f_2 = 1.04 \times 450 = 468Hz.$

14. $U = aV^3$

$$\frac{fnRT}{2} = aV^3$$

$PV = RT$

$$\frac{PV}{R} = aV^3$$

$P = CV^2$

$$W = \int_V^{2V} P\,dV = \int_V^{2V} CV^2 dV = \frac{C}{3}(8V^3 - V^3)$$

$$= \frac{7V^3C}{3}$$

$$\frac{fRT}{2} = aV^3$$

$PV = RT$

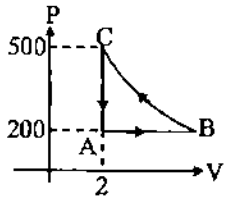
$$\frac{fPV}{2} = aV^3$$

$$P = \frac{2a}{f}V^2$$

$$C = \frac{2a}{f}$$

$$W = \frac{7}{3} \times V^3 \times \frac{2a}{f} = \frac{7}{3f} fnRT = \frac{7RT}{3}$$

15.



$W_{C\to A} = 0$

Process BC T = Constant

$P_C V_C = P_B V_B$

$500 \times 2 = 200 \times V_B$

$V_B = 5m^3$

$W_{A\to B} = 200\,[V_B - V_A] = 200\,[5-2] = 600$

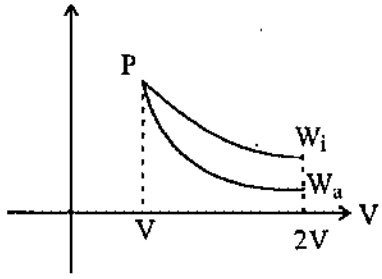
$W_{B\to C} > W_{A\to B}$

$\therefore$ Net work done is –ve

$\because W_{B\to C} < 1200$ KJ

$\therefore$ Total W < – 600 KJ

16.



$W_i > W_a > 0$

17. $$\frac{V_{sound}}{V_{av}} = \sqrt{\frac{\gamma kT}{m} \times \frac{\pi m}{8kT}}$$

18. We have, $\Delta\theta = mS\Delta T = CT$

Where C → heat capacity

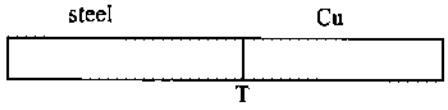
Let final temperature becomes T. then

Heat lost = Heat gain

$C(100 - T) = C'(T - 0)$

$$\text{or } T = \frac{100}{\left(1 + \frac{C'}{C}\right)} < 50 \qquad (\because C' > C)$$

19.



$$\frac{50 \times A \times (100-T)}{0.1} = \frac{400 \times A \times (T-0)}{0.2}$$

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20. Energy radiated, $U \propto AT^4t$

$$\Rightarrow \frac{U_2}{U_1} = \frac{A/2(2T)^4.t}{AT^4.t} = 8$$

$$\Rightarrow U_2 = 8U_1$$

$$\Rightarrow mS\Delta t_2 = 8mS\Delta t_1$$

$$\Rightarrow \Delta t_2 = 8\Delta t_1$$

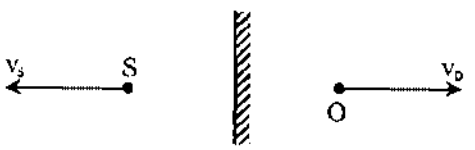
21. If v = 330 the "D" will also be correct option.
If we consider option "B" that seems to be incorrect as per given condition. So question is about incorrect statement.
So answer should be "B" and "D"

(i) $\frac{2\pi}{\lambda} = \left(\frac{4\pi}{\ell}\right)\frac{2\pi}{\lambda} = k$ = Propagation constant

or $\lambda = \ell/2$ or $\lambda = \frac{1.2}{2} = 0.6$

(ii) $v = f\lambda \Rightarrow f = v/\lambda$

23.



$$f_{app} = \frac{v - v_o}{v + v_s} f$$

$$= \frac{v - \frac{v}{500}}{v + \frac{v}{500}} f = \frac{499}{501} f = 0.990\, f$$

24. PV^γ = constant

$$P.\frac{dV^\gamma}{dV} + V^\gamma \frac{dP}{dV} = 0$$

$$P\gamma . V^{\gamma-1} + V^\gamma \frac{dP}{dV} = 0$$

$$P\gamma + V\frac{dP}{dV} = 0$$

$$-V\frac{dV}{dV} = \gamma P$$

$$B = \gamma P$$

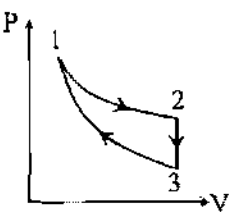
$$B \propto P^1 \quad \therefore n = 1$$

25. Average distance between molecules of an ideal gas is 3.3 nm.

26. It is clear by graph
that wave 1 have high frequency and smaller amplitude than wave 2
$v = \lambda f$ = constant.

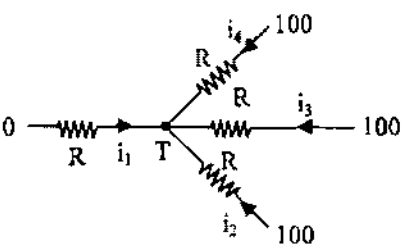
$$\therefore \quad f \propto \frac{1}{\lambda}$$

28.



$Q_{tot} = W_{12} + W_{23} + W_{31} = W_{tot}$
In isochoric process. $\Delta U = -20$
$\therefore \quad Q_{23} = \Delta U = -20$
$Q_{31} = 0$
$W_{23} = 0$ (Isochoric process)
$W_{31} = -20$ joule (given)
$Q_{12} + Q_{23} + Q_{31} = W_{tot}$
$Q_{12} - 20 + 0 = 10$
$Q_{12} = 20 + 10 = 30$ joule

29.



$i_1 + i_2 + i_3 + i_4 = 0$

$$\frac{0-T}{R} + \frac{100-T}{R} + \frac{100-T}{R} + \frac{100-T}{R} = 0$$

$300 - 4T = 0$

$$T = \frac{300}{4} \Rightarrow 75°C$$

30. The frequency of vibration of the atom increases on heating a solid body resulting into increase in intermolecular separation.

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31. Increase in length of each liquid is same

$\Delta\ell = \Delta\ell$

$$\frac{\Delta V_{Hg}}{\pi d_1^2} = \frac{\Delta V_{Bromine}}{\pi d_2^2}$$

$$\frac{(V)\gamma_{Hg}\Delta\theta}{\pi d_1^2} = \frac{V\,\gamma_{Bromine}\Delta\theta}{\pi d_2^2}$$

$$\left(\frac{d_1}{d_2}\right)^2 = \frac{\gamma_{Hg}}{\gamma_{Bromine}} = \frac{18\times10^{-5}}{108\times10^{-5}}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{1}{6}} \simeq 0.4$$

32. $PV^2 = C$

$$\Rightarrow \left(\frac{nRT}{V}\right)V^2 = C$$

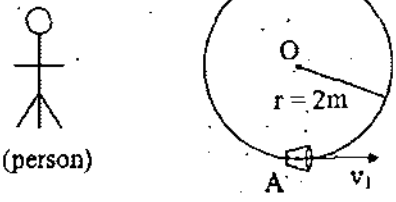
$\Rightarrow TV = C$

$\Rightarrow T_1V_1 = T_2V_2$

$\Rightarrow$ If temperature increases, volume decreases and vice versa

$\Rightarrow V_2 > V_1$ then $T_2 < T_1$.

33.



(person)

v_1 and v_2 are speed of whistle

$|v_1| = |v_2| = \omega r = 15 \times 2 = 30$ m/s

Maximum frequency heard →

Here,

f → original frequency (540 Hz)

v → speed of sound

v_s → speed of whistle

$$f_{max} = f\left[\frac{v}{v - v_s}\right]$$

Minimum frequency heard →

$$f_{min} = f\left[\frac{v}{v + v_s}\right]; \quad \frac{f_{max}}{f_{min}} = \frac{v + v_s}{v - v_s} = \frac{330+30}{330-30} = \frac{360}{300}$$

$$\frac{f_{max}}{f_{min}} = \frac{6}{5} = 1.2$$

34. Monoatomic gas $\Rightarrow \gamma = \frac{5}{3}$

n = 1

$PV^3 = C$ on comparing with $PV^\alpha = C$

Here $\alpha = 3$

Heat capacity

$$C = \frac{R}{\gamma - 1} - \frac{R}{\alpha - 1}$$

$$C = \frac{R}{\left(\frac{2}{3}\right)} - \frac{R}{(2)}$$

$$C = R\left[\frac{3}{2} - \frac{1}{2}\right]$$

$C = R$

35. Because on earth there is no atmosphere. So water will boil.
(At Boiling point vapour pressure = Atmospheric pressure, in open vessel)

36. Ideal gas equation PV = nRT

For isobaric process

$$V = \left(\frac{nR}{P}\right) T \; (V \propto T \text{ (straight line)})$$

$$\text{Slope of line} = \left(\frac{nR}{P}\right)$$

$$\text{slope} \propto \frac{1}{P}$$

$\text{slope}_3 > \text{slope}_2 > \text{slope}_1$

$P_3 < P_2 < P_1$

37. Loudness of sound in decibel $dB = 10 \log_{10}\left(\frac{I}{I_0}\right)$

when intensity of sound become 100 I then new decibel level = $dB' = 10 \log_{10}\left(\frac{100\,I}{I_0}\right)$

$dB' - dB = 10 \log_{10} 100$

$dB' - dB = 20$

∴ decibel rise by 20 dB

only one option i.e. 80 to 100 dB match with it.

38. When some body is constrained from expanding or bending then on heating thermal stress get develop in the body.

Stress = $Y\,\alpha\,\Delta T = 2 \times 10^{11} \times 1.1 \times 10^{-5} \times (40 - 25) = 3.3 \times 10^7$ N/m^2 $= 3.3 \times 10^7$ Pa

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39. $T = \frac{PV}{R}$

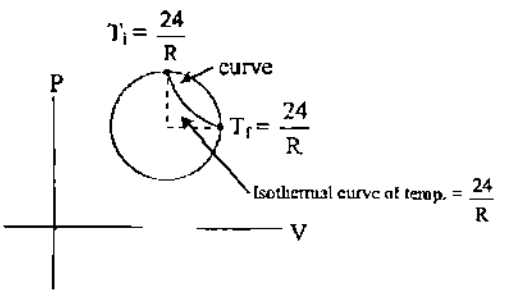
T will be maximum when PV is maximum

$T = \frac{PV}{R} = \frac{(4+2\sin\theta)(4+2\cos\theta)}{R}$

As $\sin\theta$ and $\cos\theta$ both can not be equal to 1 for same value of θ

$\therefore$ T can not be $\frac{36}{R}$

T_{max} should be less than $\frac{36}{R}$



curve is above isothermal curve

$\therefore$ temp. is more than $\frac{24}{R}$ on the given process

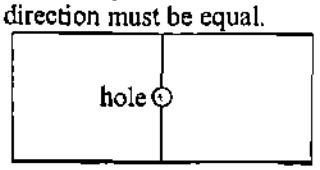
So T_{max} lie between $\frac{24}{R}$ and $\frac{36}{R}$

only one option is present

40. In sudden expansion gas do not get enough time for exchange of heat.

$\therefore$ Process is adiabatic.

41. As dimension of hole is very small than mean path, then at equilibrium effusion rate of gas in both direction must be equal.



For this $\frac{P_1}{\sqrt{T_1}} = \frac{P_2}{\sqrt{T_2}}$

Mean free path $\propto \frac{T}{P}$

$\frac{\lambda_1}{\lambda_2} = \frac{T_1}{T_2} \times \frac{P_2}{P_1}$

$\frac{T_1}{T_2} \times \frac{\sqrt{T_2}}{\sqrt{T_1}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{150}{300}} = 0.7$

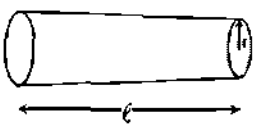
42. Blood pressure is gauge pressure = 190 mm Hg

Atmospheric pressure = 760 mm Hg

Actual pressure = 190 + 760 mm Hg = 950 mm Hg = 1.25 × 760 mm Hg

43. Thermal expansion of a solid is due to asymmetric characteristic of inter atomic potential energy curve of the solid.

44.



$\frac{dm}{dt} \times S\Delta T = \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = 5 \times 10^8 \times$ volume of rod

$= 5 \times 10^8 \times \pi \times (4)^2 \times 10^{-6} \times \frac{0.2}{10}$

$= 5 \times 10 \times \pi \times 16 \times 2 \quad = 1600\,\pi$

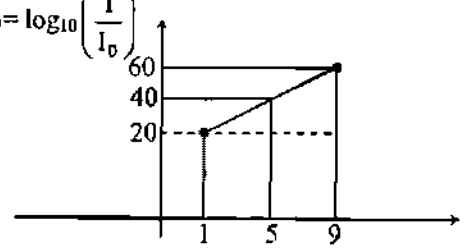
$0.2 \times 4 \times 10^3 \Delta T = 1600\pi$

$8 \times 10^2 \Delta T = 16 \times 10^2 \pi$

$\Delta T = 3.14 \times 2 \Rightarrow 6.28°C$

45.

$\beta = \log_{10}\left(\frac{I}{I_0}\right)$



at 5 KHz Hearing capacity = 40 dB

Intensity at 1 KHz

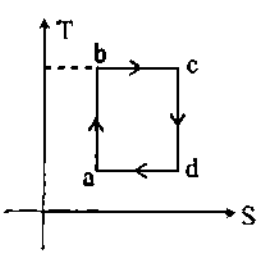
$\beta = 10 \log\left(\frac{I}{I_0}\right) \Rightarrow I = I_0 10^{\left(\frac{\beta}{10}\right)}$

$(I)_{1\ KHz} = I_0\ 10^{(20/10)} = I_0(10)^2$

$(I)_{5\ KHz} = I_0\ 10^{(40/10)} = I_0(10)^4$

$\frac{(I)_{1KHz}}{I_{5KHz}} = \frac{1}{100}$

46.



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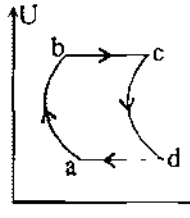
bc ⇒ Isothermal process so U remain constant

cd ⇒ Isentropic process so S remain constant

$\therefore$



bc should be straight line parallel to V & cd graph should be

47. Adiabatic process

$TV^{\gamma-1} = C$

$\gamma = 1 + \frac{2}{f}$

$TV^{\frac{2}{f}} = C$

$C_V = \frac{fR}{2} = \frac{3R(1 - aRT)}{2}$

$\frac{fR}{2} = \frac{3Re^{aRT}}{2}$

$\frac{2}{f} = \frac{2}{3e^{aRT}}$

$TV^{\frac{2}{3e^{aRT}}} = C \Rightarrow TV^{\frac{3e^{aRT}}{2}} = C$

Ans. given is $TV^{\frac{3}{2}}e^{aRT}$

So no option is matching may be due to printing mistake.

48. $y = A\sin(kx - \omega t)$

at $x = 0.025$, $y = 0.02$

$v = v\lambda$

$\lambda = \frac{100}{500} = \frac{1}{5}$ m

$y = 0.02 = A\sin\left(\frac{2\pi}{\lambda}.\frac{1}{4}\right)$

$y = 0.002 = A\sin\left(\frac{5\pi}{2}\right)$

$\Rightarrow A = 0.02$m

$\therefore y = 0.02\sin(kx - \omega t)$

$= 0.02\sin(10\pi \times 0.2 - 1000\pi \times 5 \times 10^{-4})$

$= 0.02\sin[2\pi - 0.5\pi]$

$= 0.02\sin\left(\frac{3\pi}{2}\right) = -0.02$m

49. $ds = \frac{-Qdt}{T_1} + \frac{Qdt}{T_2}$

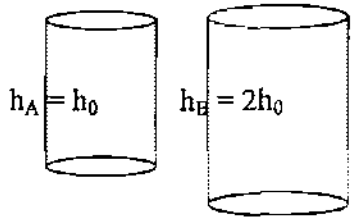
$\frac{ds}{dt} = -Q\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \quad \frac{-(T_1 - T_2)}{R} \quad \left[\frac{T_2 - T_1}{T_1T_2}\right]$

$= +\left(\frac{T_1^2 - T_2^2}{T_1T_2}\right)\frac{1}{R}$

$\frac{ds}{dt} = \left(\frac{T_1}{T_2} - \frac{T_2}{T_1}\right)\frac{1}{R}$

$= \left(x - \frac{1}{x}\right)\frac{1}{R}$

50.



$t \propto \frac{m}{A}$

$t = k.\frac{V\rho}{A} = k.\rho h$

$t \propto h$

$\therefore \frac{t_A}{t_B} = \frac{h_A}{h_B} = \frac{h_0}{2h_0} \Rightarrow t_B = 2t_A$

51. $V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{M_0}}$

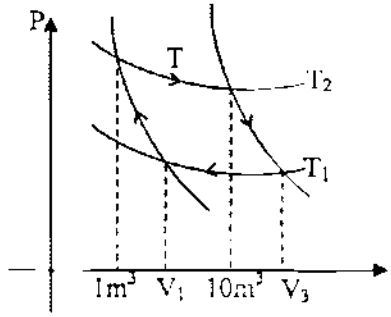
$P = \dot{N} \times 2mV_{rms}$

$1.01 \times 10^5 = \dot{N} \times 2 \times 5 \times 10^{-27} \times V_{rms}$

$\dot{N} = \frac{1.01 \times 10^5\sqrt{5 \times 10^{-27}}}{2 \times 5 \times 10^{-27}\sqrt{3 \times 1.4 \times 10^{-23} \times 293}}$

$= 6.43 \times 10^{27}$

52.



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$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{2/3}$$

$$-\frac{1}{4} = -\left(\frac{1}{V_1}\right)^{2/3} \Rightarrow V_1 = 8m^3$$

53. $PV^{5/3} = c_1 e^{\frac{PV}{E_0}}$

$$\ell n\, P + \frac{5}{3}\ \ell n\, V = \ell n\, c_1 + \frac{PV}{E_0}$$

$$\frac{dP}{P} + \frac{5}{3}\frac{dV}{V} = 0 + \frac{PdV + VdP}{E_0}$$

$$dP\left[\frac{1}{P} - \frac{V}{E_0}\right] = dV\left[P - \frac{5}{3V}\right]$$

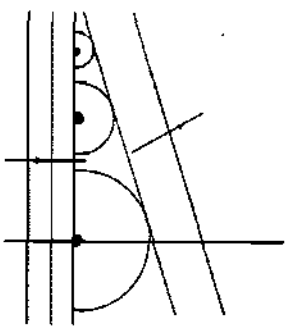
$$\frac{dV}{dP} = \frac{\frac{1}{P} - \frac{V}{E_0}}{P - \frac{5}{3V}}$$

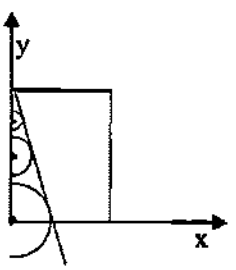
$$c = -\frac{1}{V}\frac{dV}{dP} = \frac{\left[\frac{1}{E_0} - \frac{1}{PV}\right]}{\left[P - \frac{5}{3V}\right]}$$

At very high temperature $c = \frac{1}{E_0}$

54. For ideal gas mean free path of molecules is larger than mean distance between molecules

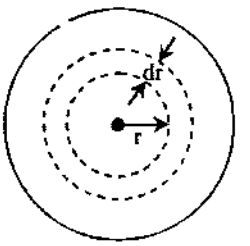
55.





In y direction refractive index is increasing therefore speed of light is decreasing

56. The energy will propagate in form of spherical blast wave which is longitudinal in nature. Hence velocity of propagate of disturbance



$$v = \sqrt{\frac{\gamma P}{\rho}}$$

Where P - Pressure

$\because$ $PV = nRT$

$$v = \sqrt{\frac{\gamma RT}{V\rho}}$$

Where $V = \frac{4\pi r^3}{3}$

& v = velocity of propagation $= \frac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt} = \sqrt{\frac{\gamma RT}{\left(\frac{4}{3}\pi r^3\right)\rho}}$$

$$\Rightarrow \frac{dr}{dt} = \frac{k}{r^{3/2}}$$

$$k = \text{constant} = \sqrt{\frac{3\gamma RT}{4\pi\rho}}$$

$$\Rightarrow r^{3/2}dr = kdt$$

$$\int r^{3/2}dr = \int kdt$$

$$r^{5/2} = kt \Rightarrow r = (kt)^{2/5}$$

$$\Rightarrow r \propto t^{2/5}$$

Hence correct answer is (B)

Alternate solution

$R \propto E^a P^b t^c$

Dimension of R = L

$L \propto (ML^2T^{-2})^a\ (ML^{-3})^b\ (T)^c$

$2a - 3b = 1$ ...(i)

$c - 2a = 0$ ...(ii)

$a + b = 0$ ...(iii)

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from (i), (ii) & (iii)

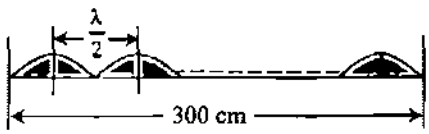
$a = \frac{1}{5}$

$c = 2a$

$\therefore \quad c = \frac{2}{5}$

$R \propto t^{2/5}$

57.



$20 \frac{\lambda}{2} = 300$

$\lambda = 30 \text{ cm} = 0.3 \text{ cm}$

$v = v\lambda$

$v = \frac{300}{0.3} = 1000 \text{ Hz}$

$v = 1\text{kHz}$

58. The planet should be in Thermal equilibrium with star.

Amount of heat energy emitted by start per second $E_1 = \left(\sigma T^{*4}\right)\left(4\pi R^{*2}\right)$

$\because$ d = distance between star and planet

Hence, amount of energy reaching planet per unit area per second $= \frac{E_1}{4\pi d^2}$

$$= \frac{\left(\sigma T^{*4}\right)\left(4\pi R^{*2}\right)}{4\pi d^2}$$

$$= \frac{\sigma T^{*4} R^{*2}}{d^2}$$

Hence energy received by planet per second

$$E_2 = \left(\frac{\sigma T^{*4} R^{*2}}{d^2}\right)\pi R_p^2$$

T = Temperature of planet then amount of energy emitted by planet per second

$E_3 = \left(\sigma T^4\right)\left(4\pi R_p^2\right)$

For thermal equilibrium $E_3 = E_2$

$$\Rightarrow \left(\sigma T^4\right)\left(4\pi R_p^2\right) = \frac{\left(\sigma T^{*4} R^{*2}\right)}{d^2}\left(\pi R_p^2\right)$$

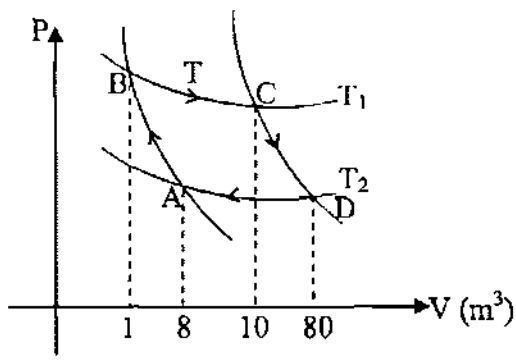
$$\Rightarrow T^4 = \frac{T^{*4} R^{*2}}{4d^2}$$

$$\Rightarrow T = T^* \sqrt{\frac{R^*}{2d}} = fT^*$$

$$\Rightarrow f \propto \sqrt{\frac{R^*}{d}}$$

Hence correct answer is (A)

59.



A to B

$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{8}\right)^{\frac{5}{3}-1}$$

$$\frac{T_1}{T_2} = \left(\frac{8}{1}\right)^{\frac{2}{3}} = \frac{4}{1}$$

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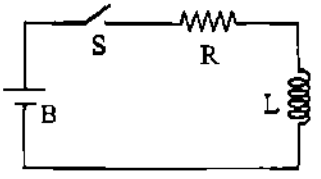
Chapter

3 Electrodynamics

1. The circuit shown consists of a switch (S), a battery (B) of emf E, a resistance R, and an inductor L. [2010]

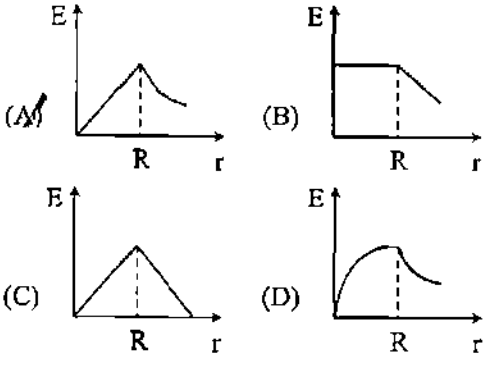


The current in the circuit at the instant the switch is closed is-

(A) E/R (B) E/R(1 – e)

(C) ∞ (D) 0

2. Consider a uniform spherical volume charge distribution of radius R. Which of the following graphs correctly represents the magnitude of the electric field E as a distance r from the center of the sphere ? [2010]



3. A charge +q is placed somewhere inside the cavity of a thick conducting spherical shell of inner radius R_1 and outer radius R_2. A charge – Q is placed at a distance $r > R_2$ from the centre of the shell. Then the electric field in the hollow cavity- [2010]

(A) depends on both +q and – Q

(B) is zero

(C) is only that due to – Q

(D) is only that due to +q

4. A charged particle of charge q and mass m, gets deflected through an angle θ upon passing through a square region of side 'a' which contains a uniform magnetic field B normal to its plane. Assuming that the particle entered the square at right angles to one side, what is the speed of the particle ? [2010]

(A) $\frac{qB}{m} a\cot\theta$ (B) $\frac{qB}{m} a\tan\theta$

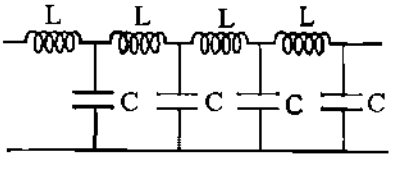
(C) $\frac{qB}{m} a\cot^2\theta$ (D) $\frac{qB}{m} a\tan^2\theta$

5. Three equal charges +q are placed at the three vertices of an equilateral triangle centered at the origin. They are held in equilibrium by a restoring force of magnitude F(r) = kr directed towards the origin, where k is a constant. What is the distance of the three charges from the origin ? [2010]

(A) $\left[\frac{1}{6\pi\varepsilon_0}\frac{q^2}{k}\right]^{1/2}$ (B) $\left[\frac{\sqrt{3}}{12\pi\varepsilon_0}\frac{q^2}{k}\right]^{1/3}$

(C) $\left[\frac{1}{6\pi\varepsilon_0}\frac{q^2}{k}\right]^{2/3}$ (D) $\left[\frac{\sqrt{3}}{4\pi\varepsilon_0}\frac{q^2}{k}\right]^{2/3}$

6. Consider the infinite ladder circuit shown below.



For which angular frequency ω will the circuit behave like a pure inductance ? [2010]

(A) $\frac{LC}{\sqrt{2}}$ (B) $\frac{1}{\sqrt{LC}}$

(C) $\frac{2}{\sqrt{LC}}$ (D) $\frac{2L}{\sqrt{C}}$

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7. Two identical particles of mass 'm' and charge q are shot at each other from a very great distance with an initial speed v. The distance of closest approach of these charges is- [2010]

(A) $\frac{q^2}{8\pi\varepsilon_0 mv^2}$ (B) $\frac{q^2}{4\pi\varepsilon_0 mv^2}$

(C) $\frac{q^2}{2\pi\varepsilon_0 mv^2}$ (D) 0

8. For a domestic AC supply of 220 V at 50 cycles per second, the potential difference between the terminals of a two pin electric outlet in a room is given by - [2011]

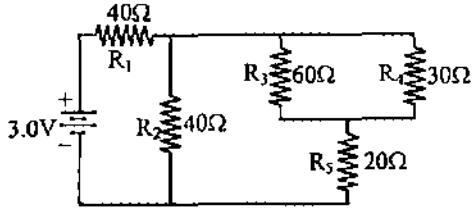
(A) $V(t) = 220\sqrt{2}\cos(100\pi t)$

(B) $V(t) = 220\cos(50\ t)$

(C) $V(t) = 220\cos(100\ \pi t)$

(D) $V(t) = 220\sqrt{2}\cos(50\ t)$

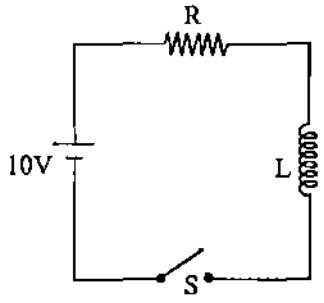
9. In the circuit shown below the resistance are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts. The resistor that dissipates the most power is - [2011]



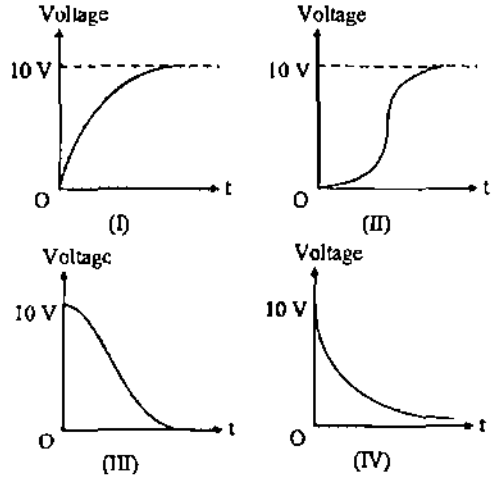
(A) R_1 (B) R_2

(C) R_3 (D) R_4

10. In the circuit shown, the switch is closed at time t = 0. [2011]



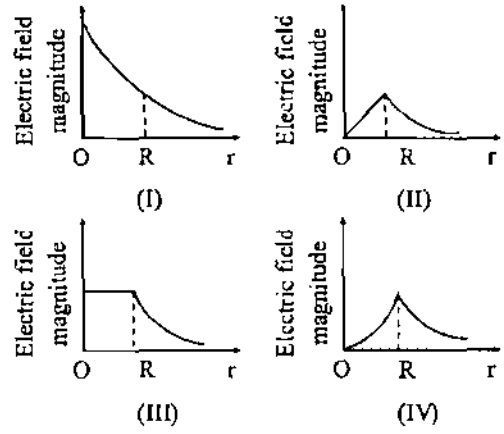
Which of the graphs shown below best represents the voltage across the inductor, as seen on an oscilloscope ?



(A) I (B) II

(C) III (D) IV

11. An isolated sphere of radius R contains uniform volume distribution of positive charge. Which of the curves shown below correctly illustrates the dependence of the magnitude of the electric field of the sphere as a function of the distance r from its centre ? [2011]



(A) I (B) II

(C) III (D) IV

12. The surface of a planet is found to be uniformly charged. When a particle of mass m and no charge is thrown at an angle from the surface of the planet, it has a parabolic trajectory as in projectile motion with horizontal range L. A particle of mass m and charge q, with the same initial conditions has a range L/2. The range of particle of mass m and charge 2q with the same initial conditions is- [2011]

(A) L (B) L/2 (C) L/3 (D) L/4

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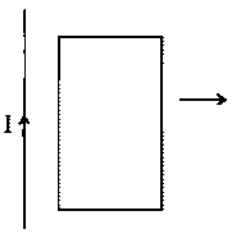
13. At a distance ℓ from a uniformly charged long wire, a charged particle is thrown radially outward with a velocity u in the direction perpendicular to the wire. When the particle reaches a distance 2ℓ from the wire its speed is found to be $\sqrt{2}\,u$. The magnitude of the velocity, when it is a distance 4ℓ away from the wire, is (ignore gravity) **[2011]**

(A) $\sqrt{3}\,u$ (B) 2u

(C) $2\sqrt{2}\,u$ (D) 4u

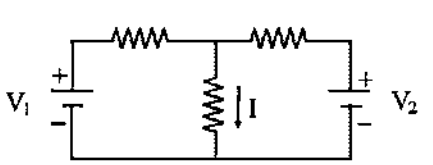
14. A rectangular loop of wire shown below is coplanar with a long wire carrying current I.



The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop ? **[2011]**

Induced current	Force on left side Force on right side
(A) Counterclockwise	To the left To the right
(B) Clockwise	To the left To the right
(C) Counterclockwise	To the right To the left
(D) Clockwise	To the right To the left

15. Two batteries V_1 and V_2 are connected to three resistors as shown below.

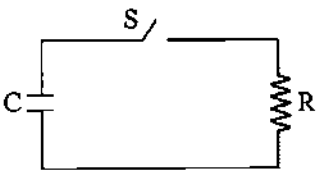


If $V_1 = 2V$ and $V_2 = 0$ V, the current I = 3 mA.
If $V_1 = 0$ V and $V_2 = 4V$, the current I = 4mA.
Now, if $V_1 = 10$ V and $V_2 = 10$ V, the current I will be- **[2011]**

(A) 7 mA (B) 15 mA

(C) 20 mA (D) 25 mA

16. The capacitor of capacitance C in the circuit shown in fully charged initially. Resistance is R.–



After the switch S is closed, the time taken to reduce the stored energy in the capacitor to half its initial value is **[2012]**

(A) RC/2 (B) 2RC ln 2

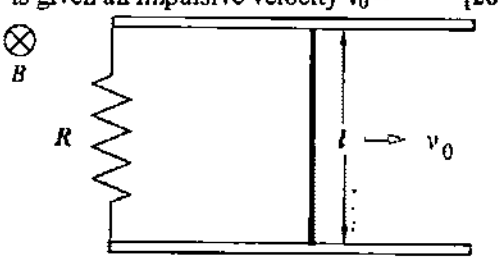
(C) RC ln 2 (D) $\frac{RC\ln 2}{2}$

17. A loop carrying current I has the shape of a regular polygon of n sides. If R is the distance from the centre to any vertex, then the magnitude of the magnetic induction vector $\vec{B}$ at the centre of the loop is – **[2012]**

(A) $n\frac{\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$ (B) $\frac{\mu_0 I}{2R}$

(C) $n\frac{\mu_0 I}{2\pi R}\tan\frac{2\pi}{n}$ (D) $\frac{\mu_0 I}{\pi R}\tan\frac{\pi}{n}$

18. A conducting rod of mass m and length l is free to move without friction on two parallel long conducting rails, as shown below. There is a resistance R across the rails. In the entire space around, there is a uniform magnetic field B normal to the plane of the rod and rails. The rod is given an impulsive velocity v_0 – **[2012]**



Finally, the initial energy $\frac{1}{2}mv_0^2$

(A) Will be converted fully into heat energy in the resistor

(B) Will enable rod to continue to move with velocity v_0 since the rails are frictionless.

(C) Will be converted fully into magnetic energy due to induced current

(D) Will be converted into the work done against the magnetic field

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19. A steady current I flows through a wire of radius r, length L and resistivity ρ. The current produces heat in the wire. The rate of heat loss in a wire is proportional to its surface area. The steady temperature of the wire is independent of– **[2012]**

(A) L (B) r
(C) I (D) ρ

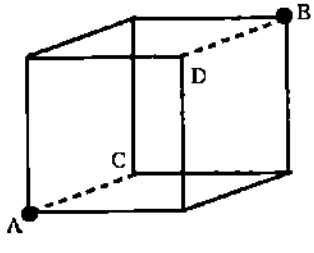
20. In one model of the electron, the electron of mass m_e is thought to be a uniformly charged shell of radius R and total charge e, whose electrostatic energy E is equivalent to its mass m_e via Einstein's mass energy relation $E = m_e c^2$. In this model, R is approximately ($m_e = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ ms^{-1}, $1/4\ \pi\varepsilon_0 = 9 \times 10^9$ Farad m^{-1}, magnitude of the electron charge = 1.6×10^{-19} C) – **[2012]**

(A) 1.4×10^{-15}m (B) 5.3×10^{-11}m
(C) 2×10^{-13}m (D) 2.8×10^{-35}m

21. Consider three concentric metallic spheres A, B and C of radii a, b, c respectively where a < b < c, A and B are connected whereas C is grounded. The potential of the middle sphere B is raised to V then the charge on the sphere C is- **[2012]**

(A) $-4\pi\varepsilon_0 V\frac{bc}{c-b}$ (B) $-4\pi\varepsilon_0 V\frac{ac}{c-a}$
(C) $+4\pi\varepsilon_0 V\frac{bc}{c-b}$ (D) zero

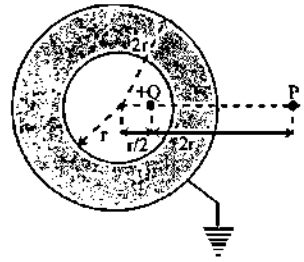
22. As shown in the figure below, a cube is formed with ten identical resistance R (thick lines) and two shorteing wires (dotted lines) along the arms AC and BD. **[2012]**



Resistance between point A and B is–

(A) R/2 (B) 5R/6
(C) 3R/4 (D) R

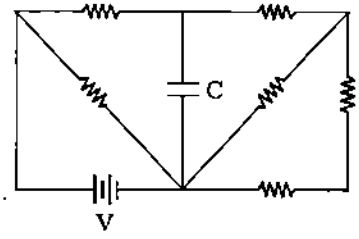
23. Consider an initially neutral hollow conducting spherical shell with inner radius r and outer radius 2r. A point charge +Q is now placed inside the shell at a distance r/2 from the centre. The shell is then grounded by connecting the outer surface to the earth. P is an external point at a distance 2r from the point charge +Q on the line passing through the centre and the point charge +Q as shown in the figure **[2013]**



The magnitude of the force on a test charge +q placed at P will be :

(A) $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{4r^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{9qQ}{100r^2}$
(C) $\frac{1}{4\pi\varepsilon_0}\frac{4qQ}{25r^2}$ (D) 0

24. Consider the circuit shown in the figure below :



All the resistors are identical. The charge stored in the capacitor, once it is fully charged, is **[2013]**

(A) 0 (B) $\frac{5}{13}$CV
(C) $\frac{2}{3}$CV (D) $\frac{5}{8}$CV

25. A blackbox (BB) which may contain a combination of electrical circuit elements (resistor, capacitor or inductor) is connected with other external circuit elements as shown below in the figure (a). After the switch (S) is closed at time t = 0, the current (I) as a function of time (t) is shown in the figure (b). **[2013]**

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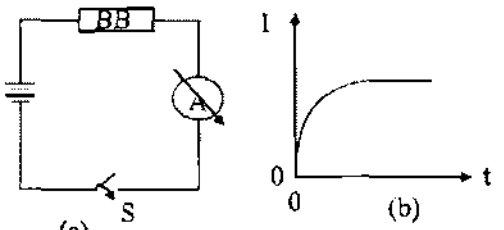
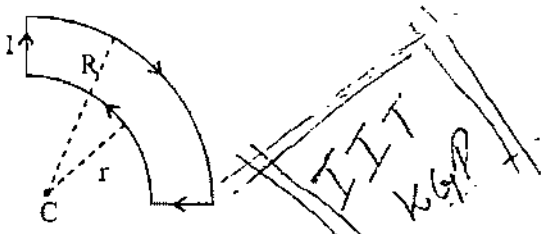


From this we can infer that the blackbox contains

(A) A resistor and a capacitor in series
(B) A resistor and a capacitor in parallel
(C) A resistor and an inductor in series
(D) A resistor and an inductor in parallel

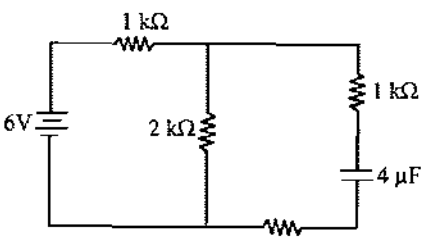
26. An arrangement with a pair of quarter circular coils of radii r and R with a common centre C and carrying a current I is shown. [2013]



The permeability of free space is μ_0. The magnetic field at C is :

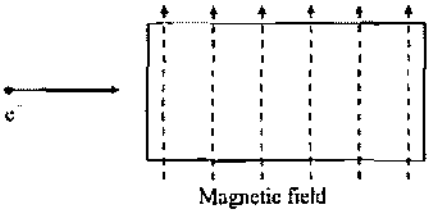
(A) $\mu_0 I (1/r - 1/R)/8$ into the page
(B) $\mu_0 I (1/r - 1/R)/8$ out of the page
(C) $\mu_0 I (1/r + 1/R)/8$ out of the page
(D) $\mu_0 I (1/r + 1/R)/8$ into the page

27. The circuit shown has been connected for a long time. The voltage across the capacitor is : [2013]



(A) 1.2 V (B) 2.0 V
(C) 2.4 V (D) 4.0 V

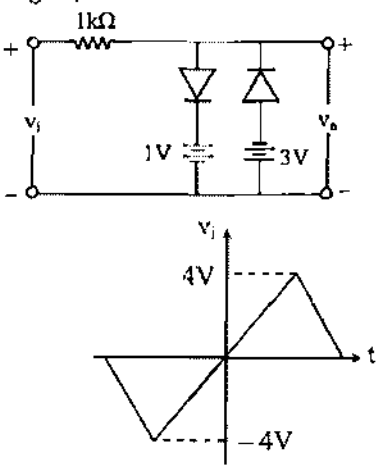
28. An electron enters a chamber in which a uniform magnetic field is present as shown. Ignore gravity.



During its motion inside the chamber [2013]

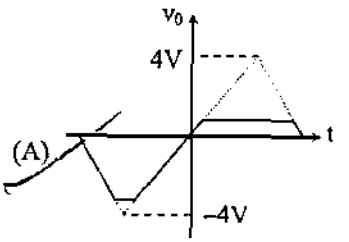
(A) the force on the electron remains constant
(B) the kinetic energy of the electron remains constant
(C) the momentum of the electron remains constant
(D) the speed of the electron increases at a uniform rate

29. The figure below shows a circuit and its input voltage v_i as function of time t. [2013]

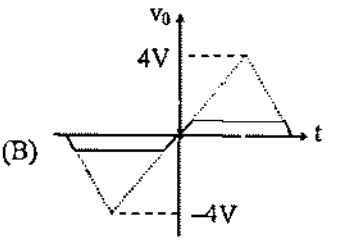


Assuming the diodes to be ideal, which of the following graphs depicts the output voltage v_0 as function of time t ?

(A)



(B)



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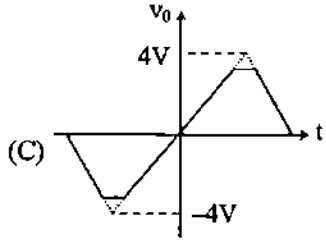
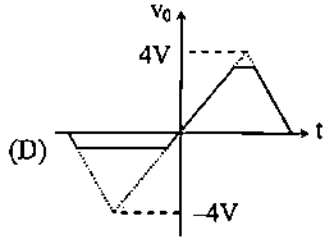
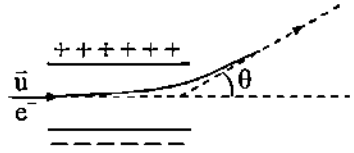
(C)


(D)


30. Two identical metallic square loops L_1 and L_2 are placed next to each other with their sides parallel on a smooth horizontal table. Loop L_1 is fixed and a current which increases as a function of time is passed through it. Then loop L_2 **[2014]**

(A) rotates about its center of mass
(B) moves towards L_1.
(C) remains stationary
(D) moves away from L_1

31. An electron enters a parallel plate capacitor with horizontal speed u and is fond to deflect by angle θ on leaving the capacitor as shown. It is found that $\tan\theta = 0.4$ and gravity is negligible **[2014]**



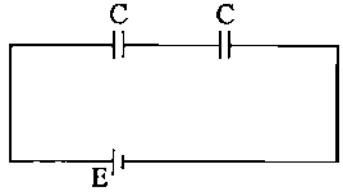
If the initial horizontal speed is doubled, then $\tan\theta$ will be

(A) 0.1 (B) 0.2 (C) 0.8 (D) 1.6

32. Consider a spherical shell of radius R with a total charge +Q uniformly spread on its surface (center of the shell lies at the origin x= 0). Two point charge, +q and – q are brought, one after the other, from far away and placed at $x = -a/2$ and $x = +a/2$ ($a < R$), respectively. Magnitude of the work done in this process is **[2014]**

(A) $(Q+q)^2/4\pi\varepsilon_0 a$ (B) zero
(C) $q^2/4\pi\varepsilon_0 a$ (D) $Qq/4\pi\varepsilon_0 a$

33. Two identical parallel plate capacitors of capacitance C each are connected in series with a battery of emf, E as shown. If one of the capacitors is now filled with a dielectric of dielectric constant k, the amount of charge which will flow through the battery is (neglect internal resistance of the battery) **[2014]**

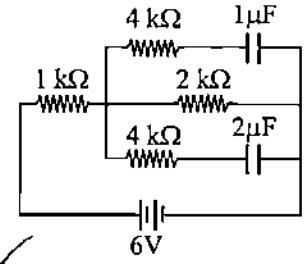


(A) $\frac{k+1}{2(k-1)}CE$ (B) $\frac{k-1}{2(k+1)}CE$
(C) $\frac{k-2}{k+2}CE$ (D) $\frac{k+2}{k-2}CE$

34. A certain p-n junction, having a depletion region of width 20 μm, was found to have a breakdown voltage of 100 V. If the width of the depletion region is reduced to 1 μm during its production, then it can be used as a Zener diode for voltage regulation of - **[2014]**

(A) 5 V (B) 10 V
(C) 7.5 V (D) 2000 V

35. What are the charges stored in the 1 μF and 2 μF capacitors in the circuit below, once the currents become steady ? **[2014]**



(A) 8 μC and 4 μC respectively
(B) 4 μC and 8 μC respectively
(C) 3 μC and 6 μC respectively
(D) 6 μC and 3 μC respectively

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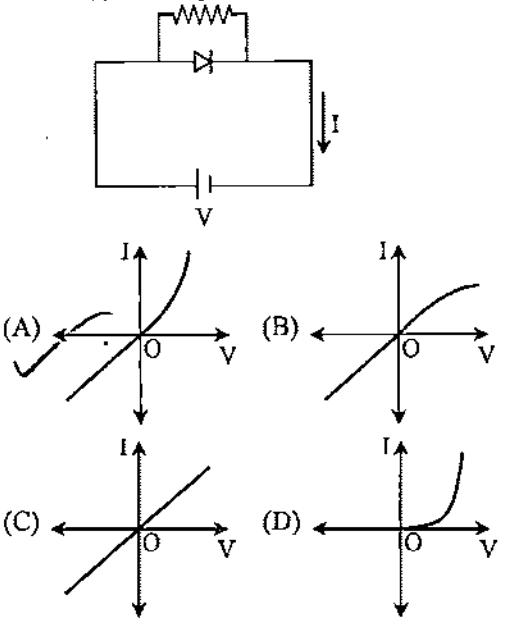
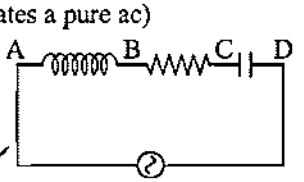
36. For a diode connected in parallel with a resistor, which is the most likely current (I) – voltage (V) characteristic ? **[2015]**



37. A point charge Q(= 3 × 10^{-12}C) rotates uniformly in a vertical circle of radius R =1 mm. The axis of the circle is aligned along the magnetic axis of the earth. At what value of the angular speed ω, the effective magnetic field at the center of the circle will be reduced to zero ? (Horizontal component of Earth's magnetic field is 30 micro Tesla) **[2015]**

(A) 10^{11} rad/s (B) 10^{9} rad/s
(C) 10^{13} rad/s (D) 10^{7} rad/s

38. An ac voltmeter connected between points A and B in the circuit below reads 36 V. If it is connected between A and C, the reading is 39 V. The reading when it is connected between B and D is 25 V. What will the voltmeter read when it is connected between A and D ? (Assume that the voltmeter reads true rms voltage values and that the source generates a pure ac) **[2015]**



(A) $\sqrt{481}$ V (B) 31V
(C) 61 V (D) $\sqrt{3361}$ V

39. A metallic ring of radius a and resistance R is held fixed with its axis along a spatially uniform magnetic field whose magnitude is $B_0 \sin(\omega t)$. Neglect gravity. Then, **[2015]**

(A) the current in the ring oscillates with a frequency of 2ω.
(B) the joule nearting loss in the ring is proportional to a^2,
(C) the force per unit length on the ring will be proportional to B_0^2.
(D) the net force on the ring is non-zero

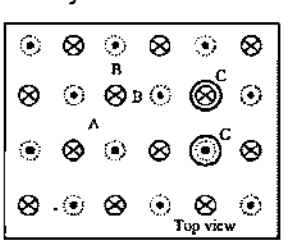
40. A wire bent in the shape of a regular n-polygonal loop carries a steady current I. Let ℓ be the perpendicular distance of a given segment and R be the distance of vertex both from the centre of the loop. The magnitude of the magnetic field at the centre of the loop is given by - **[2015]**

(A) $\frac{n\mu_0 I}{2\pi\ell}\sin(\pi/n)$ (B) $\frac{n\mu_0 I}{2\pi R}\sin(\pi/n)$
(C) $\frac{n\mu_0 I}{2\pi\ell}\cos(\pi/n)$ (D) $\frac{n\mu_0 I}{2\pi R}\cos(\pi/n)$

41. Two small metal balls of different mass m_1 and m_2 are connected by strings of equal length to a fixed point. When the balls are given equal charges, the angles that the two strings make with the vertical are 30° and 60°, respectively. The ratio m_1/m_2 is close to - **[2015]**

(A) 1.7 (B) 3.0
(C) 0.58 (D) 2.0

42. Consider the regular array of vertical identical current carrying wires (with direction of current flow as indicated in the figure below) protruding through a horizontal table. If we scatter some diamagnetic particles on the table, they are likely to accumulate - **[2015]**



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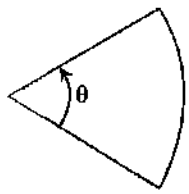
(A) around regions such as A
(B) around regions such as B
(C) in circular regions around individual wires such as C
(D) uniformly everywhere

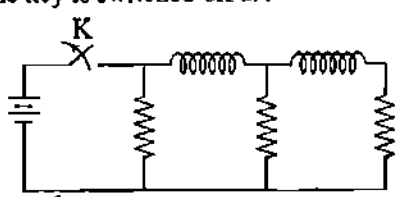
43. The distance between the vertex and the center of mass of a uniform solid planar circular segment of angular size θ and radius R is given by - **[2015]**



(A) $\frac{4}{3}R\frac{\sin(\theta/2)}{\theta}$ (B) $R\frac{\sin(\theta/2)}{\theta}$

(C) $\frac{4}{3}R\cos\left(\frac{\theta}{2}\right)$ (D) $\frac{2}{3}R\cos(\theta)$

44. In the circuit shown below, all the inductors (assumed ideal) and resistors are identical. The current through the resistance on the right is I after the key K has been switched on for along time. The currents through the three resistors (in order, from left to right) immediately after the key is switched off are - **[2015]**

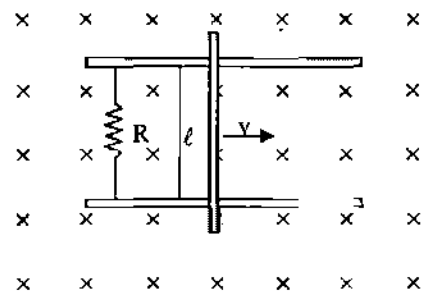


(A) 2I upwards, I downwards and I downwards
(B) 2I downwards, I downwards and I downwards
(C) I downwards, I downwards and I downwards
(D) 0, I downwards and I downwards

45. Physical processes are sometimes described visually by lines. Only the following can cross - **[2016]**

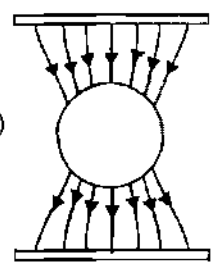
(A) Streamlines in fluid flow
(B) Lines of forces in electrostatics
(C) Rays in geometrical optics
(D) Lines of force in magnetism

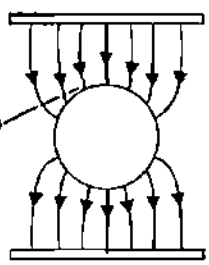
46. A conducting bar of mass m and length ℓ moves on two frictionless parallel rails in the presence of a constant uniform magnetic field of magnitude B directed into the page as shown in the figure . The bar is given an initial velocity v_0 towards the right at t = 0. Then the **[2016]**

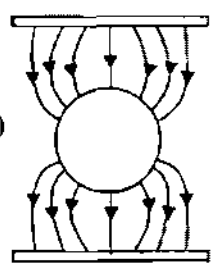


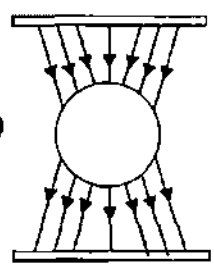
(A) Induced current in the circuit is in the clockwise direction
(B) Velocity of the bar decreases linearly with time
(C) Distance the bar travels before it comes to a complete stop is proportional to R
(D) Power generated across the resistance is proportional to ℓ

47. A metallic sphere is kept in between two oppositely charged plates. The most appropriate representation of the field lines is - **[2016]**

(A)


(B)


(C)


(D)


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48. Two infinitely long parallel wires carry currents of magnitude I_1 and I_2 and are at a distance 4 cm apart. The magnitude of the net magnetic field is found to reach a non-zero minimum values between the two wires and 1 cm away from the first wire. The ratio of the two currents and their mutual direction is **[2016]**

(A) $\frac{I_2}{I_1} = 9$, antiparallel

(B) $\frac{I_2}{I_1} = 9$, parallel

(C) $\frac{I_2}{I_1} = 3$, antiparallel

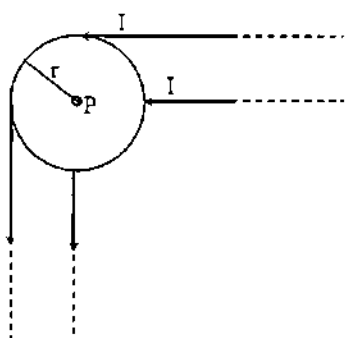
(D) $\frac{I_2}{I_1} = 3$, parallel

49. Consider a cube of uniform charge density ρ. The ratio of electrostatic potential at the centre of the cube to that at one of the corners of the cube is **[2016]**

(A) 2 (B) $\sqrt{3}/2$

(C) $\sqrt{2}$ (D) 1

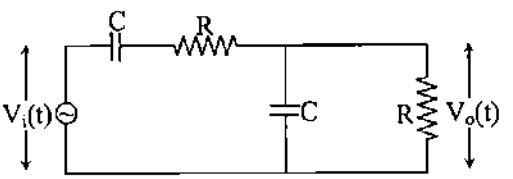
50. Two infinitely long wires each carrying current I along the same direction are made into the geometry as shown in the figure. The magnetic field at the point P is **[2016]**



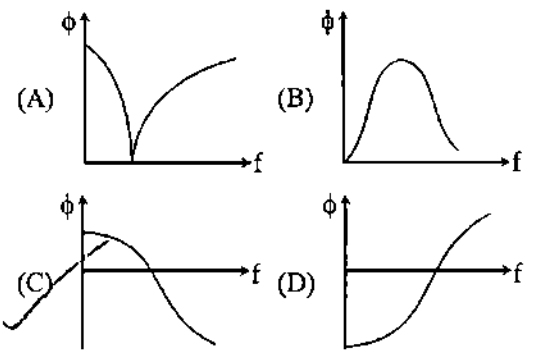
(A) $\frac{\mu_0 I}{\pi r}$ (B) $\frac{\mu_0 I}{r}\left(\frac{1}{\pi}+\frac{1}{4}\right)$

(C) Zero (D) $\frac{\mu_0 I}{2\pi r}$

51. If the input voltage V_i to the circuit below is given by $V_i(t) = A\cos(2\pi ft)$, the output voltage is given by $V_o(t) = B\cos(2\pi ft + \phi)$ -



Which one of the following four graphs best depict the variation of ϕ vs f? **[2016]**



52. If some charge is given to a solid metallic sphere, the field inside remains zero and by Gauss's law all the charge resides on the surface. Suppose now that Colomb's force between two charges varies as $1/r^3$. Then, for a charged solid metallic sphere **[5 Nov, 2017]**

(A) field inside will be zero and charge density inside will be zero.

(B) field inside will not be zero and charge density inside will not be zero.

(C) field inside will not be zero and charge density inside will be zero.

(D) field inside will be zero and charge density inside will not be zero.

53. A bird sitting on a single high tension wire does not get electrocuted because **[5 Nov, 2017]**

(A) the circuit is not complete.

(B) the bird feet has an insulating covering.

(C) capacitance of the bird is too small and the line frequency is too small.

(D) resistance of the bird is too high

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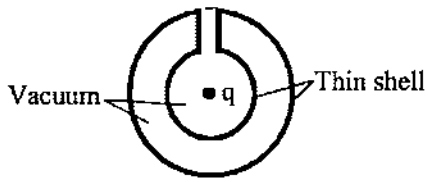
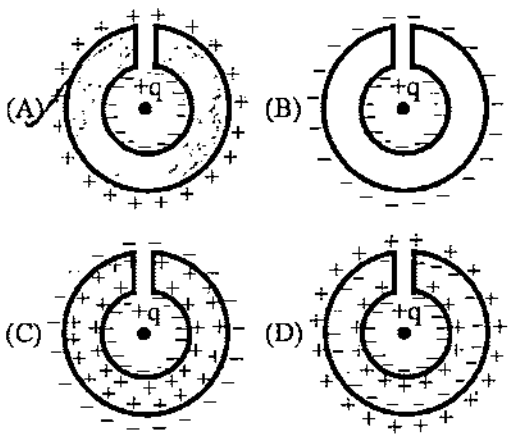
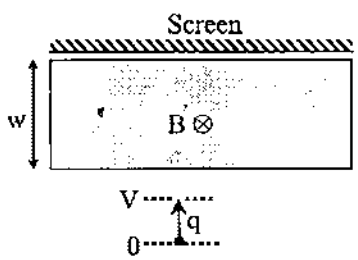
54. A positive charge q is placed at the center of a neutral hollow cylindrical conducting shell with its cross section as shown in the figure below. **[5 Nov, 2017]**



Which one of the following figures correctly indicates the induced charge distribution on the conductor (ignore edge effects).



55. A rectangular region of dimensions $w \times l$ ($w \ll l$) has a constant magnetic field into the plane of the paper as shown. On one side the region is bounded by a screen. On the other side positive ions of mass m and charge q are accelerated from rest and towards the screen by a parallel plate capacitor at constant potential difference $V < 0$, and come out through a small hole in the upper plate. Which one of the following statements is correct regarding the charge on the ions that hit the screen ? **[5 Nov, 2017]**



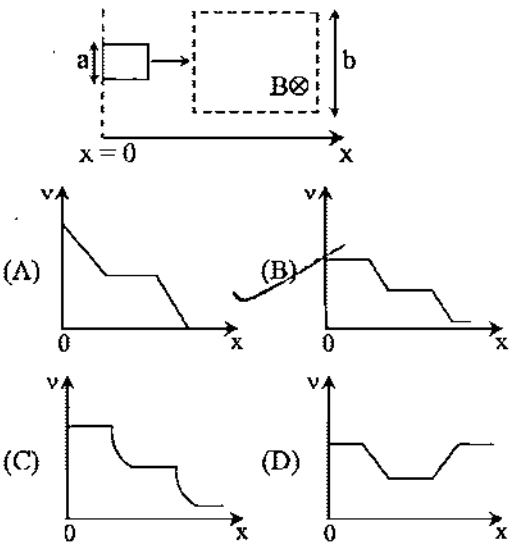
(A) Ions with $q > \frac{2|v|m}{B^2w^2}$ will hit the screen.

(B) Ions with $q < \frac{2|v|m}{B^2w^2}$ will hit the screen.

(C) All ions will hit the screen.

(D) Only ions with $q = \frac{2|v|m}{B^2w^2}$ will hit the screen.

56. A square-shaped conducting wire loop of dimension a moving parallel to the x-axis approaches a square region of size b ($a < b$) where a uniform magnetic field B exists pointing into the plane of the paper (see figure). As the loop passes through this region, the plot correctly depicting its speed (v) as a function of x is **[5 Nov, 2017]**



57. Let the electrostatic field E at distance r from a point charge q not be an inverse square but, instead an inverse cubic, e.g. $\vec{E} = k\frac{q}{r^3}\hat{r}$

Here k is a constant. Consider the following two statements **[19 Nov, 2017]**

(i) Flux through a spherical surface enclosing the charge is $\phi = q_{enclosed}/\epsilon_0$

(ii) A charge placed inside uniformly charged shell will experience a force.

Choose the correct option.

(A) Only (i) is valid

(B) Only (ii) is valid

(C) Both (i) and (ii) are invalid

(D) Both (i) and (ii) are valid

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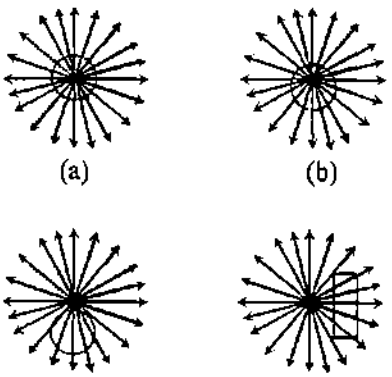
58. The black shapes in the figure below are closed surfaces. The electric field lines are in red. For which case the net flux through the surfaces is non-zero ? **[19 Nov, 2017]**



(A) In all cases net flux is non-zero
(B) Only (c) and (d)
(C) Only (a) and (b)
(D) Only (b), (c) and (d)

59. A particle of charge q and mass m enters a region of a transverse electric field of $E_0\hat{j}$ with initial velocity $v_0\hat{i}$. The time taken for the change in the de Broglie wavelength of the charge from the initial value of λ_0 to $\lambda_0/3$ is proportional to **[19 Nov, 2017]**

(A) $\frac{q}{m}$ (B) $\frac{m}{q}$

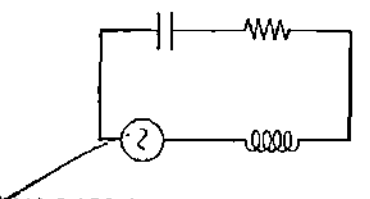
(C) $\sqrt{\frac{q}{m}}$ (D) $\sqrt{\frac{m}{q}}$

60. A square-shaped wire loop of mass m, resistance R and side 'a' moving with speed v_0, parallel to the x-axis, enters a region of uniform magnetic field B, which is perpendicular to the plane of the loop. The speed of the loop changes with distance x ($x < a$) in the filed, as **[19 Nov, 2017]**

(A) $v_0 - \frac{B^2a^2}{Rm}x$ (B) $v_0 - \frac{B^2a^2}{2Rm}x$

(C) $v_0 - \frac{B^2a}{Rm}x^2$ (D) v_0

61. Consider the R-L-C circuit given below. The circuit is driven by a 50 Hz AC source with peak voltage 220 V. If R = 400 Ω, C = 200 μF and L = 6 H, the maximum current in the circuit is closest to **[19 Nov, 2017]**



(A) 0.120 A (B) 0.55 A
(C) 1.2 A (D) 5.5 A

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ANSWERS

1. (D)	2. (A)	3. (D)	4. (A)	5. (B)	6. (C)	7. (B)
8. (C)	9. (A)	10. (D)	11. (B)	12. (C)	13. (A)	14. (B)
15. (D)	16. (D)	17. (A)	18. (A)	19. (A)	20. (A)	21. (A)
22. (A)	23. (D)	24. (D)	25. (C)	26. (B)	27. (D)	28. (B)
29. (A)	30. (D)	31. (A)	32. (C)	33. (B)	34. (A)	35. (B)
36. (A)	37. (A)	38. (A)	39. (C)	40. (A)	41. (A)	42. (A)
43. (A)	44. (A)	45. (C)	46. (C)	47. (B)	48. (A)	49. (A)
50. (D)	51. (C)	52. (D)	53. (C)	54. (A)	55. (B)	56. (B)
57. (B)	58. (C)	59. (B)	60. (A)	61. (A)		

SOLUTIONS

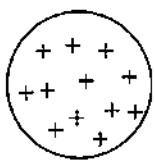
1. Using the equation $I = I_0\,(1 - e^{-\frac{t}{t_L}})$

Put $t = 0$ we get $I = 0$

Just when the battery is closed inductor provides infinite resistance to the current flow

$\therefore$ current is zero initially.

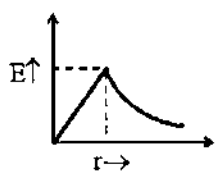
2.



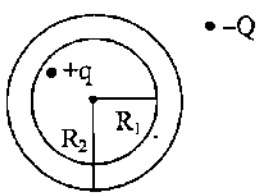
$E_r = \dfrac{\rho r}{3\varepsilon_0}$ for $0 \le r < R$; $E_r = \dfrac{kQ}{r^2}$ for $r \ge R$

$\therefore E \propto r$ for $r < R$ and $E \propto \dfrac{1}{r^2}$ for $r \ge R$

$\therefore$ curve is

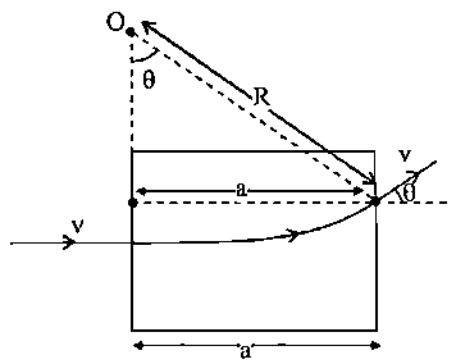


3.



For a conductor electric field inside its cavity is only due to inside charge and not due to outside charge.

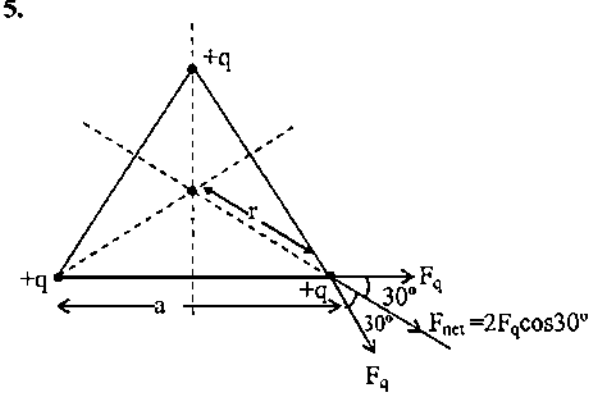
4.



Now $\sin\theta = \dfrac{a}{R}$, $R = \dfrac{mv}{qB}$

$\therefore \boxed{v = \dfrac{qBa\cot\theta}{m}}$

5.



$F(r) = kr$

now F_{net} on a particle is $2F_q \cos 30°$ due to the other two charges

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$$F_{net} = \frac{2kq^2}{a^2} \times \frac{\sqrt{3}}{2}$$

$$\text{also } r = \frac{2}{3}\left(\frac{\sqrt{3}}{2}a\right)$$

$\therefore a = \sqrt{3}\, r$ replacing it in F_{net} we get

$$F_{net} = \frac{2kq^2}{(\sqrt{3}r)^2} \times \left(\frac{\sqrt{3}}{2}\right) = \frac{kq^2}{\sqrt{3}\, r^2}$$

this is balanced by F(r)

$$\therefore F(r) = F_{net} \Rightarrow kr = \frac{1 \times q^2}{4\pi\varepsilon_0 \times \sqrt{3}\, r^2}$$

$$\therefore \quad r = \left(\frac{\sqrt{3}\, q^2}{12\pi\varepsilon_0 k}\right)^{1/3}$$

6. Let the equivalent impedance of the circuit be Z

So $Z = \omega L + Z'$

$$\text{now } Z' = \frac{ZX_C}{Z + X_C}$$

$$\therefore \quad Z = \omega L + \left(\frac{Z \times \frac{1}{\omega C}}{Z + \frac{1}{\omega C}}\right)$$

$$\text{on solving we get } Z = \frac{\omega LC \pm \sqrt{(\omega LC)^2 - 4LC}}{2C}$$

for Z to be purely inductive

$$\omega^2 L^2 C^2 - 4LC = 0 \text{ or } \boxed{\omega = \frac{2}{\sqrt{LC}}}$$

7. Using law of conservation of mechanical energy

Initial K.E. = Final P.E.

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{kq^2}{r}$$

$$\therefore r = \frac{q^2}{4\pi\varepsilon_0 mv^2}$$

8. R.M.S. value = 220 V

Peak value = $220\sqrt{2}$

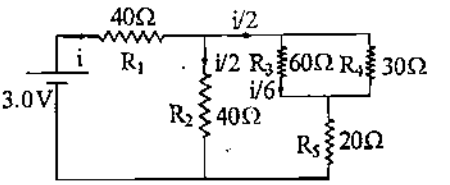
$\omega = 2\pi n$

$= 2\pi \times 50$

$= 100\,\pi$

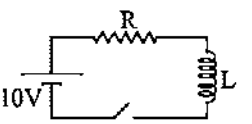
$V(t) = 220\sqrt{2}\cos(100\pi t)$

9.



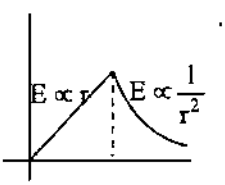
Power dissipate in R_1 is maximum as its current is maximum and its resistance is also 40Ω which is higher than R_5R_4.

10. $|e| = \frac{V}{R}e^{-\frac{R}{L}\times t}$



11. When $r < R$ $\quad E = \frac{\rho r}{3\epsilon_0}$

When $r > R$ $\quad E = \frac{Q}{4\pi\epsilon_0 r^2}$



12. For uncharged particle

$$L = \frac{u^2 \sin 2\theta}{g} \quad \ldots (i)$$

Range for particle of mass m and charge q.

$$\frac{L}{2} = \frac{u^2 \sin 2\theta}{g + \frac{qE}{m}} \quad \ldots . (ii)$$

From (i) and (ii)

$$\frac{u^2 \sin 2\theta}{2g} = \frac{u^2 \sin 2\theta}{g + \frac{qE}{m}}$$

$\Rightarrow mg = qE$

Range of particle of mass m & charge 2q.

$$R = \frac{u^2 \sin 2\theta}{g + \frac{2qE}{m}} = \frac{u^2 \sin 2\theta}{g\left(1 + \frac{2qE}{mg}\right)} = \frac{L}{3}$$

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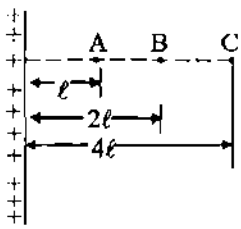
13.



energy conservation at A & B

$$qV_A + \frac{1}{2}mu^2 = qV_B + \frac{1}{2}m \times 2u^2$$

$$q[V_A - V_B] = \frac{1}{2}mu^2$$

$$q \times \frac{\lambda}{2\pi\epsilon_0}\ln 2 = \frac{1}{2}mu^2$$

energy conservation at A & C

$$qV_A + \frac{1}{2}mu^2 = qV_C + \frac{1}{2}mv^2$$

$$q[V_A - V_C] + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

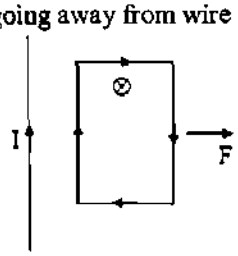
$$\frac{q\lambda}{2\pi\epsilon_0}\ln 4 + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

$$\frac{2q\lambda}{2\pi\epsilon_0}\ln 2 + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

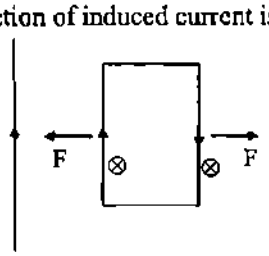
$$mu^2 + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

$$\frac{3}{2}u^2 = \frac{1}{2}v^2 \Rightarrow v = \sqrt{3}\,u$$

14. Flux is inward and it is decreasing as loop is going away from wire

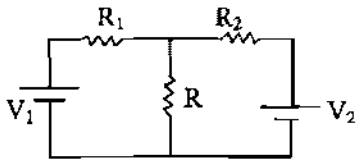


∴ direction of induced current is clockwise



Force on left side is in left and force on right side is in right.

15.



$$V_{eq} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow \frac{V_1R_2 + V_2R_1}{R_1 + R_2}$$

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

$$I = \frac{V_{eq}}{R + R_{eq}}$$

In each case R_{eq} & R is same only V_1 & V_2 is changing ∴ V_{eq} is changing

Case - 1 $V_{eq} = \dfrac{2 \times R_2 + 0 \times R_1}{R_1 + R_2}$

$[V_1 = 2\ ,\ V_2 = 0]$

$$V_{eq} = \frac{2R_2}{R_1 + R_2}$$

Case - 2 $V_{eq} = \dfrac{4R_1}{R_1 + R_2}$

$[V_1 = 0\ ,\ V_2 = 4]$

$$\frac{I_1}{I_2} = \frac{3}{4} = \frac{2R_2}{4R_1} \qquad \frac{R_2}{R_1} = \frac{3}{2}$$

Case - 3 $V_{eq} = \dfrac{10R_1 + 10R_2}{R_1 + R_2}$

$$\frac{3}{I'} = \frac{2R_2}{10(R_1 + R_2)} \Rightarrow \frac{3}{I'} = \frac{2 \times 1.5R_1}{10(2.5R_1)}$$

or $I' = 25$ mA

16. Discharging -

$$Q = Q_0 e^{-t/RC},$$

$$U = \frac{Q^2}{2C} = \frac{Q_0^2}{2C}e^{-2t/RC} \quad \text{or} \quad U = U_0e^{-2t/RC}$$

$$\frac{U_0}{2} = U_0e^{-2t/RC} \Rightarrow t = \frac{RC\ell n2}{2}$$

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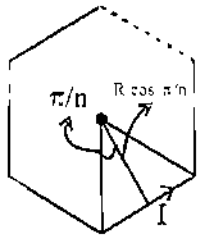
17.



$$B_{net} = n \times B_1 = n.\frac{\mu_0}{4\pi}.\frac{1}{R\cos\frac{\pi}{n}}2\sin\frac{\pi}{n}$$

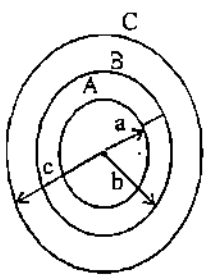
18. Due to energy conservation

19. Concept of fuse wire

20. $\frac{e^2}{8\pi\varepsilon_0 R} = m_e c^2$

solving for R

21.



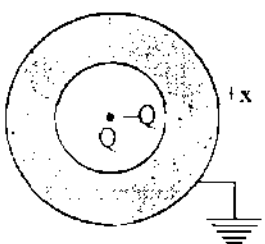
$$V_B = \frac{kq}{b} + \frac{k(-q)}{c} = V \quad \text{(Given)}$$

$$q = \frac{4\pi\varepsilon_0 bc}{c-b}.V$$

Charge on C = –q

22. The given circuit can be simplified into two wheatstone bridge in parallel

23.



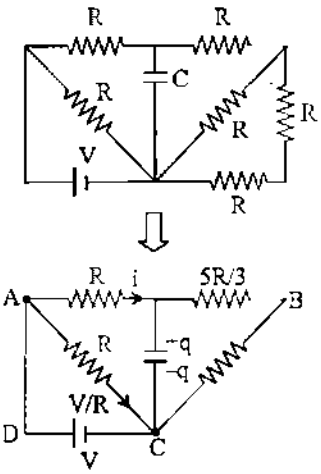
Potential of sphere = 0

$\therefore$ x = 0

Electric field of +Q & – Q get cancel out at point outside of sphere.

$\therefore$ Force on P = 0

24.



Kirchhoff voltage law in loop ABCD

$$iR + \frac{5R}{3}i - V = 0$$

$$\frac{8R}{3}i = V$$

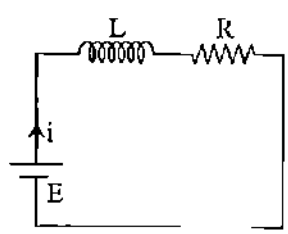
$$i = \frac{3V}{8R}$$

$$\frac{q}{C} = i \times \frac{5R}{3}$$

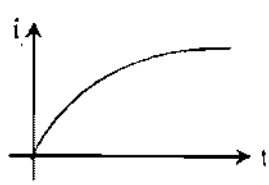
$$q = Ci \times \frac{5R}{3}$$

$$= C \times \frac{5R}{3} \times \frac{3V}{8R} = \frac{5CV}{8}$$

25.



$$i = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$



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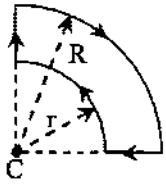
26.



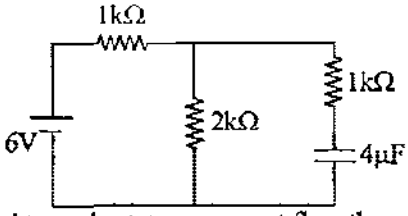
$$B_{at\,C} = \frac{\mu_0 i}{4\pi r}\times\frac{\pi}{2} \quad \frac{\mu_0 i}{4\pi R}\times\frac{\pi}{2}$$

$\odot$ $\otimes$

$$\Rightarrow \frac{\mu_0 i}{8}\left[\frac{1}{r}-\frac{1}{R}\right] \odot$$

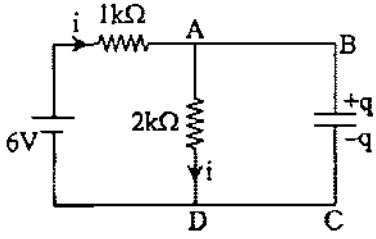
Out of page

27.



At steady state no current flow through capacitor

$\therefore$ 1kΩ behave as conducting wire.



$$i = \frac{6}{(2+1)\times 10^3} = 2\times 10^{-3}\ \text{Amp.}$$

Potential drop across c = potential difference between A & D

$\Rightarrow$ $i \times 2 \times 10^3$

$\Rightarrow$ 4 volt

28. $F = qVB \sin 90$

F remain constant in magnitude but direction change

$\therefore$ $\vec{F}$ is not constant.

F is perpendicular to electron velocity

$\therefore$ KE remain constant

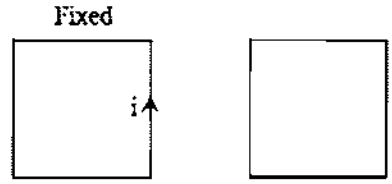
as F is acting

$\therefore$ Momentum will change.

as KE remain constant

$\therefore$ speed remain same

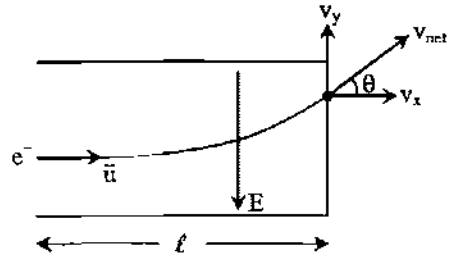
30.



When current through L_1 increases then flux linked through L_2 will increase.

$\therefore$ According to lenz law L_2 will move away.

31.



Horizontal displacement = ℓ

$$t = \frac{\ell}{u}$$

$$v_y = u_y + at$$

$$= 0 + \frac{eE}{m}\times\frac{\ell}{u}$$

$$v_y = \frac{eE}{m}\times\frac{\ell}{u}$$

v_x remain same and it is equal to u

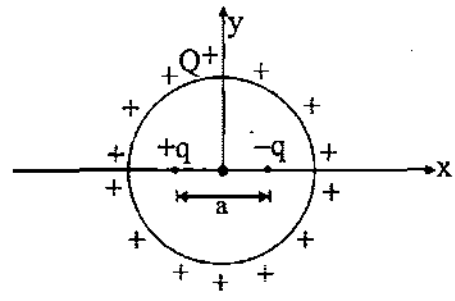
$$\tan\theta = \frac{v_y}{v_x} = \frac{eE}{m}\frac{\ell}{u}\times\frac{1}{u} = \frac{eE\ell}{mu^2}$$

$$\tan\theta \propto \frac{1}{u^2}$$

When speed u is doubled then $\tan\theta$ will become $\frac{1}{4}$th.

$$\therefore \tan\theta = \frac{0.4}{4} = 0.1$$

32.



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PE_i = Initial energy of system = $\frac{Q^2}{8\pi\varepsilon_0 R}$

(self energy of shell)

PE_f = Final energy of system

$= \frac{Q^2}{8\pi\varepsilon_0 R} + \frac{q\times(-q)}{4\pi\varepsilon_0 a} + \frac{kQ\times q}{R} + \frac{kQ(-q)}{R}$

$\Rightarrow \frac{Q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{4\pi\varepsilon_0 a}$

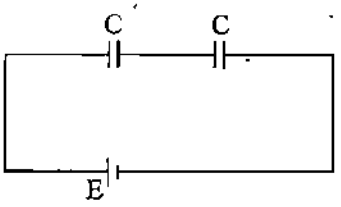
(self energy of shell) (Interaction energy between various charges)

Work done = $PE_f - PE_i$

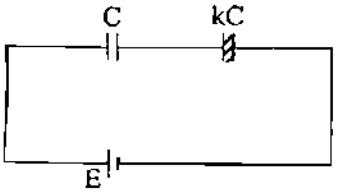
$= \frac{-q^2}{4\pi\varepsilon_0 a}$

Magnitude of work done = $\frac{q^2}{4\pi\varepsilon_0 a}$

33.



Initial charge on both C = $\frac{CE}{2}$



New charge on each C = $\left(\frac{kC}{k+1}\right)E$

Change in charge on C is supplied by battery

$\therefore$ Charge supply by battery = $\left(\frac{kC}{k+1}\right)E - \frac{CE}{2}$

$\Rightarrow CE\left[\frac{k}{k+1} - \frac{1}{2}\right]$

$\Rightarrow CE\left[\frac{k-1}{2(k+1)}\right]$

Charge passes through battery is change supply by battery

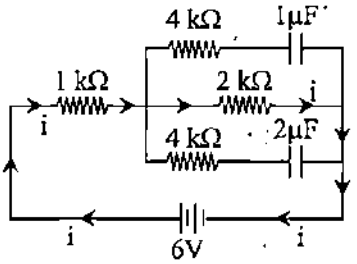
$\therefore$ Ans. $CE\left[\frac{k-1}{2(k+1)}\right]$

34. Break down voltage is proportional to width of depletion region.

$\therefore$ When width reduce to 1 μm thus become $\frac{1}{20}$ times then break down voltage also become $\frac{1}{20}$ tim. s thus it become 5 volt.

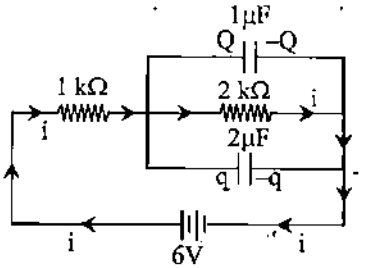
So Zener diode can used for voltage regulation of 5 volt.

35.



At steady state current does not flow in the branch of capacitor.

$\therefore$ we can replace all resistor connected in branch of capacitor with wire new circuit is



$i = \frac{6}{(2+1)\times 10^3}$

$\Rightarrow$ 2 mA

Potential drop across 2kΩ is same as potential drop across 1 μF & 2 μF.

Potential drop across 2 kΩ = $i \times 2 \times 10^3$

$= 2 \times 10^{-3} \times 2 \times 10^3 = 4$ volt.

Charge on

$1\mu F = Q = 1 \times 4 \times 10^{-6} = 4\ \mu C$

Charge on

$2\mu F = q = 2 \times 4 \times 10^{-6} = 8\ \mu C$

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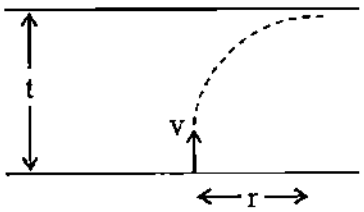
54. Option 'A' is correct option. According charge conservation & Gauss's law.

55.



Ions will hit if r > w.

$w = q\Delta V$

$\frac{1}{2}mv^2 = qV$

$v = \sqrt{\frac{2qV}{m}}$

$\frac{mv^2}{r} = qvB$

$\Rightarrow r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}}$

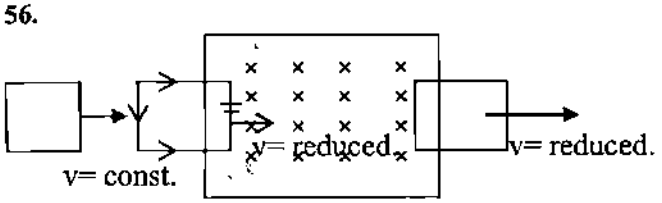
$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

$\frac{1}{B}\sqrt{\frac{2mV}{q}} > w$

$\frac{2mV}{q} > w^2B^2$

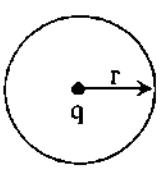
$\Rightarrow q < \frac{2mV}{w^2B^2}$

56.



inside B speed will be constant therefore B option is correct, representation of speed

57.

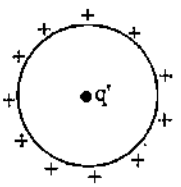


$|\vec{E}| = \frac{kq}{r^3}$

$\therefore d\phi = \vec{E} \cdot \vec{dS}$

$\phi = \int d\phi = \int \vec{E}.\vec{dS} = \frac{kq}{r^3} \cdot 4\pi r^2$

$\phi = \frac{kq4\pi}{r} \neq \frac{q_{en}}{\varepsilon_0}$

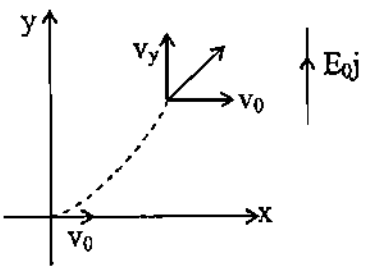


force on q' is zero

58. $\phi = \frac{q_{en}}{\varepsilon_0}$

q_{en} is not zero for a and b. therefore flux (ϕ) is not zero.

59.



$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{v}$

$v = \sqrt{v_y^2 + v_0^2}$

$v_y = u_y + a_y t$

$v_y = 0 + \frac{qE_0}{m}t$

$(3v_0) = \sqrt{v_y^2 + v_0^2}$

$\Rightarrow v_y^2 = 8v_0^2$

$\Rightarrow \frac{qE_0}{m}t = 2\sqrt{2}\,v_0$

$\Rightarrow t = \frac{2\sqrt{2}m}{qE_0}v_0$

$t \propto \frac{m}{q}$

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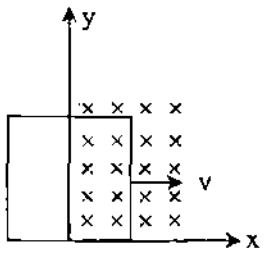
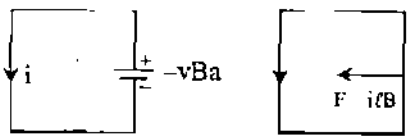
60.



$v_{emf} = v\,Ba$



$i = vBa/R$

$$F = i\ell B = \frac{vBa \cdot Ba}{R}$$

$$ma = \frac{vB^2a^2}{R}$$

$$\frac{-mdv}{dt} = \frac{vB^2a^2}{R}$$

$$\Rightarrow -m\frac{vdv}{dx} = v\,\frac{B^2a^2}{R}$$

$$-m\int_{v_0}^{v} dv = \frac{B^2a^2}{R}\int_{0}^{x} dx$$

$$-m(v - v_0) = \frac{B^2a^2x}{R}$$

$$\Rightarrow v = v_0 - \frac{B^2a^2}{mR}x$$

61. $X_L = \omega L = 2\pi \times 50 \times 6 = 600\,\pi\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi 80 \times 200 \times 10^{-6}} = \frac{100}{2\pi} = \frac{50}{\pi}\ \Omega$$

$$I_{max} = \frac{V}{z} \quad \frac{220}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{220}{\sqrt{400^2 + \left(600\pi - \frac{50}{\pi}\right)^2}}$$

$I_{max} = 0.120$ A

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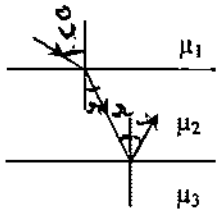
Chapter 4

Optics

1. The following travelling electromagnetic wave $E_x = 0$, $E_y = E_0\sin(kx + \omega t)$, $E_z = -2E_0\sin(kx - \omega t)$ is- **[2010]**
(A) elliptically polarized
(B) circularly polarized
(C) linearly polarized
(D) unpolarized

2. A point source of light is placed at the bottom of a vessel which is filled with water of refractive index μ to a height h. If a floating opaque disc has to be placed exactly above it so that the source is invisible from above, the radius of the disc should be- **[2010]**
(A) $\frac{h}{\sqrt{\mu-1}}$ (B) $\frac{h}{\sqrt{\mu^2-1}}$
(C) $\frac{h}{\mu^2-1}$ (D) $\frac{\mu h}{\sqrt{\mu^2-1}}$

3. Three transparent media of refractive indices μ_1, μ_2, μ_3 respectively, are stacked as shown. A ray of light follows the path shown. No light enters the third medium. **[2010]**



Then-
(A) $\mu_1 < \mu_2 < \mu_3$ (B) $\mu_2 < \mu_1 < \mu_3$
(C) $\mu_1 < \mu_3 < \mu_2$ (D) $\mu_3 < \mu_1 < \mu_2$

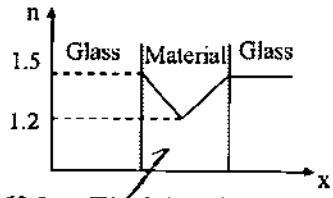
4. In Young's double slit experiment, the distance between the two slits is 0.1 mm, the distance between the slits and the screen is 1 m and the wavelength of the light used is 600 nm. The intensity at a point on the screen is 75% of the maximum intensity. What is the smallest distance of this point from the central fringe ? **[2010]**
(A) 1.0 mm (B) 2.0 mm
(C) 0.5 mm (D) 1.5 mm

5. A narrow parallel beam of light falls on a glass sphere of radius R and refractive index μ at normal incidence. The distance of the image from the outer edge is given by- **[2010]**
(A) $\frac{R(2-\mu)}{2(\mu-1)}$ (B) $\frac{R(2+\mu)}{2(\mu-1)}$
(C) $\frac{R(2-\mu)}{2(\mu+1)}$ (D) $\frac{R(2+\mu)}{2(\mu+1)}$

6. A plane polarized light passed through successive polarizers which are rotated by 30° with respect to each other in the clockwise direction. Neglecting absorption by the polarizers and given that the first polarizer's axis is parallel to the plane of polarization of the incident light, the intensity of light at the exit of the fifth polarizer is closest to - **[2011]**
(A) same as that of the incident light
(B) 17.5% of the incident light
(C) 30% of the incident light
(D) zero

7. In a Young's double slit set-up, light from a laser source falls on a pair of very narrow slits separated by 1.0 micrometer and bright fringes separated by 1.0 millimeter are observed on a distant screen. If the frequency of the laser light is doubled, what will be the separation of the bright fringes ? **[2011]**
(A) 0.25 mm (B) 0.5 mm
(C) 1.0 mm (D) 2.0 mm

8. A material is embedded between two glass plates. Refractive index n of the material varies with thickness as shown below. The maximum incident angle (in degrees) on the material for which beam will pass through the material is- **[2011]**



(A) 60.0 (B) 53.1 (C) 43.5 (D) 32.3

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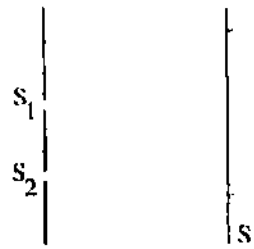
9. In a Young's double slit experiment the intensity of light at each slit is I_0. Interference pattern in observed along a direction parallel to the line $S_1 S_2$, on screen S.– **[2012]**



The minimum, maximum, and the intensity averaged over the entire screen are respectively

(A) $0, 4I_0, 2I_0$ (B) $0, 4I_0, I_0$

(C) $I_0, 2I_0, 3I_0/2$ (D) $0, 2_0, I_0$

10. An unpolarized beam of light of intensity I_0 passes through two linear polarizers making an angle of 30° with respect to each other. The emergent beam will have an intensity – **[2012]**

(A) $\frac{3I_0}{4}$ (B) $\frac{\sqrt{3}I_0}{4}$

(C) $\frac{3I_0}{8}$ (D) $\frac{I_0}{8}$

11. On a bright sunny day a diver of height h stands at the bottom of a lake of depth H. Looking upward, he can see objects outside the lake in a circular region of radius R. Beyond this circle he sees the image of objects lying on the floor of the lake. If refractive index of water is 4/3, then the value of R is– **[2012]**

(A) $3(H-h)/\sqrt{7}$ (B) $(H-h)/\sqrt{7/3}$

(C) $3h\sqrt{7}$ (D) $(H-h)/\sqrt{5/3}$

12. A ray of light incident on a glass sphere (refractive index $\sqrt{3}$) suffers total internal reflection before emerging out exactly parallel to the incident ray. The angle of incidence was **[2013]**

(A) 75° (B) 30°

(C) 45° (D) 60°

13. Monochromatic light passes through a prism. Compared to that in air, inside the prism the light's **[2014]**

(A) speed and wavelength are different but frequency remains same.

(B) speed and frequency are different but wavelengtL remains same.

(C) wavelength and frequency are different, but speed remains same.

(D) speed, wavelength and frequency are all different.

14. The flat face of a plano-convex lens of focal length 10 cm is silvered. A point source placed 30 cm in front of the curved surface will produce a **[2014]**

(A) real image 15 cm away from the lens

(B) real image 6 cm away from the lens

(C) virtual image 15 cm away from the lens

(D) virtual image 6 cm away from the lens

15. A concave mirror of radius of curvature R has a circular outline of radius r. A circular disc is to be placed normal to the axis at the focus so that it collects all the light that is reflected from the mirror from a beam parallel to the axis. For $r \ll R$, the area of this disc has to be at least **[2014]**

(A) $\frac{\pi r^6}{4R^4}$ (B) $\frac{\pi r^4}{4R^2}$

(C) $\frac{\pi r^5}{4R^3}$ (D) $\frac{\pi r^4}{R^2}$

16. The angles of incidence and refraction of a monochromatic ray of light of wavelength λ at an air-glass interface are i and r, respectively. A parallel beam of light with a small spread $\delta\lambda$ in wavelength about a mean wavelength λ is refracted at the same air-glass interface. The refractive index μ of glass depends on the wavelength λ as $\mu(\lambda) = a + b/\lambda^2$ where a and b are constants. Then the angular spread in the angle of refraction of the beam is **[2014]**

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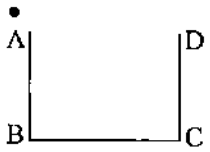
(A) $\left|\frac{\sin i}{\lambda^3 \cos r}\delta\lambda\right|$ (B) $\left|\frac{2b}{\lambda^3}\delta\lambda\right|$

(C) $\left|\frac{2b\tan r}{a\lambda^3 + b\lambda}\delta\lambda\right|$ (D) $\left|\frac{2b(a+b/\lambda^2)\sin i}{\lambda^3}\delta\lambda\right|$

17. A beam of monoenergetic electrons, which have been accelerated from rest by a potential U, is used to form an interference pattern in a Young's Double slit experiment. The electrons are now accelerated by potential 4U. The fringe width - **[2015]**

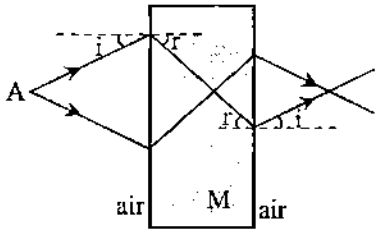
(A) remains the same
(B) is half the original fringe width
(C) is twice the original fringe width
(D) is one-fourth the original fringe width

18. A cubical vessel has opaque walls. An observer (dark circle in figure below) is located such that she can see only the wall CD but not the bottom. Nearly to what height should water be poured so that she can see an object placed at the bottom at a distance of 10 cm from the corner C ? Refractive index of water is 1.33. **[2015]**



(A) 10 cm (B) 16 cm
(C) 27 cm (D) 45 cm

19. Electromagnetic waves emanating from a point A (in air) are incident on a rectangular block of material M and emerge from the other side as shown. The angles i and r are angles of incidence and refraction when the wave travels from air to the medium. Such paths for the rays are possible **[2015]**

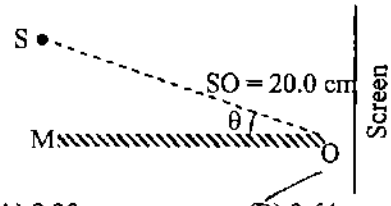


(A) if the material has a refractive index very nearly equal to zero.
(B) only with gamma rays with a wavelength smaller than the atomic nuclei of the material
(C) if the material has a refractive index less than zero.
(D) only if the wave travels in M with a speed faster than the speed of light in vacuum.

20. Electrons accelerated from rest by an electrostatic potential are collimated and sent through a Young's double slit setup. The fringe width is w. If the accelerating potential is doubled then the width is now close to - **[2016]**

(A) 0.5 w (B) 0.7 w
(C) 1.0 w (D) 2.0 w

21. A monochromatic light source S of wavelength 440 nm is placed slightly above a plane mirror M as shown . Image of S in M can be used as a virtual source to produce interference fringes on the screen. The distance of source S from O is 20.0 cm, and the distance of screen from O is 100.0 cm (figure is not to scale). If the angle $\theta = 0.50 \times 10^{-3}$ radians, the width of the interference fringes observed on the screen is – **[2016]**



(A) 2.20 mm (B) 2.64 mm
(C) 1.10 mm (D) 0.55 mm

22. A glass prism has a right-triangular cross section ABC, with $\angle A = 90°$. A ray of light parallel to the hypotenuse BC and incident on the side AB emerges grazing the side AC. Another ray, again parallel to the hypotenuse BC, incident on the side AC suffers total internal reflection at the side AB. Which one of the following must be true about the refractive index μ of the material of the prism ? **[2016]**

(A) $\sqrt{\frac{3}{2}} < \mu < \sqrt{2}$ (B) $\mu > \sqrt{3}$

(C) $\mu < \sqrt{\frac{3}{2}}$ (D) $\sqrt{2} < \mu < \sqrt{3}$

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23. A carrot looks orange in colour because of the β carotene molecule in it. This means that the β carotene molecule absorbs light of wavelengths **[5 Nov, 2017]**

(A) longer than 550 nm.

(B) shorter than 550 nm.

(C) longer than 700 nm.

(D) shorter than 700 nm.

24. It was found that the refractive index of material of a certain prism varied as $1.5 + 0.004/\lambda^2$, where λ is the wavelength of light used to measure the refractive index. The same material was then used to construct a thin prism of apex angle 10°. Angles of minimum deviation (δ_m) of the prism were recorded for the sources with wavelengths λ_1 and λ_2 respectively. Then **[5 Nov, 2017]**

(A) $\delta_m(\lambda_1) < \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$.

(B) $\delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 > \lambda_2$.

(C) $\delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$.

(D) δ_m is the same in both the cases.

25. Two circularly shaped linear polarisers are placed coaxially. The transmission axis of the first polarizer is at 30° from the vertical while the second one is at 60°, both in the clockwise sense. If an unpolarised beam of light of intensity I = 20 W/m^2 is incident on this pair of polarisers, then the intensities I_1 and I_2 transmitted by the first and the second polarisers, respectively, will be close to **[5 Nov, 2017]**

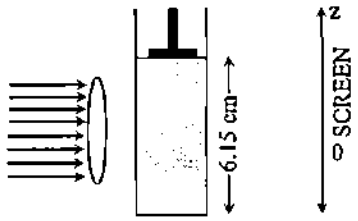
(A) $I_1 = 10.0$ W/m^2 and $I_2 = 7.5$ W/m^2

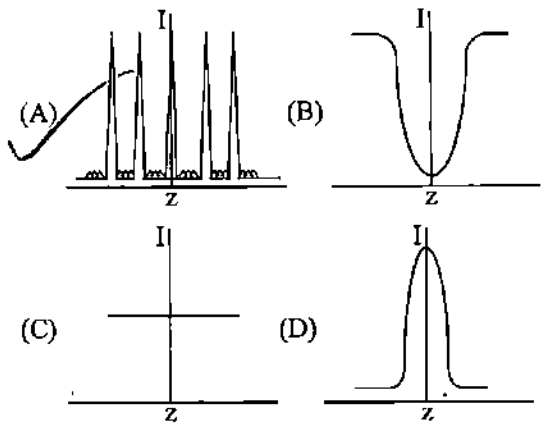
(B) $I_1 = 20.0$ W/m^2 and $I_2 = 15$ W/m^2

(C) $I_1 = 10.0$ W/m^2 and $I_2 = 8.6$ W/m^2

(D) $I_1 = 15.0$ W/m^2 and $I_2 = 0.0$ W/m^2

26. A parallel beam of light is incident on a tank filled with water up to a height of 61.5 mm as shown in the figure below. Ultrasonic waves of frequency 0.5 MHz are sent along the length of the water column using a transducer placed at the top, and they form longitudinal standing waves in the water. Which of the schematic plots below best describes the intensity distribution of the light as seen on the screen ? Take the speed of sound in water to be 1,500 m/s. **[5 Nov, 2017]**



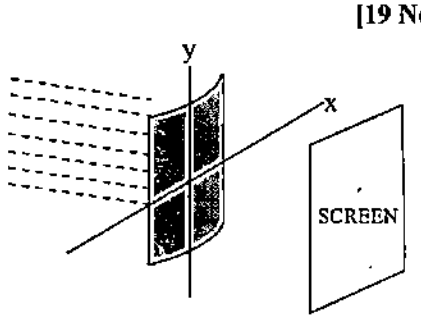


27. Three circularly shaped linear polarisers are placed coaxially. The transmission axis of the first polariser is at 30°, the second one is at 60° and the third at 90° to the vertical all in the clockwise sense. Each polariser additionally absorbs 10% of the light. If a vertically polarised beam of light of intensity I = 100 W/m^2 is incident on this assembly of polarisers, then the final intensity of the transmitted light will be close to **[19 Nov, 2017]**

(A) 10 W/m^2 (B) 20 W/m^2

(C) 30 W/m^2 (D) 50 W/m^2

28. Consider a plane parallel beam of light incident on a plano-cylindrical lens as shown below. Which of the following will you observe on a screen placed at the focal plane of the lens ? **[19 Nov, 2017]**



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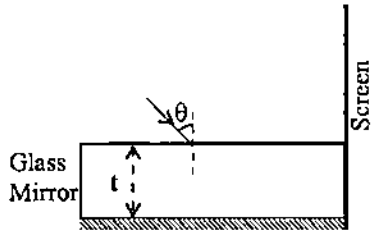
(A) The screen will be uniformly illuminated.
(B) There will be a single bright spot on the screen.
(C) There will be a single bright line on the screen parallel to the x-axis
(D) There will be a single bright line on the screen parallel to the y-axis

29. Back surface of a glass (refractive index n and thickness t) is polished to work as a mirror as shown below. A laser beam falls on it and is partially reflected and refracted at the air-glass interface and fully reflected at the mirror surface respectively. A pattern of discrete spots of light is observed on the screen.

[19 Nov, 2017]



The spacing between the spots on the screen will be

(A) $\frac{2t\cos\theta}{\sqrt{n^2-\sin^2\theta}}$ (B) $\frac{2t\sin\theta}{\sqrt{n^2-\sin^2\theta}}$

(C) $\frac{2t\tan\theta}{\sqrt{n^2-\sin^2\theta}}$ (D) $\frac{2t\sin\theta}{\sqrt{1-\frac{\sin^2\theta}{n^2}}}$

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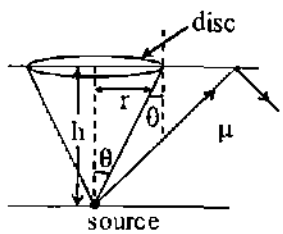
ANSWERS

1. (B)	2. (B)	3. (D)	4. (A)	5. (A)	6. (C)	7. (B)
8. (B)	9. (A)	10. (C)	11. (A)	12. (D)	13. (A)	14. (B)
15. (B)	16. (C)	17. (B)	18. (C)	19. (C)	20. (B)	21. (B)
22. (A)	23. (B)	24. (C)	25. (A)	26. (A)	27. (C)	28. (D)
29. (A)						

SOLUTIONS

1. From the equation of E_y & E_z it is evident that wave is circularly polarized.

2.



r should be such that rays beyond it got totally internally reflected

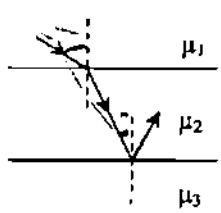
For this $\theta > C$ or $\sin\theta > \sin C$

also $\mu = \dfrac{1}{\sin C}$ $\therefore$ $\dfrac{r}{\sqrt{h^2+r^2}} > \dfrac{1}{\mu}$

In limiting case $\dfrac{r}{\sqrt{h^2+r^2}} = \dfrac{1}{\mu}$

solving we get $\boxed{r = \dfrac{h}{\sqrt{\mu^2-1}}}$

3. At first incidence light is deviated towards the normal therefore $\mu_2 > \mu_1$. Also at second incidence TIR takes place therefore $\mu_2 > \mu_3$, also $\mu_1 > \mu_3$ because for the same angle in medium μ_2, angle in μ_1 medium is less.



$\therefore \mu_3 < \mu_1 < \mu_2$

4. $d = 0.1$ mm, $D = 1$ m, $\lambda = 600$ nm

$I_P = 75\%$ of maximum or $I_P = 3I_0$

Where I_0 is the intensity of a single wave

now

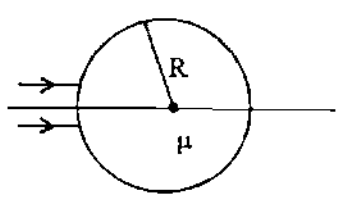
$$I_P = 3I_0 = (\sqrt{I_0})^2 + (\sqrt{I_0})^2 + 2\sqrt{I_0 \times I_0}\cos\phi$$

$$\therefore \cos\phi = \cos\frac{\pi}{3}, \text{ also } \Delta x = \frac{yd}{D}$$

$$\text{now } \Delta x = \frac{\lambda}{2\pi}\times\frac{\pi}{3} = \frac{\lambda}{6}$$

$$\therefore \quad y = \frac{\lambda D}{6d} = \frac{600\times10^{-9}\times1}{6\times0.1\times10^{-3}} \text{ or } \boxed{y = 1\text{m}}$$

5.



$$\text{now } \frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu-1}{R} \Rightarrow V_1 = \frac{\mu R}{\mu-1}$$

$$\text{now } \frac{1}{V_f} - \frac{\mu}{-(2R-V_1)} = \frac{1-\mu}{-R}$$

replace V_1 by $\dfrac{\mu R}{\mu-1}$ and solving for V_f

we get $V_f = \dfrac{R(\mu-2)}{2(\mu-1)}$

First image is real and second is virtual. 85

6. $I = I_0(\cos^2\phi)^4$

$$= I_0 \times \left(\frac{3}{4}\right)^4 = 30\% \text{ of } I_0$$

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7. Separation

$$\text{Bright fringe} = \frac{\lambda D}{d}$$

$f\lambda = c$

If f is doubled

λ become halved

$\therefore \beta$ become half

$$\beta = \frac{1}{2} = 0.5 \text{ mm}$$

8. $1.5 \times \sin i = 1.2 \sin r$

$$\sin r = \frac{1.5}{1.2} \sin i$$

T/R should not take place

$\therefore \sin r < 1$

$$\frac{1.5}{1.2} \sin i < 1;\ \sin i < \frac{12}{15};\ \sin i < 0.8$$

$$\sin 45 = \frac{1}{\sqrt{2}} = 0.707$$

$i_{max} > 45$

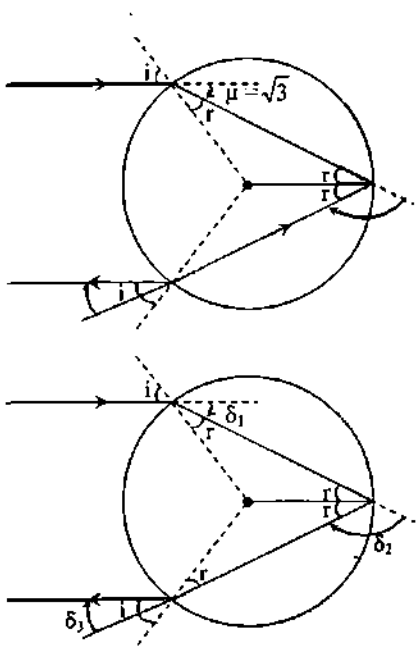
9. $I_{min} = 0$; $I_{max} = 4I_0$

$I_{av} = 2I_0$

10. $$I_0 \xrightarrow[\text{Polarizer}]{\text{First}} I_0/2 \xrightarrow[\text{II polarizer}]{\text{Malus law}} \left(\frac{I_0}{2}\right)\cos^2 30°$$

11. $$R = \frac{h'}{\sqrt{\mu^2 - 1}}$$

12.



$\delta_{net} = 180°$

$\delta_1 + \delta_2 + \delta_3 = 180°$

$i - r + 180° - 2r + i - r = 180°$

$2i = 4r$

$i = 2r$ ...(1)

Snell law

$1 \times \sin i = \sqrt{3} \sin r$ ...(2)

from (1) & (2) $\sin i = \sqrt{3} \sin \frac{i}{2}$

$$2\sin\frac{i}{2}\cos\frac{i}{2} = \sqrt{3}\sin\frac{i}{2}$$

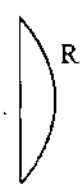
$$\Rightarrow \cos\frac{i}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{i}{2} = 30$$

$\Rightarrow i = 60°$

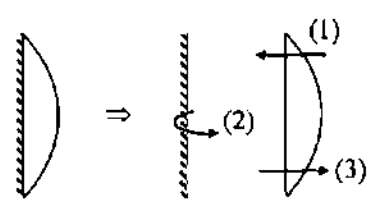
13. On refraction of light, frequency remain unchanged. However speed and wavelength get change.

14.



f = 10 cm

After silvering of flat face lens behave as mirror of focal length f_{eq}.



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{2}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_{eq}} = \frac{2}{10} + \frac{1}{\infty}$$

$f_{eq} = 5$

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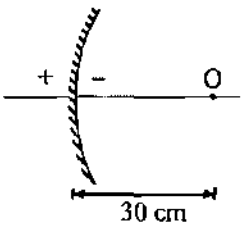


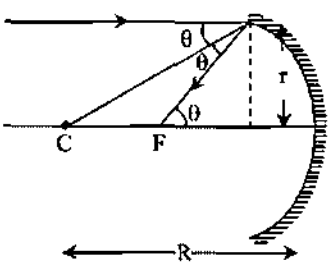
mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

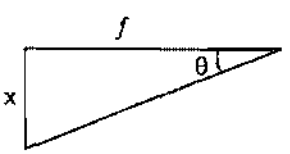
$$\Rightarrow \frac{1}{-5} = \frac{1}{-30} + \frac{1}{v}$$

$$v = -6 \text{ cm}$$

Image is real and 6 cm away from silvered lens.

15.





Area of disc

$$= \pi x^2 = \pi f^2 \tan^2\theta$$

$$= \pi \left\{\frac{R}{2}[1-\sec\theta]\right\}^2 \tan^2\theta$$

$$= \pi \frac{R^2}{4}[1-\sec\theta]^2 \tan^2\theta$$

$$\Rightarrow \simeq \pi \frac{R^2}{4}\left(\frac{r^2}{R^2}\right)\left(\frac{r^2}{R^2}\right)$$

$$\simeq \frac{\pi}{4}\frac{r^4}{R^2}$$

16. Snell law

$\sin i = \mu \sin r$

$$\sin i = \left(a + \frac{b}{\lambda^2}\right)\sin r$$

Differentiating with respect to λ

$$0 = \cos r\, dr\left(a + \frac{b}{\lambda^2}\right) + \sin r\left(\frac{b}{\lambda^3}(-2)\right)d\lambda$$

$$0 = \cos r\, dr\left(\frac{a\lambda^2 + b}{\lambda^2}\right) + \sin r\left(\frac{-2b}{\lambda^3}\right)d\lambda$$

$$\frac{d\lambda}{\lambda}\,2b\sin r = \cos r\, dr\,(a\lambda^2 + b)$$

$$dr = \frac{2b\,d\lambda}{\lambda}\,\frac{\tan r}{(a\lambda^2 + b)}$$

$$\delta r = \frac{(2b\tan r)\delta\lambda}{(a\lambda^3 + b\lambda)}$$

17. Debroglie wavelength of electron

$$\lambda = \frac{h}{\sqrt{2mK}}$$ Kinetic energy $\boxed{K = qV}$

$$\lambda \propto \frac{1}{\sqrt{V}}$$

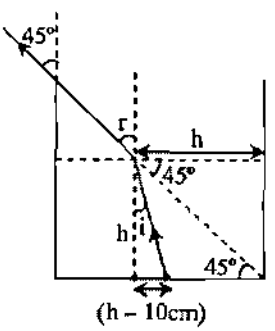
Potential becomes four time so wavelength becomes half.

Fringe width $(\beta) = \frac{\lambda D}{d}$

$\boxed{\beta \propto \lambda}$

Fringe width becomes half.

18.

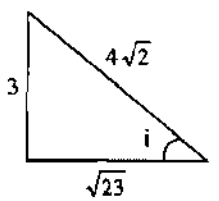


From diagram $r = 45°$

using snell law

$$\frac{4}{3}\sin i = \sin r$$

$$\sin i = \frac{3}{\sqrt{2}\times 4}$$



$$\tan i = \frac{3}{\sqrt{23}} \Rightarrow \tan i = \frac{h-10}{h}$$

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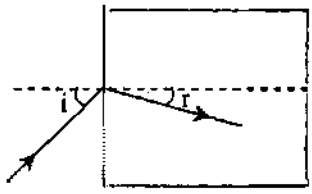
$h \tan i = h - 10$

$10 = h[1 - \tan i]$

$h = \dfrac{10}{1 - \tan i}$

$\Rightarrow$ 27 approx. = 27 cm

19.



Meta materials are the material for which refractive index is negative for them. Refraction diagram is shown,
here. In question same type of diagram is given.

20. $\beta = \dfrac{\lambda D}{d}$

$\lambda = \dfrac{h}{mV} = \dfrac{h}{\sqrt{2mq\Delta V}}$

$\beta \propto \lambda$

$\therefore \beta \propto \dfrac{1}{\sqrt{\Delta V}}$

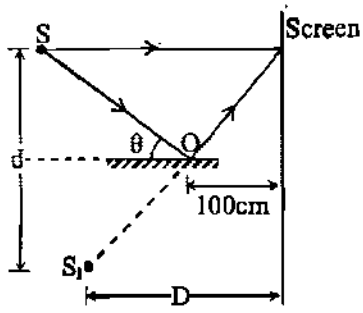
as ΔV is double

$\therefore$ β is $\dfrac{1}{\sqrt{2}}$ times of β_{old}

$\therefore \beta_{new} = 0.7\beta$

$= 0.7w$

21.



S and S_1 are source of YDSE

$\theta = 0.5 \times 10^{-3}$ radian (very small)

$D = SO \cos\theta + 100$

$= 20 \times 1 + 100$

$= 120$ cm

$d = 2 \times SO \sin\theta$

$\Rightarrow 2 \times 20 \times \theta$

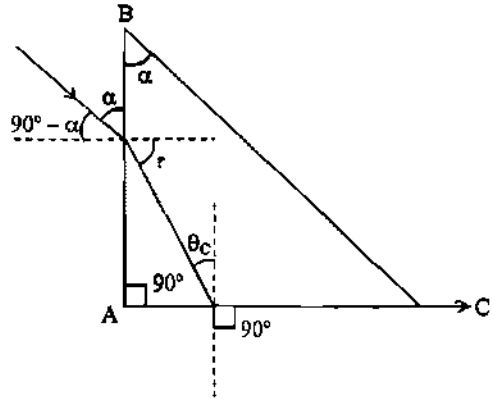
$\Rightarrow 40 \times 0.5 \times 10^{-3}$ cm

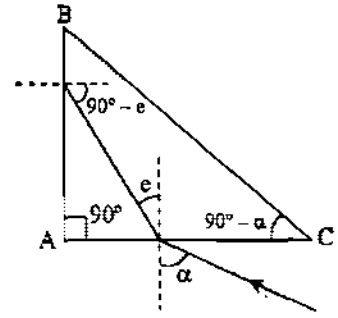
2×10^{-2} cm

$\beta = \dfrac{\lambda D}{d} = \dfrac{440 \times 10^{-6} \times 120 \times 10^{2}}{2 \times 10^{-2} \times 10^{2}}$

$264 \times 10^{-2} \Rightarrow 2.64$ mm

22.





$r + \theta_C = 90°$ .....(1)

$1 \times \sin(90° - \alpha) = \mu \sin r$

$\cos\alpha = \mu \sin r$ .....(2)

$90° - e > \theta_C$ .....(3)

$\mu \sin e = 1 \times \sin\alpha$ .....(4)

(3) & (4)

$90° - \theta_C > e$

$\cos\theta_C > \sin e$

$\cos\theta_C > \dfrac{\sin\alpha}{\mu}$

$1 - \sin^2\theta_C > \dfrac{1}{\mu^2}[1 - \mu^2 \sin^2 r]$

$1 - \dfrac{1}{\mu^2} > \dfrac{1}{\mu^2}[1 - \mu^2 \sin^2(90° - \theta_C)]$

$1 - \dfrac{1}{\mu^2} > \dfrac{1}{\mu^2} - \cos^2\theta_C$

$1 - \dfrac{2}{\mu^2} > -\left[1 - \dfrac{1}{\mu^2}\right]$

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$2 > \frac{3}{\mu^2}$

$\mu > \sqrt{\frac{3}{2}}$

(1) & (2)

$\cos\alpha = \mu \sin(90^\circ - \theta_C)$

$\cos\alpha = \mu \cos\theta_C$

$\cos\alpha < 1$

$\mu\cos\theta_C < 1$

$\sqrt{1-\frac{1}{\mu^2}} < \frac{1}{\mu}$

$1-\frac{1}{\mu^2} < \frac{1}{\mu^2}$

$1 < \frac{2}{\mu^2}$

$\mu < \sqrt{2}$

$\therefore \sqrt{\frac{3}{2}} < \mu < \sqrt{2}$

23. VIBG | YOR
400 nm | 700 nm
Absorbed Reflected
therefore seen

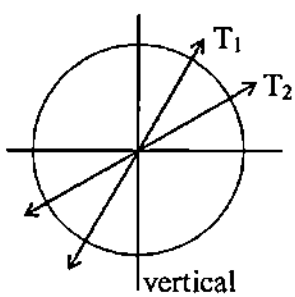
24. $n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$1.5 + \frac{0.004}{\lambda^2} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\delta_m \propto \frac{1}{\lambda}$

$\therefore \delta_m(\lambda_1) > \delta_m(\lambda_2)$ if $\lambda_1 < \lambda_2$

25.



$I_0 = 20\ W/m^2$

$I_1 = \frac{I_0}{2} = \frac{20}{2} = 10\ W/m^2$

$I_2 = I_1 \cos^2 30^\circ$

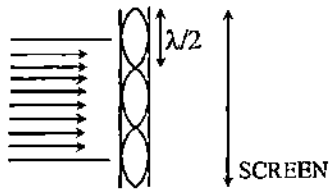
$= 10\left(\frac{\sqrt{3}}{2}\right)^2 = 10.\frac{3}{4}$

$= 7.5\ w/m^2$

26. $v = \nu\lambda$

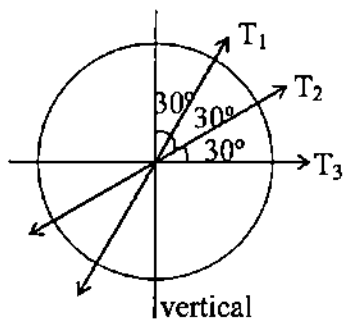
$\Rightarrow \lambda = \frac{1500}{0.5\times10^6} = 3000 \times 10^{-6}$

$\lambda = 3mm$



intensity vary according to graph 'A'

27.



$I_1 = I_0 \times 0.9\cos^2 30^\circ = I_0 \times 0.9 \times \frac{3}{4}$

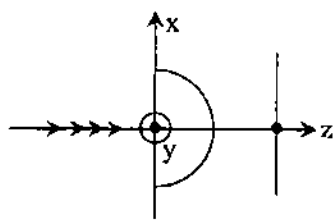
$I_2 = I_1 \times 0.9\cos^2 30^\circ = I_1 \times 0.9 \times \frac{3}{4}$

$I_3 = I_2 \times 0.9\cos^2 30^\circ = I_2 \times 0.9 \times \frac{3}{4}$

$\Rightarrow I_3 = I_0(0.9)^3\left(\frac{3}{4}\right)^3$

$I_3 = 30.75\ w/m^2.$

28.



no deflection of light beam

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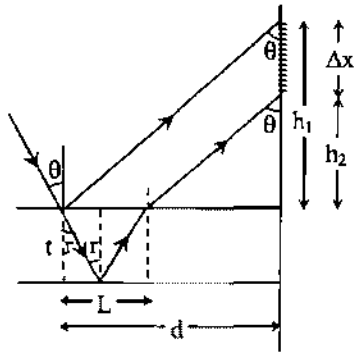
29.



Therefore $\Delta x = h_1 - h_2$

$\tan\theta = \frac{d}{h_1}$ & $\tan\theta = \frac{d-L}{h_2}$

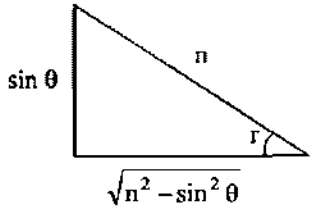
$\Delta x = \frac{d}{\tan\theta} - \frac{d-L}{\tan\theta}$

$\Delta x = \frac{L}{\tan\theta}$

Snell's Law

$1 \cdot \sin\theta = n \cdot \sin r \Rightarrow 1 \sin r = \frac{\sin\theta}{n}$

$\tan r = \frac{L}{2.t} \Rightarrow L = 2\,t \tan r$



$L = \frac{2t \cdot \sin\theta}{\sqrt{n^2 - \sin^2\theta}}$

$\therefore \Delta x = \frac{2t\sin\theta}{\sqrt{n^2 - \sin^2\theta}\tan\theta}$

$\tan\theta = \frac{2t\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$

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Chapter

5 Modern Physics

1. A nucleus has a half-life of 30 minutes. At 3 PM its decay rate was measured as 120,000 counts/sec. What will be the decay rate at 5 PM ? **[2010]**

(A) 120,000 counts/sec
(B) 60,000 counts/sec
(C) 30,000 counts/sec
(D) 7,500 counts/sec

2. At time t = 0, a container has N_0 radioactive atoms with a decay constant λ. In addition, c numbers of atoms of the same type are being added to the container per unit time. How many atoms of this type are there at t = T ? **[2010]**

(A) $\frac{c}{\lambda}\exp(-\lambda T) - N_0\exp(-\lambda T)$

(B) $\frac{c}{\lambda}\exp(-\lambda T) + N_0\exp(-\lambda T)$

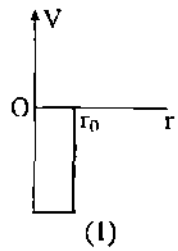
(C) $\frac{c}{\lambda}\{1 - \exp(-\lambda T)\} + N_0\exp(-\lambda T)$

(D) $\frac{c}{\lambda}\{1 + \exp(-\lambda T)\} + N_0\exp(-\lambda T)$

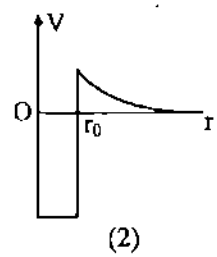
3. An electron collides with a free molecules initially in its ground state. The collision leaves the molecules in an excited state that is metastable and does not decay to the ground state by radiation. Let K be the sum of the initial kinetic energies of the electron and the molecule, and $\vec{P}$ the sum of their initial momenta. Let K' and $\vec{P}'$ represent the same physical quantities after the collision. Then - **[2011]**

(A) $K = K'$, $\vec{P} = \vec{P}'$ (B) $K' < K$, $\vec{P} = \vec{P}'$
(C) $K = K'$, $\vec{P} \neq \vec{P}'$ (D) $K' < K$, $\vec{P} \neq \vec{P}'$

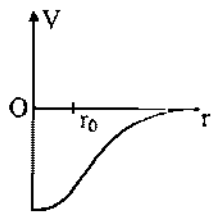
4. Given below are three schematic graphs of potential energy V(r) versus distance r for three atomic particles : electron (e), proton (p^+) and neutron (n), in the presence of a nucleus at the origin O. The radius of the nucleus is r_0. The scale on the V-axis may not be the same for all figures. The correct pairing of each graph with the corresponding atomic particle is - **[2011]**



(1)



(2)



(3)

(A) (1, n), (2, p^+), (3, e^-)
(B) (1, p^+), (2, e^-), (3, n)
(C) (1, e^-), (2, p^+), (3, n)
(D) (1, p^+), (2, n), (3, e^-)

5. Due to transitions among its first three energy levels, hydrogenic atom emits radiation at three discrete wavelengths λ_1, λ_2, and λ_3 ($\lambda_1 < \lambda_2 < \lambda_3$). Then - **[2011]**

(A) $\lambda_1 - \lambda_2 + \lambda_3$
(B) $\lambda_1 + \lambda_2 = \lambda_3$
(C) $1/\lambda_1 + 1/\lambda_2 = 1/\lambda_3$
(D) $1/\lambda_1 = 1/\lambda_2 + 1/\lambda_3$

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6. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. The half lives of X and Y are 40000 years and 20 years respectively. In a certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20}, the number of Y nuclei present in its is– [2012]

(A) 2×10^{17} (B) 2×10^{20}
(C) 4×10^{23} (D) 4×10^{20}

7. A radioactive nucleus A has a single decay mode with half life τ_A. Another radioactive nucleus B has two decay modes 1 and 2. If decay mode 2 were absent, the half life of B would have been $\tau_A/2$. If decay mode 1 were absent, the half life of B would have been $3\tau_A$, then the ratio τ_B/τ_A is– [2012]

(A) 3/7 (B) 7/2
(C) 7/3 (D) 1

8. A stream of photons having energy 3 eV each impinges on a potassium surface. The work function of potassium is 2.3 eV. The emerging photo-electrons are slowed down by a copper plate placed 5 mm away. If the potential difference between the two metal plates is 1 V, the maximum distance the electrons can move away from the potassium surface before being turned back is– [2012]

(A) 3.5 mm (B) 1.5 mm
(C) 2.5 mm (D) 5.0 mm

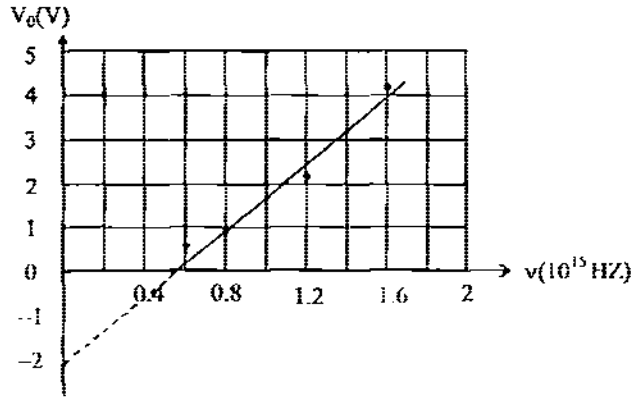
9. A nuclear decay is possible if the mass of the parent nucleus exceeds the total mass of the decay particles. If M(A,Z) denotes the mass of a single neutral atom of an element with mass number A and atomic number Z, then the minimal condition that the β decay

$$X_Z^A \rightarrow Y_{Z+1}^A + \beta^- + \bar{\nu}_e$$

will occur is (m_e denotes the mass of the β particle and the neutrino mass m_ν can be neglected) : [2013]

(A) $M(A,Z) > M(A,Z+1) + m_e$
(B) $M(A,Z) > M(A,Z+1)$
(C) $M(A,Z) > M(A,Z+1) + Zm_e$
(D) $M(A,Z) > M(A,Z+1) - m_e$

10. In a photocell circuit the stopping potential, V_0, is a measure of the maximum kinetic energy of the photoelectrons. The following graph shown experimentally measured values of stopping potential versus frequency ν of incident light.



The values of Planck's constant and the work function as determined from the graph are (taking the magnitude of electronic charge to be $e = 1.6 \times 10^{-19}$ C) [2013]

(A) 6.4×10^{-34} Js, 2.0 eV
(B) 6.0×10^{-34} Js, 2.0 eV
(C) 6.4×10^{-34} Js, 3.2 eV
(D) 6.0×10^{-34} Js, 3.2 eV

11. Two species of radioactive atoms are mixed in equal number. The disintegration constant of the first species is λ and of the second is λ/3. After a long time the mixture will behave as a species with mean life of approximately [2013]

(A) 0.70/λ (B) 2.10/λ
(C) 1.00/λ (D) 0.52/λ

12. Photons of energy 7 eV are incident on two metals A and B with work functions 6 eV and 3 eV respectively. The minimum de Broglie wavelength of the emitted photoelectrons with maximum energies are λ_A and λ_B, respectively where λ_A/λ_B is nearly [2013]

(A) 0.5 (B) 1.4
(C) 4.0 (D) 2.0

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13. A singly ionized helium atom in an excited state (n = 4) emits a photon of energy 2.6 eV. Given that the ground state energy of hydrogen atom is – 13.6 eV, the energy (E_f) and quantum number (n) of the resulting state are respectively- **[2013]**

(A) $E_f = -13.6$ eV, n=1
(B) $E_f = -6.0$ eV, n = 3
(C) $E_f = -6.0$ eV, n = 2
(D) $E_f = -13.6$ eV, n = 2

14. The half life of a particle of mass 1.6×10^{-26} kg is 6.9 s and a stream of such particles is travelling with the kinetic energy of a particle being 0.05 eV. The fraction of particles which will decay when they travel a distance of 1 m is - **[2014]**

(A) 0.1 (B) 0.01
(C) 0.001 (D) 0.0001

15. A 160 watt light source is radiating light of wavelength 6200 Å uniformly in all directions. The photon flux at a distance of 1.8 m is of the order of (Planck's constant 6.63×10^{-34} J-s) **[2014]**

(A) $10^2\ m^{-2}\ s^{-1}$ (B) $10^{12}\ m^{-2}\ s^{-1}$
(C) $10^{19}\ m^{-2}\ s^{-1}$ (D) $10^{25}\ m^{-2}\ s^{-1}$

16. The wavelength of the first Balmer line caused by a transition from the n = 3 level to the n = 2 level in hydrogen is λ_1. The wavelength of the line caused by an electronic transition from n = 5 to n = 3 is - **[2014]**

(A) $\frac{375}{128}\lambda_1$ (B) $\frac{125}{64}\lambda_1$
(C) $\frac{64}{125}\lambda_1$ (D) $\frac{128}{375}\lambda_1$

17. The binding energy per nucleon of $_5B^{10}$ is 8.0 MeV and that of $_5B^{11}$ is 7.5 MeV. The energy required to remove a neutron from $_5B^{11}$ is (mass of electron and proton are 9.11×10^{31} kg and 1.67×10^{27} kg, respectively) - **[2014]**

(A) 2.5 MeV (B) 8.0 MeV
(C) 0.5 MeV (D) 7.5 MeV

18. A 1.5 kW (kilo-watt) laser beam of wavelength 6400 Å is used to levitate a thin aluminium disc of same area as the cross section of the beam. The laser light is reflected by the aluminium disc without any absorption. The mass of the foil is close to **[2014]**

(A) 10^{-9} kg (B) 10^{-3} kg
(C) 10^{-4} kg (D) 10^{-6} kg

19. When ultraviolet radiation of a certain frequency falls on a potassium target, the photoelectrons released can be stopped completely by a retarding potential of 0.6 V. If the frequency of the radiation is increased by 10%, this stopping potential rises to 0.9 V. The work function of potassium is **[2014]**

(A) 2.0 eV (B) 2.4 eV
(C) 3.0 eV (D) 2.8 eV

20. Carbon-II decays to boron-II according to the following formula.

$$^{11}_{6}C \rightarrow {}^{11}_{5}B + e^+ + \nu_e + 0.96\ MeV$$

Assume that positrons (e^+) produced in the decay combine with free electrons in the atmosphere and annihilate each other almost immediately. Also assume that the neutrinos (ν_e) are massless and do not interact with the environment. At t = 0 we have 1 μg of $^{12}_{6}C$. If the half-life of the decay process is t_0, the net energy produced between time t = 0 and t = $2t_0$ will be nearly - **[2015]**

(A) 8×10^{18} MeV (B) 8×10^{16} MeV
(C) 4×10^{18} MeV (D) 4×10^{16} MeV

21. Two uniform plates of the same thickness and area but of different materials, one shaped like an isosceles triangle and the other shaped like a rectangle are joined together to form a composite body as shown in the figure. If the centre of mass of the composite body is located at the midpoint of their common side, the ratio between masses of the triangle to that of the rectangle is - **[2015]**

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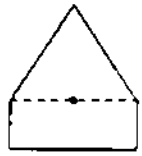

(A) 1 : 1 (B) 4 : 3
(C) 3 : 4 (D) 2 : 1

22. A 160 watt infrared source is radiating light of wavelength 50000Å uniformly in all directions. The photon flux at a distance of 1.18 m is of the order of - **[2015]**
(A) $10\ m^{-2}s^{-1}$ (B) $10^{10}m^{-2}s^{-1}$
(C) $10^{15}\ m^{-2}s^{-1}$ (D) $10^{20}m^{-2}s^{-1}$

23. Photons of wavelength λ are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius R by a perpendicular magnetic field having a magnitude B. The work function of the metal is (Where symbols have their usual meanings) - **[2016]**

(A) $\frac{hc}{\lambda} - m_e + \frac{e^2B^2R^2}{2m_e}$

(B) $\frac{hc}{\lambda} + 2m_e\left(\frac{eBR}{2m_e}\right)^2$

(C) $\frac{hc}{\lambda} - m_eC^2 - \frac{e^2B^2R^2}{2m_e}$

(D) $\frac{hc}{\lambda} - 2m_e\left(\frac{eBR}{2m_e}\right)^2$

24. An electron with kinetic energy E collides with a hydrogen atom in the ground state. The collision will be elastic **[2016]**
(A) For all values of E
(B) For E < 10.2 eV
(C) For 10.2 eV < E < 13.6 eV only
(D) For 0 < E < 3.4 eV only

25. The continuous part of X-ray spectrum is a result of the **[2016]**
(A) Photoelectric effect
(B) Raman effect
(C) Compton effect
(D) Inverse photoelectric effect

26. An electron and a photon have same wavelength of 10^{-9} m. If E is the energy of the photon and p is the momentum of the electron, the magnitude of E/p in SI units is **[2016]**
(A) 1.00×10^{-9} (B) 1.50×10^{8}
(C) 3.00×10^{8} (D) 1.20×10^{7}

27. If one takes into account finite mass of the proton, the correction to the binding energy of the hydrogen atom is approximately (mass of proton = 1.60×10^{-27} kg, mass of electron = 9.10×10^{-31} kg)- **[2016]**
(A) 0.06 % (B) 0.0006 %
(C) 0.02 % (D) 0.00 %

28. A photon of wavelength λ is absorbed by an electron confined to a box of length $\sqrt{35h\lambda/8mc}$. As a result, the electron makes a transition from state k = 1 to the state n. Subsequently the electron transits from the state n to the state m by emitting a photon of wavelength $\lambda' = 1.85\lambda$. Then **[2016]**
(A) n = 4; m = 2
(B) n = 5; m = 3
(C) n = 6; m = 4
(D) n = 3; m = 1

29. The magnitude of acceleration of the electron in the n^{th} orbit of hydrogen atom is a_H and that of singly ionized helium atom is a_{He}. The ratio $a_H : a_{He}$ is **[5 Nov, 2017]**
(A) 1 : 8
(B) 1 : 4
(C) 1 : 2
(D) dependent on n

30. Using dimensional analysis the resistivity in terms of fundamental constants h, m_e, c, e, ε_0 can be expressed as **[5 Nov, 2017]**

(A) $\frac{h}{\varepsilon_0 m_e ce^2}$ (B) $\frac{\varepsilon_0 m_e ce^2}{h}$

(C) $\frac{h^2}{m_e ce^2}$ (D) $\frac{m_e \varepsilon_0}{ce^2}$

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31. An electron in an electron microscope with initial velocity $v_0\hat{i}$ enters a region of a stray transverse electric field $E_0\hat{j}$. The time taken for the change in its de-Broglie wavelength from the initial value of λ to $\lambda/3$ is proportional to **[5 Nov, 2017]**

(A) E_0 (B) $\frac{1}{E_0}$ (C) $\frac{1}{\sqrt{E_0}}$ (D) $\sqrt{E_0}$

32. A particle of mass m moves around the origin in a potential $\frac{1}{2}m\omega^2r^2$, where r is the distance from the origin. Applying the Bohr model in this case, the radius of the particle in its n^{th} orbit in terms of $a = \sqrt{h/(2\pi m\omega)}$ is **[5 Nov, 2017]**

(A) $a\sqrt{n}$ (B) an
(C) an^2 (D) $an\sqrt{n}$

33. A neutron star with magnetic moment of magnitude m is spinning with angular velocity ω about its magnetic axis. The electromagnetic power P radiated by it is given by $\mu_0^x m^y \omega^z c^u$ where μ_0 and c are the permeability and speed of light in free space, respectively. Then **[5 Nov, 2017]**

(A) x = 1, y = 2, z = 4 and u = –3
(B) x = 1, y = 2, z = 4 and u = 3
(C) x = –1, y = 2, z = 4 and u = –3
(D) x = –1, y = 2, z = 4 and u = 3

34. To calculate the size of a hydrogen anion using the Bohr model, we assume that its two electrons move in an orbit such that they are always on diametrically opposite sides of the nucleus. With each electron having the angular momentum $\hbar = h/2\pi$, and taking electron interaction into account the radius of the orbit in terms of the Bohr radius of hydrogen atom $a_B = \frac{4\pi\varepsilon_0 h^2}{me^2}$ is **[5 Nov, 2017]**

(A) a_B (B) $\frac{4}{3}a_B$ (C) $\frac{2}{3}a_B$ (D) $\frac{3}{2}a_B$

35. Consider the following nuclear reactions :

I. $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + X$

II. $^{9}_{4}Be + ^{4}_{2}H \rightarrow ^{12}_{6}He + Y$

Then **[19 Nov, 2017]**

(A) X and Y are both protons.
(B) X and Y are both neutrons.
(C) X is a proton and Y is a neutron.
(D) X is a neutron and Y is a proton.

36. The n-side of the depletion layer of a p-n junction : **[19 Nov, 2017]**

(A) always has same width as of the p-side.
(B) has no bound charges.
(C) is negatively charged.
(D) is positively charged.

37. Some of the wavelength observed in the emission spectrum of neutral hydrogen gas are 912, 1026, 1216, 3646, 6563 Å. If broad band light is passing through neutral hydrogen gas at room temperature, the wavelength that will not be absorbed strongly is **[19 Nov, 2017]**

(A) 1026 Å (B) 1216 Å
(C) 912 Å (D) 3646 Å

38. The emission series of hydrogen atom is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is the Rydberg constant. For a transition from n_2 to n_1, the relative change $\Delta\lambda/\lambda$ in the emission wavelength if hydrogen is replaced by deuterium (assume that the mass of proton and neutron are the same and approximately 2000 times larger than that of electrons) is **[19 Nov, 2017]**

(A) 0.025 % (B) 0.005 %
(C) 0.0025 % (D) 0.05 %

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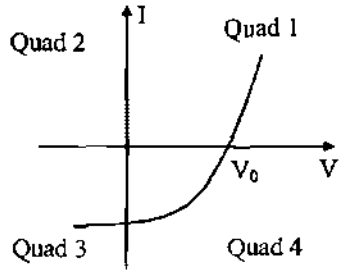
39. When light shines on a p-n junction diode, the current (I) vs, voltage (V) is observed as in the figure below : **[19 Nov, 2017]**



In which quadrant(s) does the diode generate power, so that it can be used as a solar cell ?

(A) Quad 1 only

(B) Quad 1 and 3 only

(C) Quad 4 only

(D) Quad 1 and 4 only

40. Consider the following statements regarding the photoelectric effect experiment :

(I) Photoelectrons are emitted as soon as the metal is exposed to light

(II) There is a minimum frequency below which no photo-current is observed

(III) The stopping potential is proportional to the frequency of light

(IV) The photo-current varies linearly with the intensity of the light

Which of the above statements indicate that light consists of quanta (photons) with energy proportional to frequency ? **[19 Nov, 2017]**

(A) I and III only

(B) II and III only

(C) II, III and IV only

(D) I, II and III only

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ANSWERS

1. (D)	2. (C)	3. (B)	4. (A)	5. (C)	6. (A)	7. (A)
8. (A)	9. (A)	10. (B)	11. (B)	12. (D)	13. (B)	14. (D)
15. (C)	16. (B)	17. (A)	18. (D)	19. (B)	20. (B)	21. (D)
22. (D)	23. (D)	24. (B)	25. (D)	26. (C)	27. (A)	28. (C)
29. (A)	30. (C)	31. (B)	32. (A)	33. (A)	34. (B)	35. (C)
36. (D)	37. (D)	38. (A)	39. (C)	40. (D)		

SOLUTIONS

1. Given $T = 30$ minutes. $\frac{dN}{dt} = 120\ K\ \frac{\text{counts}}{\text{sec}}$

After each half life, activity is reduced to half therefore after n half lives activity reduces to $\left(\frac{1}{2}\right)^n$.

Also $\frac{dN}{dt} \propto N$

$\frac{dN}{dt}$ at 5 P.M. will be equal to activity remaining after four half lives.

i.e. $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{16}\text{th}\right)$ of the initial activity

$\left(\frac{dN}{dt}\right)_{\text{at 5 P.M.}} = \left(\frac{1}{16}\text{th}\right)$ of the initial activity

$\left(\frac{dN}{dt}\right)_{5\text{ P.M.}} = \left(\frac{1}{16}\right)\left(\frac{dN}{dt}\right)_{3\text{ P.M.}}$

$$\boxed{\left(\frac{dN}{dt}\right)_{5\text{ P.M.}} = 7500 \text{ counts/sec}}$$

2. N_0 – initial nucleon
at $t = 0$, N_0
Addition is at a constant rate

$$(\lambda N - C) = -\frac{dN}{dt}$$

$$\int_0^k dt = \int_{N_0}^{N} \frac{dN}{\lambda N - C}$$

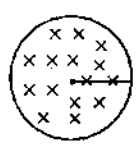
Integrating we get

$$N = \frac{C}{\lambda} + \frac{e^{-\lambda t}}{\lambda}(\lambda N_0 - C)$$

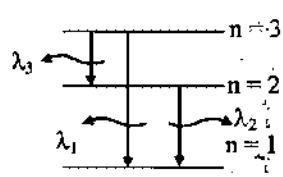
$$\therefore \boxed{N = \frac{C}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}}$$

3. Collision of e^- lead to excitation of molecules
so Collision is inelastic
$\therefore K' < K$ and loss of kinetic energy go for excitation
of molecules. Momentum remain conserved during collision.

$\vec{P} = \vec{P'}$

4. 

outside the nucleus electric potential decreases
e^- is negativity charged
$\therefore$ its PE is negative even outside the nucleus where
nuclear attractive force is negligible
$(3) \rightarrow e^-$
outside the nucleus
neutron will not
experience electric force
as it is neutral. So no potential energy associated with
it outside nucleus.
$1 \rightarrow$ neutron

5. 

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3}$$

$$\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

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6. In radioactive equilibrium
rate of decay of X = rate of decay of Y

$$\lambda_x N_x = \lambda_y N_y, \quad \frac{N_x}{T_x} = \frac{N_y}{T_y}.$$

7.
$$\tau_B = \frac{\tau_{A/2} \cdot 3\tau_A}{\tau_{A/2} + 3\tau_A}$$

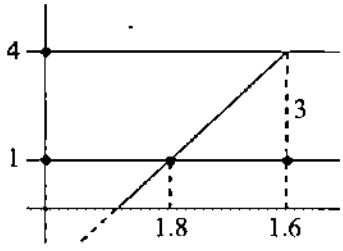
$$\frac{\tau_B}{\tau_A} = \frac{3}{7}$$

8. $K = 3 - 2.3 = 0.7eV$

$$S = \frac{K}{eE} \text{ and } E = V/d$$

9. β decay occur only when rest mass energy of reactant > rest mass energy of product.
as rest mass energy $E = mc^2$
$\therefore \quad M(A, Z) > M(A, Z+1) + m_e$

10.



$KE_{max} = eV_0$
$hf = \phi + KE_{max}$

$$V_0 = \frac{hf}{e} - \phi$$

$$V_0 = \frac{h}{e}\nu - \phi$$

when $\nu = 0$
$V_0 = -\phi$
$V_0 = -2$ given
$\therefore \quad \phi = 2eV$

$$\frac{h}{e} = \text{slope of graph}$$

$$\frac{h}{e} = \frac{3}{0.8 \times 10^{15}}$$

$$h = \frac{1.6 \times 10^{-19} \times 3}{0.8 \times 10^{15}}$$

$h = 6 \times 10^{-34}$

11.
$$\lambda_{eu} = \frac{\lambda_1 N + \lambda_2 N}{N + N}$$

$$\Rightarrow \frac{\lambda_1 + \lambda_2}{2}$$

$$\lambda_{eu} \Rightarrow \frac{4\lambda}{6}$$

$$\text{Mean life} = \frac{6}{4\lambda}$$

$$\Rightarrow \frac{3}{2\lambda}$$

$$\Rightarrow \frac{1.5}{\lambda}$$

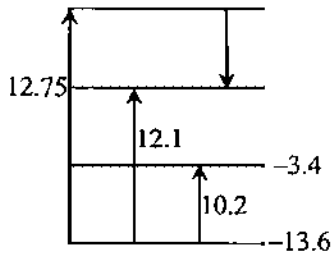
approx. 2.0λ

12. De-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mKE}}$

When KE is max. the λ is minimum
$KE_{max} = hf - \phi$
In case of A, $K_1 \Rightarrow 7 - 6 \Rightarrow 1eV$
In case of B, $K_2 \Rightarrow 7 - 3 \Rightarrow 4eV$

$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{KE_B}{KE_A}} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \sqrt{\frac{4}{1}} = 2:1$$

13.



$$E_n = -13.6 \times \frac{Z^2}{n^2}$$

$$\therefore \quad E_4 = -\frac{13.6 \times 2^2}{4^2}$$

$$E_4 - E_n = 2.6 - 4 \times 13.6 \left[\frac{1}{4^2} - \frac{1}{n^2}\right] = 2.6$$

$n = 3$

$$E_f = -\frac{13.6 \times 2^2}{3^2}$$

$\Rightarrow -6.0\ eV$

14.
$$KE = \frac{1}{2}mv^2$$

$$0.05 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 1.6 \times 10^{-26} \times v^2$$

$0.05 \times 2 \times 10^7 = v^2$
$10^6 = v^2$
$v = 1000$ m/sec

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time taken to travel a distance of 1 m is $\frac{1}{v}$

$\Rightarrow \frac{1}{1000} = 0.001$ sec

Half life of radioactive material = 6.9 sec

$T_{1/2} = \frac{0.693}{\lambda}$

$\lambda = \frac{0.693}{6.9} \Rightarrow 0.1$

fraction of particle decay in 0.001 sec

or $\frac{1}{1000}$ sec $= 1 - e^{-\lambda t}$

$\Rightarrow 1 - e^{-0.1\times\frac{1}{1000}} \Rightarrow 1 - e^{-\frac{1}{10000}}$

$\Rightarrow 0.0001$

15. Intensity of light at 1.8 m $= \frac{P}{4\pi(1.8)^2}$

$I \Rightarrow \frac{160}{4\times\pi\times(1.8)^2}$

Photon flux = Number of photon per unit area.

$\Rightarrow \frac{I}{\frac{hc}{\lambda}}$

$\Rightarrow \frac{I\lambda}{hc}$

$\Rightarrow \frac{160\times6200\times10^{-10}}{4\times\pi\times(1.8)^2\times6.63\times10^{-34}\times3\times10^8}$

$\Rightarrow 1.22\times10^{19}$

16. $\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$\frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$

$\frac{1}{\lambda_1} = R\left[\frac{5}{36}\right]$ ....(1)

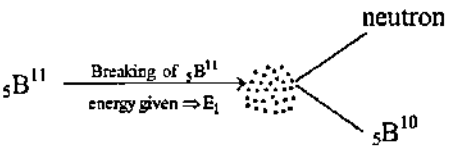
$\frac{1}{\lambda_2} = R\left[\frac{1}{9} - \frac{1}{25}\right]$

$\frac{1}{\lambda_2} = R\left[\frac{16}{9\times25}\right]$ ....(2)

From (1) & (2)

$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \div \frac{16}{9\times25}$

$\lambda_2 = \frac{5}{36} \times \frac{9\times25}{16} = \frac{125}{64}\lambda_1$

17. 

formation of $_5B^{10}$ will release energy $\Rightarrow E_2$

E_1 = Binding energy of $_5B^{11}$

$\Rightarrow 7.5 \times 11$ M. V = 82.5 MeV

E_2 = Binding energy of $_5B^{10} = 8.0 \times 10$

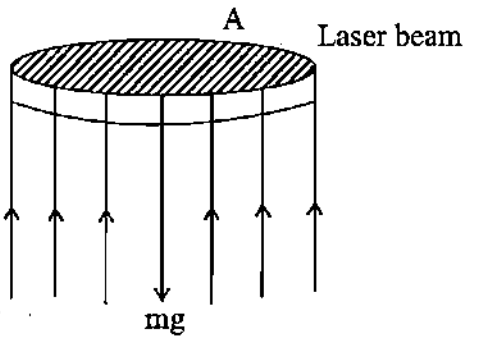
= 80 MeV

Energy given = $E_1 - E_2$

= 82.5 – 80

= 2.5 MeV

18. Power of light = P



Force acting on Al disc = $\frac{2P}{C}$

$= \frac{2\times1.5\times10^3}{3.0\times10^8} = 10^{-5}$

Force acting on Al disc = mg

$10^{-5} = m \times 10$

$m = 10^{-6}$ kg

19. $KE_{max} = e \times V_{retarding}$

$= e \times 0.6$

= 0.6 eV

Photon energy = hf = E

When frequency increase by 10% energy of photon also increases by 10%

New energy = E' = 1.1 E

New $KE_{max.} = e \times V_{retarding} = e \times 0.9 = 0.9$ eV

Einstein photoelectric equation

$hf = KE_{max} + \phi$

$E = 0.6 + \phi$ ...(1)

$1.1\ E = 0.9 + \phi$ ...(2)

$1.1 = \frac{0.9+\phi}{0.6+\phi}$

$1.1\ \phi + 0.66 = 0.9 + \phi$

$0.1\ \phi = 0.24$

$\phi = 2.4$ Ev

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20. $^{12}_{6}C \longrightarrow {}^{11}_{5}B + \beta^{\oplus} + \nu + 0.96\ MeV$

$$M = \frac{M_0}{2^n} \Rightarrow M = \frac{1\ \mu g}{2^2}$$

[Half life = t_0 ; $n = t/t_0 \Rightarrow n = \frac{2t_0}{t} \Rightarrow n = 2$]

$M = 0.25\ \mu g$ (remained)

Carbon used $\Rightarrow M_0 - M \Rightarrow 0.75\ \mu g$

Number of moles $= \left(\frac{0.75\times10^{-6}}{12}\right)$

Number of reaction

$$= \frac{0.75\times10^{-6}}{12} \times 6.023 \times 10^{23}$$

$$= 0.37 \times 10^{17} \text{ reaction}$$

Energy from reaction

$$= 0.376 \times 10^{17} \times 0.96\ MeV$$
$$\approx 0.36 \times 10^{17}$$
$$= 3.6 \times 10^{16}\ MeV$$
$$\approx 4 \times 10^{16}\ MeV$$

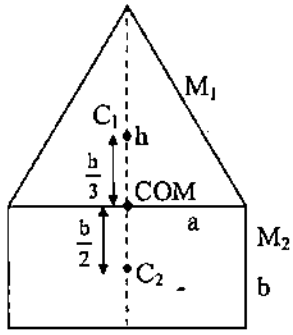
Energy from annihilation

$$= 2\, m_0c^2\, (0.376 \times 10^{17})$$
$$\approx 1.02\, (0.376 \times 10^{17})\ MeV$$
$$\approx 4 \times 10^{16}\ MeV$$

Total energy = $E_{reaction} + E_{anhiltation}$

$$E_T \approx 8 \times 10^{16}\ MeV$$

21.



Equal are

$$\frac{1}{2}ah = ab$$

$$h = 2b$$

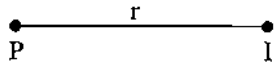
$M_1\frac{h}{3} = M_2\frac{b}{2}$ [centre of mass of combination at the mid-point of their common edge]

$$\frac{M_1}{M_2} = \frac{3}{2}\frac{b}{h}$$

$$\frac{M_1}{M_2} = \frac{3}{2}\left[\frac{1}{2}\right]$$

$$\frac{M_1}{M_2} = \frac{3}{4}$$

22. $P = 160$ watt, $\lambda = 50000Å$



$$I = \frac{P}{4\pi r^2}$$

$$nh\nu = \frac{P}{4\pi r^2}$$

$$\Rightarrow n = \frac{P}{4\pi r^2(h\nu)}$$

(n = no. of photons per sec per m^2)

$$n = \frac{P\lambda}{4\pi r^2 hc}$$

$$n = 10^{20}\ m^{-2}\ s^{-1}$$

23. $$R = \frac{mv}{qB}$$

$$V = \frac{qBR}{m} = \frac{eBR}{m_e}$$

$$\frac{hc}{\lambda} - \phi = KE_{max}$$

(Einstein photo electric equation)

$$\phi = \frac{hc}{\lambda} - KE_{max} \Rightarrow = \frac{hc}{\lambda} - \frac{1}{2}m_e\left(\frac{eBR}{m_e}\right)^2$$

$$= \frac{hc}{\lambda} - 2m_e\left(\frac{eBR}{2m_e}\right)^2$$

24. When e^- collide with atom which is massive in comparison to e^-. Max possible loss of KE = KE of e^- (initial KE) = E

if this E is less than min excitation energy then collision is elastic

$\therefore$ E < 10.2 eV (Minimum excitation energy)

25. Continuous X Ray is inverse of photoelectric effect

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26. Energy of photon = $E = \frac{hc}{\lambda}$

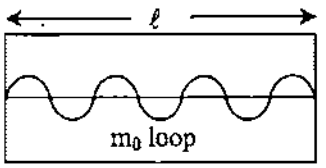
Momentum of photon = $P = \frac{h}{\lambda}$

$E = PC$

$\therefore \frac{E}{P} = C = 3 \times 10^8$ m/s

27. 0.06%

28.



KE of $e^- = \frac{hc}{\lambda}$

$$\lambda_{dc\text{-}broglie} = \frac{h}{\sqrt{2mKE}} = \frac{h}{\sqrt{2m \times \frac{hc}{\lambda}}} = \frac{\sqrt{h\lambda}}{\sqrt{2mc}}$$

$$\frac{m_0 \lambda_{debroglie}}{2} = \ell$$

$$\frac{m_0}{2} \times \frac{\sqrt{h\lambda}}{\sqrt{2mc}} = \frac{\sqrt{35h\lambda}}{\sqrt{8mc}} \Rightarrow \frac{m_0}{2\sqrt{2}} = \frac{\sqrt{35}}{2\sqrt{2}}$$

$m_0 = \sqrt{35} = 5.8 \approx 6$

i.e. e^- get excite to state 6.

$\therefore n = 6$

29. $r \propto \frac{1}{z}$ & $v \propto z$

$\therefore$ according $\left(\frac{v^2}{r}\right) \propto z^2 \times \frac{1}{(1/z)} = z^3$

$$\frac{a_H}{a_{He}} = \left(\frac{z_H}{z_{He}}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

30. $\rho = \frac{E}{J}$

$E = [M\,L\,T^{-3}\,I^{-1}]$

$J = [I\,L^{-2}]$

$h = M\,L^2\,T^{-1}$

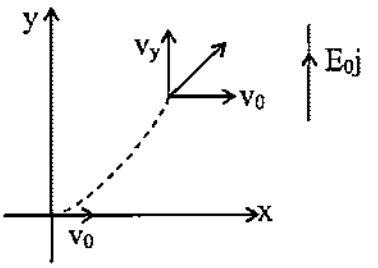
$m_e = M$

$c = LT^{-1}$

$\varepsilon_0 = M^{-1}\,L^{-3}\,T^4\,I^2$

$\therefore \rho = \frac{h^2}{m_e c e^2}$

31.



$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{v}$

$v = \sqrt{v_y^2 + v_0^2}$

$v_y = u_y + a_y t$

$v_y = 0 + \frac{qE_0}{m} t$

$(3v_0) = \sqrt{v_y^2 + v_0^2}$

$\Rightarrow v_y^2 = 8v_0^2$

$\Rightarrow \frac{qE_0}{m} t = 2\sqrt{2}\, v_0$

$t = \frac{2\sqrt{2}m}{qE_0} v_0 \Rightarrow t \propto \frac{1}{E_0}$

32. $mvr = n\frac{h}{2\pi}$

$v = r\omega$

$mr^2\omega = n\frac{h}{2\pi}$

$r = \sqrt{\frac{nh^2}{2\pi m\omega}}$

$r = \sqrt{n}\sqrt{\frac{h^2}{2\pi m\omega}}$

$r = \sqrt{n}\,.a$

33. $P = [M\,L^2\,T^{-3}]$

$\mu_0 = [M\,L\,T^{-2}\,I^{-2}]$

$m = [I\,L^2]$

$\omega = [T^{-1}]$

$C = [L\,T^{-1}]$

$\therefore [M\,L^2\,T^{-3}] = [M\,L\,T^{-2}\,I^{-2}]^x [I\,L^2]^y [T^{-1}]^z [LT^{-1}]^u$

$x = 1,\ y = 2, z = 4, u = -3$

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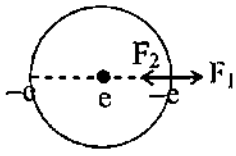
34.



$$mvr = n\frac{h}{2\pi} \quad ...(1)$$

$$\& \quad \frac{mv^2}{r} = \frac{ke^2}{r^2} = \frac{ke^2}{(2r)^2}$$

$$\frac{mv^2}{r} = \frac{3}{4}\frac{ke^2}{r^2}$$

$$\Rightarrow mv^2r = \frac{3}{4}ke^2 \quad ...(2)$$

Solving (1) + (2)

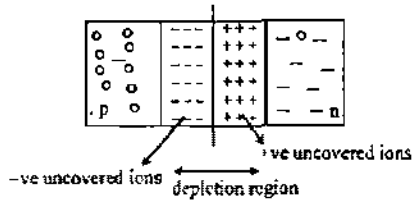
$$r = \frac{4\pi\varepsilon_0 h^2}{me^2} \times \frac{4}{3}$$

$$r = \frac{4}{3}a_B$$

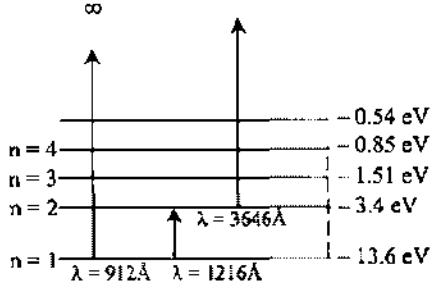
35. $_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$

$_4Be^9 + {}_2H^4 \rightarrow {}_6He^{12} + {}_0n^1$

36



37.



Since hydrogen is in it's neutral state. Therefore $\lambda = 3646$ Å will not strongly absorbed

38. $R = \frac{m_e e^4}{8\varepsilon_0^2 ch^3}$

where m_e = mass of electron

When we consider mass of nucleus also then we replace m with reduced mass

$$\mu = \frac{m_e m}{m_e + m}$$

where m = mass of nucleus

In case of hydrogen atom

$$\mu_1 = \frac{(m_e)(2000m_e)}{m_e + (2000)m_e} = \frac{2000}{2001}m_e$$

In case of deuterium

$$\mu_2 = \frac{(m_e)(4000m_e)}{m_e + (4000)m_e} = \frac{4000}{4001}m_e$$

Hence,

$$\frac{\lambda_2}{\lambda_1} = \frac{R_2}{R_1} = \frac{\mu_2}{\mu_1}$$

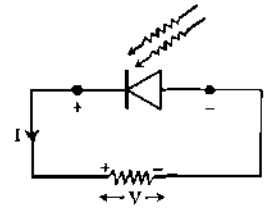
$$= \left(\frac{4000m_e}{4001}\right)\frac{(2001)}{(2000m_e)}$$

$$= \frac{4002}{4001}$$

$$\Rightarrow \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{1}{4001}$$

$$\therefore \quad \frac{\Delta\lambda}{\lambda_1} \times 100 = \frac{1}{4001} \times 100 = 0.025\%$$

39.



both i and V are of same sign

40. Statement I, II & III are correct

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Chemistry

[TOPIC- WISE KVPY [STREAM-SX] Questions with Solutions]

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Chapter 1

Physical Chemistry

1. The energy of a photon of wavelength k = 1 meter is (Planck's constant = 6.625×10^{-34} Js, speed of light = 3×10^{8} m/s) [2010]
(A) 1.988×10^{-25} J (B) 1.988×10^{-28} J
(C) 1.988×10^{-30} J (D) 1.988×10^{-25} J

2. The concentration of a substance undergoing a chemical reaction becomes one-half of its original value after time t regardless of the initial concentration. The reaction is an example of a - [2010]
(A) zero order reaction
(B) first order reaction
(C) second order reaction
(D) third order reaction

3. In the nuclear reaction
$^{234}_{90}Th \rightarrow ^{234}_{91}Pa + X$. X is – [2010]
(A) $^{0}_{-1}e$ (B) $^{0}_{1}e$
(C) H (D) $^{2}_{1}H$

4. A concentrated solution of copper sulphate, which is dark blue in colour, a mixed at room temperature with a dilute solution of copper sulphate, which is light blue. For this process – [2010]
(A) Entropy change is positive, but enthalpy change is negative
(B) Entropy and enthalpy changes are both positive
(C) Entropy change is positive and enthalpy does not change
(D) Entropy change is negative and enthalpy change is positive

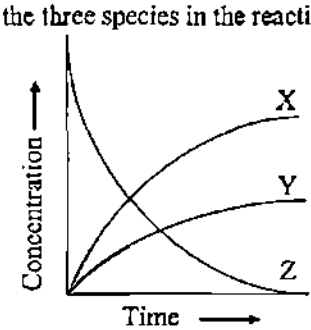
5. Increasing the temperature increases the rate of reaction but does not increase the - [2010]
(A) number of collisions
(B) activation energy
(C) average energy of collisions
(D) average velocity of the reactant molecules

6. In metallic solids, the number of atoms for the face-centered and the body centered cubic unit cells, are, respectively- [2010]
(A) 2, 4 (B) 2, 2 (C) 4, 2 (D) 4, 4

7. From equations 1 and 2,
$CO_2 \rightleftharpoons CO + 1/2\ O_2$
$[K_{cl} = 9.1 \times 10^{-13}$ at 1000°C] (eq. 1)
$H_2O \rightleftharpoons H_2 + 1/2\ O_2$
$[K_{cl} = 7.1 \times 10^{-12}$ at 1000°C] (eq. 2)
the equilibrium constant for the reaction $CO_2 + H_2 \rightleftharpoons CO + H_2O$ at the same temperature, is- [2010]
(A) 0.78 (B) 2.0 (C) 16.2 (D) 1.28

8. For a first order reaction R → P, the rate constant is k. If the initial concentration of R is $[R_0]$, the concentration of R at any time 't' is given by the expression- [2010]
(A) $[R_0]\ e^{kt}$ (B) $[R_0](1 - e^{-kt})$
(C) $[R_0]\ e^{-kt}$ (D) $[R_0]\ (1 - e^{kt})$

9. Consider the reaction :
$2\ NO_2(g) \rightarrow 2\ NO(g) + O_2\ (g)$. In the figure below, identify the curves X, Y and Z associated with the three species in the reaction - [2010]



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(A) X = NO, Y = O_2, Z = NO_2
(B) X = O_2, Y = NO, Z = NO_2
(C) X = NO_3, Y = NO, Z = O_2
(D) X = O_2, Y = NO_2, Z = NO

10. 2.52 g of oxalic acid *dehydrate* was dissolved in 100 ml of water, 10 mL of this solution was diluted to 500 mL. The normality of the final solution and the amount of oxalic acid (mg/mL) in the solution are respectively- **[2010]**
(A) 0.16 N, 5.04 (B) 0.08 N, 3.60
(C) 0.04 N, 3.60 (D) 0.02 N, 10.08

11. A solution containing 8.0 g of nicotine in 92 g of water freezes 0.925 degrees below the normal freezing point of water. If the molal freezing point depression constant $K_f = -1.85°C\ mol^{-1}$ then the molar mass of nicotine is - **[2010]**
(A) 16 (B) 80 (C) 320 (D) 160

12. For the reaction A → B, $\Delta H° = 7.5\ mol^{-1}$ and $\Delta S° = 2.5\ J\ mol^{-1}$. The value of $\Delta G°$ and the temperature at which the reaction reaches equilibrium are, respectively, **[2010]**
(A) 0 kJ mol^{-1} and 400 K
(B) –2.5 kJ mol^{-1} and 400 K
(C) 2.5 kJ mol^{-1} and 200 K
(D) 0 kJ mol^{-1} and 300 K

13. The solubility product of $Mg(OH)_2$ is 1.0×10^{-12}. Concentrated aqueous NaOH solution is added to a 0.01 M aqueous solution of $MgCl_2$. The pH at which precipitation occur is- **[2010]**
(A) 7.2 (B) 7.8 (C) 8.0 (D) 9.0

14. A metal with an atomic radius of 141.4 pm crystallizes in the face centred cubic structure. The volume of the unit cell in pm is- **[2010]**
(A) 2.74×10^7 (B) 2.19×10^7
(C) 6.40×10^7 (D) 9.20×10^7

15. Assuming ideal behaviour, the enthalpy and volume of mixing of two liquids, respectively, are **[2011]**
(A) zero and zero (B) + ve and zero
(C) –ve and zero (D) – ve and – ve

16. At 298 K, the ratio of osmotic pressures of two solutions of a substance with concentrations of 0.01 M and 0.001 M, respectively, is **[2011]**
(A) 1 (B) 100 (C) 10 (D) 1000

17. The rate of gas phase chemical reactions generally increases rapidly with rise in temperature. This is mainly because **[2011]**
(A) the collision frequency increases with temperature
(B) the fraction of molecules having energy in excess of the activation energy increases with temperature
(C) the activation energy decreases with temperature
(D) the average kinetic energy of molecules increases with temperature

18. For a process to occur spontaneously **[2011]**
(A) only the entropy of the system must increase
(B) only the entropy of the suroundings must increase
(C) either the entropy of the system or that of the surroundings must increase
(D) the total entropy of the system and the surroundings must increase

19. When the size of a spherical nanoparticle decreases from 30 nm to 10nm, the ratio surface area/volume becomes **[2011]**
(A) 1/3 of the original
(B) 3 times the original
(C) 1/9 of the original
(D) 9 times the original

20. The value of the limiting molar conductivity ($\Lambda°$) for NaCl, HCl and NaOAc are 126.4, 425.9 and 91.0 S $cm^2\ mol^{-1}$, respectively. For HOAc, $\Lambda°$ in S $cm^2\ mol^{-1}$ is **[2011]**
(A) 390.5 (B) 299.5 (C) 208.5 (D) 217.4

21. To obtain a diffraction peak, for a crystalline solid with interplane distance equal to the wavelength of incident X-ray radiation, the angle of incidence should be **[2011]**
(A) 90° (B) 0° (C) 30° (D) 60°

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22. The standard Gibbs free energy change (ΔG° in kJ mol^{-1}), in a Daniel cell ($E^\circ_{cell} = 1.1$ V), when 2 moles of Zn(s) is oxidized at 298 K, is closest to **[2011]**

(A) – 212.3 (B) – 106.2
(C) – 424.6 (D) – 53.1

23. In a zero-order reaction, if the initial concentration of the reactant is doubled, the time required for half the reactant to be consumed **[2011]**

(A) increases two-fold
(B) increases four-fold
(C) decreases by half
(D) does not change

24. The adsorption isotherm for a gas is given by the relation x = ap/(1 + bp) where x is moles of gas adsorbed per gram of the adsorbent, p is the pressure of the gas, and a and b are constants. Then x **[2011]**

(A) increases with p
(B) remains unchanged with p
(C) decreases with p
(D) increases with p at low pressures and then remains the same at high pressure

25. MnO_4^- oxidizes (i) oxalate ion in acidic medium at 333 K and and (ii) HCl. For balanced chemical equations, the ratios $[MnO_4^- : C_2O_4^{2-}]$ in (i) and $[MnO_4^- : HCl]$ in (ii), respectively, are - **[2011]**

(A) 1 : 5 and 2 : 5 (B) 2 : 5 and 1 : 8
(C) 2 : 5 and 1 : 5 (D) 5 : 2 and 1 : 8

26. If $E^\circ_{Fe^{2+}/Fe} = -0.440$ V and $E^\circ_{Fe^{3+}/Fe^{2+}} = 0.770$ V, then $E^\circ_{Fe^{3+}/Fe}$ is- **[2011]**

(A) 0.330 V (B) –0.037 V
(C) –0.330 V (D) –1.210 V

27. The electron in hydrogen atom is in the first Bohr orbit (n = 1). The ratio of transition energies, E(n = 1 → n = 3) to E(n = 1 → n = 2), is - **[2011]**

(A) 32/27 (B) 16/27
(C) 32/9 (D) 8/9

28. At 300 K the vapour pressure of two pure liquids, A and B are 100 and 500 mm Hg, respectively. If in a mixture of A and B, the vapour pressure is 300 mm Hg, the mole fractions of A in the liquid and in the vapour phase, respectively, are - **[2011]**

(A) 1/2 and 1/3 (B) 1/4 and 1/6
(C) 1/4 and 1/10 (D) 1/2 and 1/6

29. Emulsification of 10 ml of oil in water produces 2.4×10^{18} droplets. If the surface tension at the oil-water interface is 0.03 Jm^{-2} and the area of each droplet is 12.5×10^{-16} m^2, the energy spent in the formation of oil droplets is- **[2011]**

(A) 90 J (B) 30 J
(C) 900 J (D) 10 J

30. For a one-electron atom, the set of allowed quantum numbers is – **[2012]**

(A) $n = 1, l = 0, m_l = 0, m_s = +½$
(B) $n = 1, l = 1, m_l = 0, m_s = +½$
(C) $n = 1, l = 0, m_l = -1, m_s = -½$
(D) $n = 1, l = 1, m_l = 1, m_s = -½$

31. Typical electronic energy gaps in molecules are about 1.0 eV. In terms of temperature, the gap is closed to – **[2012]**

(A) 10^2 K (B) 10^4 K
(C) 10^3 K (D) 10^5 K

32. A zero-order reaction, A → Product, with an initial concentration $[A]_0$ has a half-life of 0.2 s. If one starts with the concentration $2[A]_0$, then the half-life is – **[2012]**

(A) 0.1 s (B) 0.4 s
(C) 0.2 s (D) 0.8 s

33. The isoelectronic pair of ions is – **[2012]**

(A) Sc^{2+} and V^{3+} (B) Mn^{3+} and Fe^{2+}
(C) Mn^{2+} and Fe^{3+} (D) Ni^{3+} and Fe^{2+}

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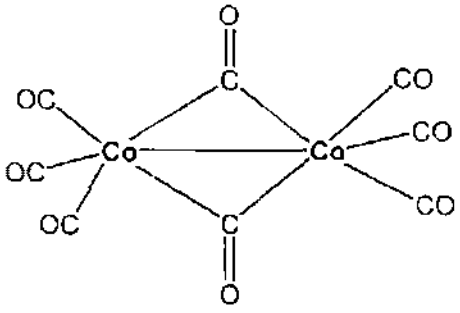
34. The oxidation state of cobalt in the following molecule is – **[2012]**

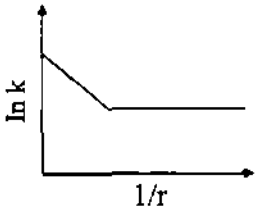


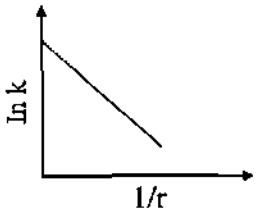
(A) 3 (B) 1 (C) 2 (D) 0

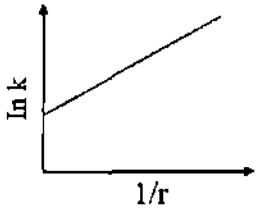
35. The pK_a of a weak acid is 5.85. The concentrations of the acid and its conjugate base are equal at a pH of – **[2012]**

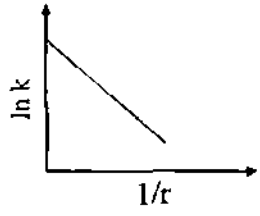
(A) 6.85 (B) 5.85 (C) 4.85 (D) 7.85

36. Among the following graphs showing variation of rate (k) with temperature (T) for a reaction, the one that exhibits arrhenius behavior over the entire temperature range is – **[2012]**

(A) 

(B) 

(C) 

(D) 

37. The C–O bond length in CO, CO_2 and CO_3^{2-} follows the order – **[2012]**

(A) $CO < CO_2 < CO_3^{2-}$
(B) $CO_2 < CO_3^{2-} < CO$
(C) $CO > CO_2 > CO_3^{2-}$
(D) $CO_3^{2-} < CO_2 < CO$

38. The equilibrium constant for the following reactions are K_1 and K_2, respectively. **[2012]**

$2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g)$
$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$

Then the equilibrium constant for the reaction $2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g)$ is –

(A) K_1K_2 (B) $K_1K_2^2$ (C) $K_1^2K_2^2$ (D) $K_1^2K_2$

39. Doping silicon with boron produces a - **[2012]**

(A) n-type semiconductor
(B) Metallic conductor
(C) p-type semiconductor
(D) Insulator

40. In the DNA of E. Coli the mole ratio of adenine to cytosine is 0.7. If the number of moles of adenine in the DNA is 350000, the number of moles of guanine is equal to – **[2012]**

(A) 350000 (B) 500000
(C) 225000 (D) 700000

41. A metal is irradiated with light of wavelength 660 nm. Given that the work that the work function of the metal is 1.0 eV, the de Broglie wavelength of the ejected electron is close to – **[2012]**

(A) 6.6×10^{-7} m (B) 8.9×10^{-11} m
(C) 1.3×10^{-9} m (D) 6.6×10^{-13} m

42. The inter-planar spacing between the (2 2 1) planes of a cubic lattice of length 450 pm is - **[2012]**

(A) 50 pm (B) 150 pm
(C) 300 pm (D) 450 pm

43. The ΔH for vaporization of a liquid is 20 kJ/mol. Assuming ideal behaviour, the change in internal energy for the vaporization of 1 mol of the liquid at 60°C and 1 bar is close to – **[2012]**

(A) 13.2 kJ/mol (B) 17.2 kJ/mol
(C) 19.5 kJ/mol (D) 20.0 kJ/mol

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44. Three moles of an ideal gas expands reversibly under isothermal condition form 2 L to 20 L at 300 K. The amount of heat-change (in kJ/mol) in the process is – **[2012]**

(A) 0 (B) 7.2 (C) 10.2 (D) 17.2

45 The following data are obtained for a reaction, $X + Y \rightarrow$ Products. **[2012]**

Expt.	$[X_0]$/mol	$[Y_0]$/mol	rate/mol $L^{-1}s^{-1}$
1	0.25	0.25	1.0×10^{-6}
2	0.50	0.25	4.0×10^{-6}
3	0.25	0.50	8.0×10^{-6}

The overall order of the reaction is

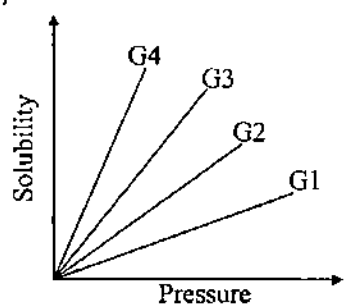
(A) 2 (B) 4 (C) 3 (D) 5

46. For a zero-order reaction with rate constant k, the slope of the plot of reactant concentration against time is – **[2013]**

(A) k/2.303 (B) k

(C) –k/2.303 (D) – k

47. The variation of solubility of four different gases (G1, G2, etc.) in a given solvent with pressure at a constant temperature is shown in the plot. - **[2013]**



The gas with the highest value of Henry's law constant is - **[2013]**

(A) G4 (B) G2

(C) G3 (D) G1

48. For the reaction, A $\rightleftharpoons$ nB the concentration of A decreases from 0.06 to 0.03 mol L^{-1} and that of B rises from 0 to 0.06 mol L^{-1} at equilibrium. The values of n and the equilibrium constant for the reaction, respectively, are - **[2013]**

(A) 2 and 0.12 (B) 2 and 1.2

(C) 3 and 0.12 (D) 3 and 1.2

49. The entropy change in the isothermal reversible expansion of 2 moles of an ideal gas from 10 to 100 L at 300 K is - **[2013]**

(A) 42.3 JK^{-1} (B) 35.8 JK^{-1}

(C) 38.3 JK^{-1} (D) 32.3 JK^{-1}

50. For the isothermal reversible expansion of an ideal gas - **[2013]**

(A) $\Delta H > 0$ and $\Delta U = 0$

(B) $\Delta H > 0$ and $\Delta U < 0$

(C) $\Delta H = 0$ and $\Delta U = 0$

(D) $\Delta H = 0$ and $\Delta U > 0$

51. If the angle of incidence of X-ray of wavelength 3Å which produces a second order diffracted beam from the (100) planes in a simple cubic lattice with interlayer spacing a = 6Å is 30°, the angle of incidence that produces a first-order diffracted beam from the (200) planes is – **[2013]**

(A) 15° (B) 45° (C) 30° (D) 60°

52. The number of ions produced in water by dissolution of the complex having the empirical formula, $CoCl_3.4NH_3$, is - **[2013]**

(A) 1 (B) 2 (C) 4 (D) 3

53. The spin-only magnetic moments of $[Fe(NH_3)_6]^{3+}$ and $[FeF_6]^{3-}$ in BM are, respectively, **[2013]**

(A) 1.73 and 1.73 (B) 5.92 and 1.73

(C) 1.73 and 5.92 (D) 5.92 and 5.92

54. An ionic compound is formed between a metal M and a non-metal Y. If M occupies half the octahedral voids in the cubic close-packed arrangement formed by Y, the chemical formula of the ionic compound is - **[2013]**

(A) MY (B) MY_2 (C) M_2Y (D) MY_3

55. In 108 g of water, 18 g of a non-voltile compound is dissolved. At 100°C the vapour pressure of the solution is 750 mm Hg. Assuming that the compound does not undergo association or dissociation, the molar mass of the compound in g mol^{-1} is - **[2013]**

(A) 128 (B) 182 (C) 152 (D) 228

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56. The standard electrode potential of Zn^{2+}/Zn is – 0.76 V and that of Cu^{2+}/Cu is 0.34 V. The emf(V) and the free energy change (kJ mol^{-1}), respectively, for a Daniel cell will be - **[2013]**

(A) – 0.42 and 81 (B) 1.1 and – 213
(C) – 1.1 and 213 (D) 0.42 and – 81

57. Consider the equilibria (1) and (2) the with equilibrium constants K_1 and K_2, respectively. **[2013]**

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g) \quad \ldots (1)$$

$$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g) \quad \ldots (2)$$

K_1 and K_2 are related as -

(A) $2K_1 = K_2^2$ (B) $K_1^2 = \frac{1}{K_2}$

(C) $K_2^2 = \frac{1}{K_1}$ (D) $K_2 = \frac{2}{K_1^2}$

58. The density and eq. wt. of a metal are 10.5 g cm^{-3} and 100, respectively. The time required for a current of 3 amp to deposit a 0.005 mm thick layer of the same metal on an area of 80 cm^2 is closest to- **[2013]**

(A) 120 s (B) 135 s
(C) 67.5 s (D) 270 s

59. The amount of $Na_2S_2O_3.5H_2O$ required to completely reduce 100 mL of 0.25 N iodine solution, is - **[2013]**

(A) 6.20 g (B) 9.30 g
(C) 3.10 g (D) 7.75 g

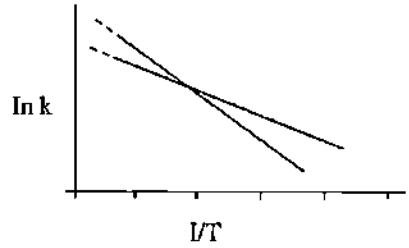
60. When 1.88 g of AgBr(s) is added to a 10^{-3} M aqueous solution of KBr, the concentration of Ag is 5×10^{-10} M. If the same amount of AgBr(s) is added to a 10^{-2} M aqueous solution of $AgNO_3$, the concentration of Br^- is **[2014]**

(A) 9.4×10^{-9} M (B) 5×10^{-10} M
(C) 1×10^{-11} M (D) 5×10^{-11} M

61. The metal with the highest oxidation state present in K_2CrO_4, $NbCl_5$ and MnO_2 is - **[2014]**

(A) Nb (B) Mn
(C) K (D) Cr

62. The Arrhenius plots of two reactions, I and II are shown graphically - **[2014]**



The graph suggests that -

(A) $E_I > E_{II}$ and $A_I > A_{II}$
(B) $E_{II} > E_I$ and $A_{II} > A_I$
(C) $E_I > E_{II}$ and $A_{II} > A_I$
(D) $E_{II} > E_I$ and $A_I > A_{II}$

63. In a cubic close packed structure, fractional contributions of an atom at the corner and at the face in the unit cell are, respectively - **[2014]**

(A) 1/8 and 1/2 (B) 1/2 and 1/4
(C) 1/4 and 1/2 (D) 1/4 and 1/8

64. The equilibrium constant K_c of the reaction, $2A \rightleftharpoons B+C$ is 0.5 at 25°C and 1 atm. The reaction will proceed in the backward direction when concentrations [A], [B] and [C] are, respectively - **[2014]**

(A) 10^{-3}, 10^{-2} and 10^{-2} M
(B) 10^{-1}, 10^{-2} and 10^{-2} M
(C) 10^{-2}, 10^{-2} and 10^{-3} M
(D) 10^{-2}, 10^{-3} and 10^{-3} M

65. If the molar conductivities (in S cm^2 mol^{-1}) of NaCl, KCl and NaOH at infinite dilution are 126, 150 and 250 respectively, the molar conductivity of KOH (in S cm^2 mol^{-1}) is - **[2014]**

(A) 526 (B) 226 (C) 26 (D) 274

66. Two elements, X and Y, have atomic numbers 33 and 17, respectively. The molecular formula of a stable compound formed between them is - **[2014]**

(A) XY (B) XY_2 (C) XY_3 (D) XY_4

67. The number of moles of $KMnO_4$ required to oxidize one equivalent of KI in the presence of sulfuric acid is – **[2014]**

(A) 5 (B) 2 (C) 1/2 (D) 1/5

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68. Three successive measurements in an experiment gave the values 10.9, 11.4042 and 11.42. The correct way of reporting the average value is - **[2014]**
(A) 11.2080 (B) 11.21
(C) 11.2 (D) 11

69. The latent heat of melting of ice at 0 °C is 6 kJ mol^{-1}. The entropy change during the melting in J K^{-1} mol^{-1} is closest to - **[2014]**
(A) 22 (B) 11 (C) –11 (D) –22

70. The density of acetic acid vapour at 300 K and 1 atm is 5 mg cm^{-3}. The number of acetic acid molecules in the cluster that is formed in the gas phase is closest to **[2014]**
(A) 5 (B) 2 (C) 3 (D) 4

71. The molar enthalpy change for $H_2O(l) \rightleftharpoons H_2O(g)$ at 373 K and 1 atm is 41 kJ/mol. Assuming ideal behaviour, the internal energy change for vaporization of 1 mol of water at 373 K and 1 atm in kJ mol^{-1} is : **[2014]**
(A) 30.2 (B) 41.0 (C) 48.1 (D) 37.9

72. The equilibrium constant (K_c) of two reactions $H_2 + I_2 \rightleftharpoons 2HI$ and $N_2 + 3H_2 \rightleftharpoons 2NH_3$ are 50 and 1000, respectively. The equilibrium constant of the reaction $N_2 + 6HI \rightleftharpoons 2NH_3 + 3I_2$ is closest to : **[2014]**
(A) 50000 (B) 20
(C) 0.008 (D) 0.005

73. Given that the bond energies of : N≡N is 946 kJ mol^{-1}, H–H is 435 kJ mol^{-1}, N–N is 159 kJ mol^{-1}, and N–H is 389 kJ mol^{-1}, the heat of formation of hydrazine in the gas phase in kJ mol^{-1} is : **[2014]**
(A) 833 (B) 101
(C) 334 (D) 1268

74. The radius of K^+ is 133 pm and that of Cl is 181 pm. The volume of the unit cell of KCl expressed in 10^{-22} cm^3 is : **[2014]**
(A) 0.31 (B) 1.21
(C) 2.48 (D) 6.28

75. The reaction, $K_2Cr_2O_7 + m\ FeSO_4 + n\ H_2SO_4 \rightarrow Cr_2(SO_4)_3 + p\ Fe_2(SO_4)_3 + K_2SO_4 + q\ H_2O$ when balanced, m, n, p and q are, respectively : **[2014]**
(A) 6, 14, 3, 14 (B) 6, 7, 3, 7
(C) 3, 7, 2, 7 (D) 4, 14, 2, 14

76. The standard free energy change (in J) for the reaction $3Fe^{2+}(aq) + 2Cr(s) = 2Cr^{3+}(aq) + 3Fe(s)$ given $E^0_{Fe^{2+}/Fe} = -0.44$ V and $E^0_{Cr^{3+}/Cr} = -0.74$ V is (F = 96500 C) **[2014]**
(A) 57,900 (B) –57,900
(C) 173,700 (D) 173, 700

77. The rate constant of a chemical reaction at a very high temperature will approach **[2015]**
(A) Arrhenius frequency factor divided by the ideal gas constant
(B) activation energy
(C) Arrhenius frequency factor
(D) activation energy divided by the ideal gas constant

78. The standard reduction potentials (in V) of a few metal ion/metal electrodes are given below. $Cr^{3+}/Cr = -0.74$; $Cu^{2+}/Cu = +0.34$; $Pb^{2+}/Pb = -0.13$; $Ag^+/Ag = +0.8$. The reducing strength of the metals follows the order **[2015]**
(A) Ag > Cu > Pb > Cr
(B) Cr > Pb > Cu > Ag
(C) Pb > Cr > Ag > Cu
(D) Cr > Ag > Cu > Pb

79. The molar conductivities of HCl, NaCl, CH_3COOH, and CH_3COONa at infinite dilution follow the order **[2015]**
(A) HCl > CH_3COOH > NaCl > CH_3COONa
(B) CH_3COONa > HCl > NaCl > CH_3COOH
(C) HCl > NaCl > CH_3COOH > CH_3COONa
(D) CH_3COOH > CH_3COONa > HCl > NaCl

80. The spin only magnetic moment of $[ZCl_4]^{2-}$ is 3.87 BM where Z is **[2015]**
(A) Mn (B) Ni
(C) Co (D) Cu

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81. The pH of 1N aqueous solutions of HCl, CH_3COOH and HCOOH follows the order **[2015]**
(A) HCl > HCOOH > CH_3COOH
(B) HCl = HCOOH > CH_3COOH
(C) CH_3COOH > HCOOH > HCl
(D) CH_3COOH = HCOOH > HCl

82. Schottky defect in a crystal arises due to **[2015]**
(A) creation of equal number of cation and anion vacancies
(B) creation of unequal number of cation and anion vacancies
(C) migration of cations to interstitial voids
(D) migration of anions to interstitial voids

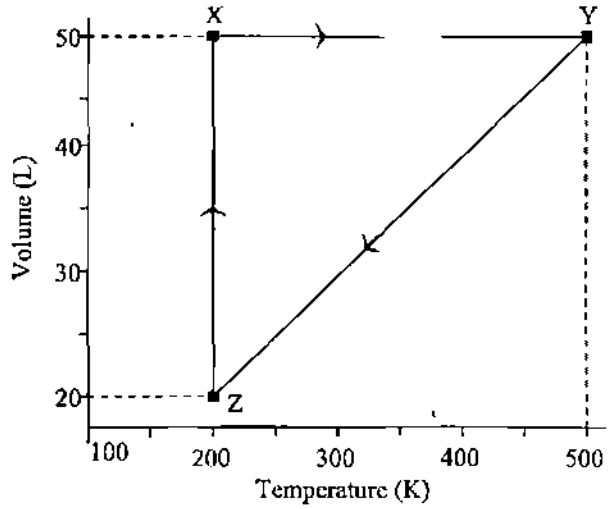
83. For the reaction $N_2 + 3X_2 \rightarrow 2NX_3$ where X = F, Cl (the average bond energies are F-F = 155 kJ mol^{-1} N-F = 272 kJ mol^{-1}, Cl-Cl = 242 kJ mol^{-1}, N-Cl 200 kJ mol^{-1} and N ≡ N -- 941 kJ mol^{-1}), the heats of formation of NF_3 and NCl_3 in kJ mol^{-1}, respectively, are closest to **[2015]**
(A) – 226 and +467
(B) + 226 and – 467
(C) – 151 and + 311
(D) + 151 and – 311

84. The equilibrium constants for the reactions X = 2Y and Z = P + Q are K_1 and K_2, respectively. If the initial concentrations and the degree of dissociation of X and Z are the same, the ratio K_1/K_2 is **[2015]**
(A) 4 (B) 1
(C) 0.5 (D) 2

85. The geometry and the number of unpaired electron(s) of $[MnBr_4]^{2-}$, respectively, are **[2015]**
(A) tetrahedral and 1
(B) square planar and 1
(C) tetrahedral and 5
(D) square planar and 5

86. The standard cell potential for $Zn|Zn^{2+}||Cu^{2+}|Cu$ is 1.10 V. When the cell is completely discharged, log $[Zn^{2+}]/[Cu^{2+}]$ is closest to **[2015]**
(A) 37.3 (B) 0.026
(C) 18.7 (D) 0.052

87. The volume vs. temperature graph of 1 mole of an ideal gas is given below



The pressure of the gas (in atm) at X, Y and Z, respectively, are **[2015]**
(A) 0.328, 0.820, 0.820
(B) 3.28, 8.20, 3.28
(C) 0.238, 0.280, 0.280
(D) 32.8, 0.280, 82.0

88. The pK_a of oxoacids of chlorine in water follows the order **[2016]**
(A) HClO < $HClO_3$ < $HClO_2$ < $HClO_4$
(B) $HClO_4$ < $HClO_3$ < $HClO_2$ < HClO
(C) $HClO_4$ < $HClO_2$ < $HClO_3$ < HClO
(D) $HClO_2$ < HClO < $HClO_3$ < $HClO_4$

89. The packing efficiency of the face centered cubic (fcc), body centered cubic (bcc) and simple / primitive cubic (pc) lattices follows the order **[2016]**
(A) fcc > bcc > pc (B) bcc > fcc > pc
(C) pc > bcc > fcc (D) bcc > pc > fcc

90. The ratio of root mean square velocity of hydrogen at 50 K to that of nitrogen at 500 K is closest to **[2016]**
(A) 1.18 (B) 0.85
(C) 0.59 (D) 1.40

91. It takes 1h for a first order reaction to go to 50% completion. The total time required for the same reaction to reach 87.5% completion will be **[2016]**
(A) 1.75 h (B) 6.00 h
(C) 3.50 h (D) 3.00 h

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92. A unit cell of calcium fluoride has four calcium ions. The number of fluoride ions in the unit cell is **[2016]**

(A) 2 (B) 4 (C) 6 (D) 8

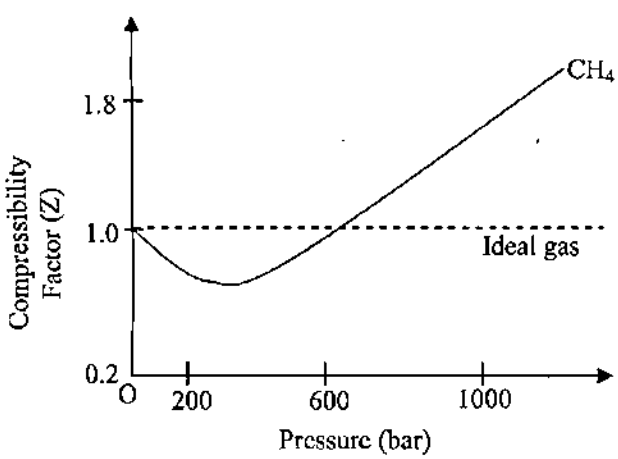
93. The equilibrium constant of a 2 electron redox reaction at 298 K is 3.8×10^{-3}. The cell potential E° (in V) and the free energy change ΔG° (in kJ mol^{-1}) for this equilibrium respectively, are **[2016]**

(A) – 0.071, –13.8 (B) – 0.071, 13.8
(C) 0.71, –13.8 (D) 0.071, –13.8

94. In the radioactive disintegration series $^{232}_{90}Th \longrightarrow {}^{208}_{82}Pb$, involving α and β decay, the total number of α and β particles emitted are **[2016]**

(A) 6 α and 6 β (B) 6 α and 4 β
(C) 6 α and 5 β (D) 5 α and 6 β

95. In the following compressibility factor (Z) vs pressure graph at 300 K, the compressibility of CH_4 at pressure < 200 bar deviates from ideal behaviour because **[2016]**



(A) The molar volume of CH_4 is less than its molar volume in the ideal state
(B) The molar volume of CH_4 is same as that in its ideal state
(C) Intermolecular interactions between CH_4 molecules decresases
(D) The molar volume of CH_4 is more than its molar volume in the ideal state

96. Copper (atomic mass = 63.5) crystallizers in a FCC lattice and has density 8.93 $g.cm^{-3}$. **[2016]** The radius of copper atom is closest to

(A) 361.6 pm (B) 511.4 pm
(C) 127.8 pm (D) 102.8 pm

97. Given the standard potentials $E^\circ(Cu^{2+}/Cu)$ and $E^\circ(Cu^{+}/Cu)$ as 0.340 V and 0.522 V respectively, the value of $E^\circ(Cu^{2+}/Cu^{+})$ is **[2016]**

(A) 0.364 V (B) 0.158 V
(C) – 0.182 V (D) – 0.316 V

98. For electroplating, 1.5 amp current is passed for 250 s through 250 mL of 0.15 M solution of MSO_4. Only 85% of the current was utilized for electrolysis. The molarity of MSO_4 solution after electrolysis is closest to
[Assume that the volume of the solution remained constant] **[2016]**

(A) 0.14 (B) 0.014
(C) 0.07 (D) 0.035

99. 2.33 g of compound X (empirical formula $CoH_{12}N_4Cl_3$) upon treatment with excess $AgNO_3$ solution produces 1.435 g of a white precipitate. The primary and secondary valences of cobalt in compound X, respectively, are **[2016]**
[Given : Atomic mass : Co = 59, Cl = 35.5, Ag = 108]

(A) 3, 6 (B) 3, 4 (C) 2, 4 (D) 4, 3

100. The specific conductance (κ) of 0.02 M aqueous acetic acid solution at 298 K is 1.65×10^{-4} S cm^{-1}. The degree of dissociation of acetic acid is **[2016]**
[Given : equivalent conductance at infinite dilution of H^+ = 349.1 S cm^2 mol^{-1} and CH_3COO^- = 40.9 S cm^2 mol^{-1}]

(A) 0.021 (B) 0.21
(C) 0.012 (D) 0.12

101. Among the following metals, the strongest reducing agent is **[5 Nov, 2017]**

(A) Ni (B) Cu (C) Zn (D) Fe

102. Chlorine has two naturally occurring isotopes, ^{35}Cl and ^{37}Cl. If the atomic mass of Cl is 35.45, the ratio of natural abundance of ^{35}Cl and ^{37}Cl is closest to **[5 Nov, 2017]**

(A) 3.5 : 1 (B) 3 :1
(C) 2.5 : 1 (D) 4 : 1

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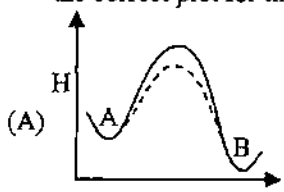
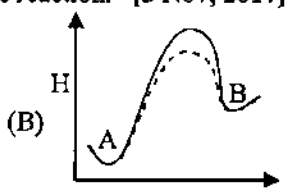
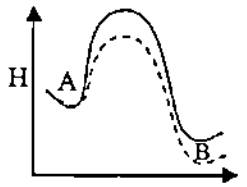
103. The reaction $C_2H_6(g) \rightleftharpoons C_2H_4(g) + H_2(g)$ is at equilibrium in a closed vessel at 1000 K. The enthalpy change (ΔH) for the reaction is 137.0 kJ mol^{-1}. Which one of the following actions would shift the equilibrium to the right ? **[5 Nov, 2017]**

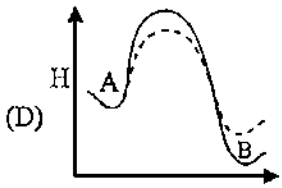
(A) Decreasing the volume of the closed reaction vessel
(B) Decreasing the temperature at which the reaction is performed
(C) Adding an inert gas to the closed reaction vessel
(D) Increasing the volume of the closed reaction vessel

104. The enthalpy (H) of an elementary exothermic reaction A $\rightleftharpoons$ B is schematically plotted against the reaction coordinate. The plots in the presence and absence of a catalyst are shown in dashed and solid lines, respectively. Identify the correct plot for the reaction. **[5 Nov, 2017]**

(A) 

(B) 

(C) 

(D) 

105. $Mg(OH)_2$ is precipitated when NaOH is added to a solution of Mg^{2+}. If the final concentration of Mg^{2+} is 10^{-10} M, the concentration of OH^-(M) in the solution is
[Solubility product for $Mg(OH)_2 = 5.6 \times 10^{-12}$] **[5 Nov, 2017]**

(A) 0.056 (B) 0.12
(C) 0.24 (D) 0.025

106. A constant current (0.5 amp) is passed for 1 hour through (i) aqueous $AgNO_3$, (ii) aqueous $CuSO_4$ and (iii) molten AlF_3, separately. The ratio of the mass of the metals deposited on the cathode is
[M_{Ag}, M_{Cu}, M_{Al} are molar masses of the respective metals] **[5 Nov, 2017]**

(A) $M_{Ag} : 2\,M_{Cu} : 3\,M_{Al}$
(B) $M_{Ag} : M_{Cu} : M_{Al}$
(C) $6\,M_{Ag} : 3\,M_{Cu} : 2\,M_{Al}$
(D) $3\,M_{Ag} : 2\,M_{Cu} : M_{Al}$

107. A reaction has an activation energy of 209 kJ mol^{-1}. The rate increases 10–fold when the temperature is increased from 27°C to X °C. The temperature X is closest to
[Gas constant, R = 8.314 J mol^{-1} K^{-1}] **[5 Nov, 2017]**

(A) 35 (B) 40 (C) 30 (D) 45

108. A mineral consists of a cubic close-packed structure formed by O^{2-} ions where half the octahedral voids are occupied by Al^{3+} and one-eighth of the tetrahedral voids are occupied by Mn^{2+}. The chemical formula of the mineral is **[5 Nov, 2017]**

(A) $Mn_3Al_2O_6$ (B) $MnAl_2O_4$
(C) $MnAl_4O_7$ (D) $Mn_2Al_2O_5$

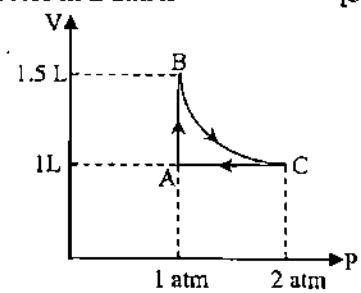
109. For a 4p orbital, the number of radial and angular nodes, respectively, are **[5 Nov, 2017]**

(A) 3,2 (B) 1,2
(C) 2, 4 (D) 2,1

110. ^{227}Ac has a half-life of 22 years with respect to radioactive decay. The decay follows two parallel paths : $^{227}Ac \rightarrow {}^{227}Th$ and $^{227}Ac \rightarrow {}^{223}Fr$. If the percentage of the two daughter nuclides are 2.0 and 98.0, respectively, the decay constant (in $year^{-1}$) for $^{227}Ac \rightarrow {}^{227}Th$ path is closest to **[5 Nov, 2017]**

(A) 6.3×10^{-2} (B) 6.3×10^{-3}
(C) 6.3×10^{-1} (D) 6.3×10^{-4}

111. A system consisting of 1 mol of an ideal gas undergoes a reversible process, A → B → C → A (schematically indicated in the figure below). If the temperature at the starting point A is 300 K and the work done in the process B → C is 1 L atm, the heat exchanged in the entire process in L atm is **[5 Nov, 2017]**



(A) 1.0 (B) 0.0
(C) 1.5 (D) 0.5

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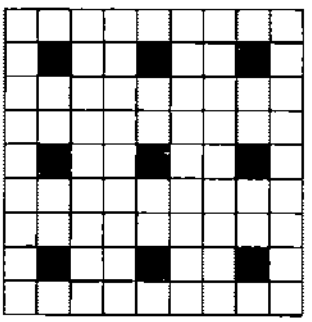
112. A mixture of toluene and benzene boils at 100°C. Assuming ideal behaviour, the mole fraction of toluene in the mixture is closest to
[Vapour pressures of pure toluene and pure benzene at 100°C are 0.742 and 1.800 bar respectively. 1 atm = 1.013 bar] **[5 Nov, 2017]**

(A) 0.824 (B) 0.744
(C) 0.544 (D) 0.624

113. A two-dimensional solid pattern formed by two different atoms X and Y is shown below. The black and white squares represent atoms X and Y, respectively. The simplest formula for the compound based on the unit cell from the pattern is **[5 Nov, 2017]**



(A) XY_8 (B) X_4Y_9
(C) XY_2 (D) XY_4

114. The oxidising ability of the given anions follows the order **[19 Nov, 2017]**

(A) $TiO_4^{4-} < VO_4^{3-} < CrO_4^{2-} < MnO_4^-$
(B) $VO_4^{3-} < CrO_4^{2-} < MnO_4^- < TiO_4^{4-}$
(C) $CrO_4^{2-} < MnO_4^- < VO_4^{3-} < TiO_4^{4-}$
(D) $VO_4^{3-} < TiO_4^{4-} < CrO_4^{2-} < MnO_4^-$

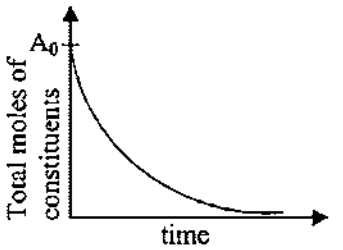
115. An excess of $Ag_2CrO_4(s)$ is added to a 5×10^{-3} M K_2CrO_4 solution. The concentration of Ag^+ in the solution is closest to
[Solubility product for $Ag_2CrO_4 = 1.1 \times 10^{-12}$] **[19 Nov, 2017]**

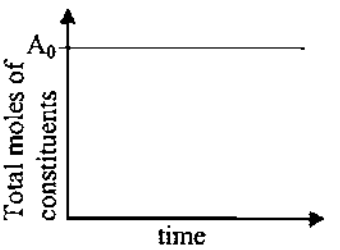
(A) 2.2×10^{-10} M (B) 1.5×10^{-5} M
(C) 1.0×10^{-6} M (D) 5.0×10^{-3} M

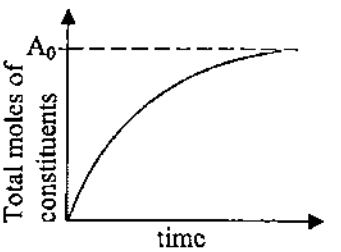
116. The packing efficiency in a body-centred cubic (bcc) structure is closet to **[19 Nov, 2017]**

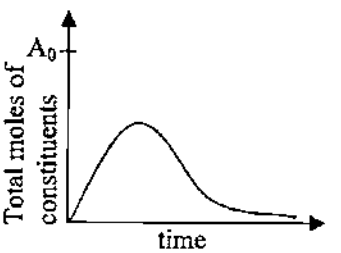
(A) 74 % (B) 63%
(C) 68 % (D) 52%

117. The consecutive reaction $X \rightarrow Y \rightarrow Z$ takes place in a closed container. Initially, the container has A_0 moles of X (and no Y and Z). The plot of total moles of the constituents in the container as a function of time will be **[19 Nov, 2017]**

(A)


(B)


(C)


(D)


118. The particle emitted during the sequential radioactive decay of $^{238}U_{92}$ to $^{206}Pb_{82}$ are **[19 Nov, 2017]**

(A) 5 α and 6 β (B) 6 α and 8 β
(C) 8 α and 4 β (D) 8 α and 6 β

119. The allowed set of quantum numbers for an electron in a hydrogen atom is **[19 Nov, 2017]**

(A) $n = 4, l = 2, m_l = 0, m_s = 0$
(B) $n = 3, l = 1, m_l = -3, m_s = -1/2$
(C) $n = 3, l = 3, m_l = -1, m_s = 1/2$
(D) $n = 2, l = 1, m_l = -1, m_s = 1/2$

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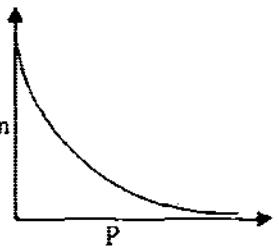
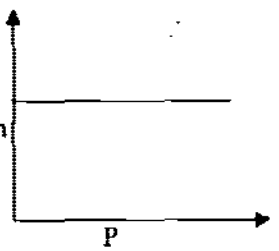
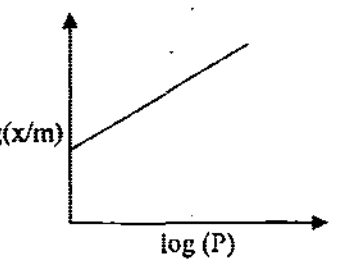
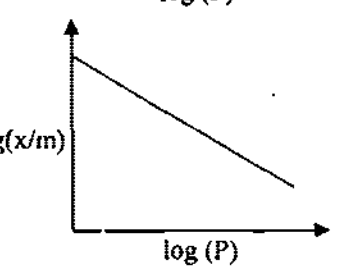
120. The plot that best represents the relationship between the extent of adsorption (x/m) and pressure (P) is **[19 Nov, 2017]**

(A)


(B)


(C)


(D)


121. The pH of 0.1 M acetic acid solution is closest to [Dissociation constant of acid $K_a = 1.8 \times 10^{-5}$] **[19 Nov, 2017]**

(A) 2.87 (B) 1.00
(C) 2.07 (D) 4.76

122. The limiting molar conductivities of the given electrolytes at 298 K follow the order
[$\lambda^0(K^+) = 73.5$, $\lambda^0(Cl^-) = 76.3$,
$\lambda^0(Ca^{2+}) = 119.0$, $\lambda^0(SO_4^{2-}) = 160.0$ S cm^2 mol^{-1}] **[19 Nov, 2017]**

(A) $KCl < CaCl_2 < K_2SO_4$
(B) $KCl < K_2SO_4 < CaCl_2$
(C) $K_2SO_4 < CaCl_2 < KCl$
(D) $CaCl_2 < K_2SO_4 < KCl$

123. The maximum work (in kJ mol^{-1}) that can be derived from complete combustion of 1 mol of CO at 298 K and 1 atm is
[Standard enthalpy of combustion of CO = –283.0 kJ mol^{-1} ; standard molar entropies at 298 K ; $S_{O_2} = 205.1$ J mol^{-1}, $S_{CO} = 197.7$ J mol^{-1}, $S_{CO_2} = 213.7$ J mol^{-1}] **[19 Nov, 2017]**

(A) 257 (B) 227
(C) 57 (D) 127

124. 18 g of glucose ($C_6H_{12}O_6$) dissolved in 1 kg of water is heated to boiling. The boiling point (in K) measured at 1 atm pressure is closest to [Ebullioscopic constant, K_b for water is 0.52 K kg mol^{-1}. Consider absolute zero to be –273.15°C] **[19 Nov, 2017]**

(A) 373.15 (B) 373.10
(C) 373.20 (D) 373.25

125. Polonium (atomic mass = 209) crystallizes in a simple cubic structure with a density of 9.32 g cm^{-3}. Its lattice parameter (in pm) is closest to **[19 Nov, 2017]**

(A) 421 (B) 334
(C) 481 (D) 193

126. The following reaction takes place at 298 K in an electrochemical cell involving two metals A and B, $A^{2+}(aq.) + B(s) \rightarrow B^{2+}(aq.) + A(s)$
With $[A^{2+}] = 4 \times 10^{-3}$ M and $[B^{2+}] = 2 \times 10^{-3}$M in the respective half-cells, the cell EMF is 1.091 V. The equilibrium constant of the reaction is closest to **[19 Nov, 2017]**

(A) 4×10^{36} (B) 2×10^{37}
(C) 2×10^{34} (D) 4×10^{37}

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ANSWERS

1. (A)	2. (D)	3. (A)	4. (C)	5. (B)	6. (B)	7. (D)
8. (B)	9. (A)	10. (C)	11. (D)	12. (D)	13. (D)	14. (C)
15. (A)	16. (C)	17. (B)	18. (D)	19. (D)	20. (A)	21. (C)
22. (C)	23. (A)	24. (D)	25. (B)	26. (B)	27. (A)	28. (D)
29. (A)	30. (A)	31. (B)	32. (B)	33. (C)	34. (D)	35. (B)
36. (D)	37. (A)	38. (B)	39. (C)	40. (B)	41. (C)	42. (B)
43. (B)	44. (D)	45. (D)	46. (D)	47. (D)	48. (A)	49. (C)
50. (C)	51. (C)	52. (B)	53. (C)	54. (B)	55. (D)	56. (B)
57. (B)	58. (B)	59. (A)	60. (D)	61. (D)	62. (A)	63. (A)
64. (A)	65. (D)	66. (C)	67. (D)	68. (C)	69. (A)	70. (B)
71. (D)	72. (C)	73. (B)	74. (C)	75. (B)	76. (C)	77. (C)
78. (B)	79. (A)	80. (C)	81. (C)	82. (A)	83. (A)	84. (A)
85. (C)	86. (A)	87. (A)	88. (B)	89. (A)	90. (A)	91. (D)
92. (D)	93. (B)	94. (B)	95. (A)	96. (C)	97. (B)	98. (A)
99. (A)	100. (A)	101. (C)	102. (B)	103. (D)	104. (A)	105. (C)
106. (C)	107. (A)	108. (B)	109. (D)	110. (D)	111. (C)	112. (B)
113. (A)	114. (A)	115. (B)	116. (C)	117. (B)	118. (D)	119. (D)
120. (C)	121. (A)	122. (A)	123. (A)	124. (C)	125. (B)	126. (B)

SOLUTIONS

1. $E = \frac{hc}{\lambda} = \frac{6.02\times10^{-34}\times3\times10^{8}}{1}$

$E = 1.988 \times 10^{-23}$ J

2. Informative

3. $^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa + {}_{-1}e^{0}$

4. Informative

5. Informative

6. $Fcc = 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

$Bcc = 8 \times \frac{1}{8} + 1 \times 1 = 2$

7. (i) $CO_2 \rightleftharpoons CO + \frac{1}{2}O_2 \quad K_1 = 9.1 \times 10^{-13}$

(ii) $H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2 \quad K_2 = 7.1 \times 10^{-12}$

Object $CO_2 + H_2 \rightleftharpoons CO + H_2O$

equation (i) – (ii)

$\therefore K^1 = \frac{K_1}{K_2} = 1.28$

$C_t = C_0 e^{-kt}$

$\therefore R = R_0 e^{-kt}$

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9. $r = -\frac{1}{2}\frac{d[NO_2]}{dt} = +\frac{1}{2}\frac{d[NO]}{dt} = +\frac{d[O_2]}{dt}$

$\therefore$ NO_2 is reactant so (Z) rate of dis appearance of NO_2 = rate of formation of of NO
So NO is (X)

10. Initial Normality

$$N = \frac{2.52 \times 1000}{63 \times 100} = 0.4$$

$$\begin{matrix} COOH \\ | \\ COOH \end{matrix} \cdot 2H_2O$$

$\therefore N_1V_1 = N_2V_2$

$0.4 \times 10 = N_2 \times 500$

$$N_2 = \frac{0.4}{50} = 0.08\ N$$

Then final weight

$$N = \frac{w \times 1000}{E \times V_{ml}}$$

$$0.08 = \frac{w \times 1000}{63 \times 500}$$

12. At equation $\Delta G^\circ = 0$

$$\therefore T = \frac{\Delta H}{\Delta S} = \frac{7.5 \times 1000}{25} = 300\ K$$

13. $MgCl_2 \longrightarrow Mg^{+2} + 2Cl^-$

?

0.01 M

$K_{sp} = Q = [Mg^{+2}][OH^-]^2$

$10^{-12} = [0.01][OH^-]^2$

$[OH^-]^2 = 10^{-10}$

$[OH^-] = 10^{-5}$

pOH = 5 $\quad \therefore$ pH = 9

14. $r = \frac{a}{2\sqrt{2}} = 141.4\ pm$

$a = 2 \times \sqrt{2} \times 141.4$

$\therefore V = a^3 = (2 \times \sqrt{2} \times 141.4)^3$

15. For Ideal Solution

ΔH Mix = 0 $\quad \Delta V$ Mix = 0

16. $\pi = CRT$

$\Rightarrow \pi \propto C$ at const. T

$$\frac{\pi_1}{\pi_2} = \frac{C_1}{C_2} = \frac{0.01}{0.001} = 10$$

18. $(\Delta S)_{system} + (\Delta S)_{surrounding} > 0$ (irreversible process)

19. $$\frac{\text{Surface area}}{\text{Volume}} = \frac{\pi d^2}{\frac{\pi d^3}{6}} = \frac{6}{d}$$

$d_1 = 30\ nm \quad d_2 = 10\ nm$

$$\frac{\left(\frac{\text{Surface area}}{\text{Volume}}\right)_2}{\left(\frac{\text{Surface area}}{\text{Volume}}\right)_1} = \frac{\frac{6}{d_2}}{\frac{6}{d_1}} = \frac{d_1}{d_2} = \frac{30}{10} = 3$$

20. $\Lambda(HOAc) = \Lambda^\circ(NaOAc) + \Lambda^\circ(HCl) - \Lambda^\circ(NaCl)$
$= 91 + 425.9 - 126.4 = 390.5$

21. From Bragg's equation

$n\lambda = 2d \sin\theta$

$d = \lambda, n = 1$

$1 \times \lambda = 2 \times \lambda \sin\theta$

$$\sin\theta = \frac{1}{2}$$

$\theta = 30^\circ$

22. $Zn + Cu^{+2} \rightarrow Zn^{+2} + Cu$

$2Zn + 2Cu^{+2} \rightarrow 2Zn^{-2} + 2Cu$

For 2 moles of Zn, n = 4

$\Delta G^\circ = -nFE^\circ_{Cell} = -4 \times 96500 \times 1.1 = -424.6\ kJ$

23. $$K = \frac{A_0 - A_t}{t}$$

$$K = \frac{A_2}{2t_{1/2}}$$

$$t_{1/2} = \frac{A_0}{2K}$$ Zero order

$t_{1/2} \propto$ initial concentration

so double times

24. According to Lagmuir curve

$$x = \frac{ap}{1 + bp}$$

$p \rightarrow \infty \qquad x = \frac{a}{b}$

$p \rightarrow 0 \qquad x \propto p$

25. $16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 2Mn^{+2} + 10CO_2 + 8H_2O$

$MnO_4^- : C_2O_4^{2-} = 2 : 5$

$2KMnO_4 + 16\ HCl \rightarrow 2KCl + 2MnCl_2 + 5Cl_2 + 8H_2O$

$MnO_4^- : HCl = 2 : 16 = 1 : 8$

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26. Fe^{+3}/Fe

$Fe^{+2} + 2e^- \rightarrow Fe \quad E^0 = -0.44$

$Fe^{+3} + e^- \rightarrow Fe^{+2} \quad E^0 = 0.770$

$Fe^{+3} + 3e^- \rightarrow Fe \quad E^0 = ?$

$$E^0 = \frac{n_1E_1^0 + n_2E_2^0}{n_3} = \frac{2(-0.44)+1\times(0.770)}{3}$$

$$= \frac{-0.88+0.770}{3} = -0.037 \text{ volt}$$

27. $\Delta E = 13.6\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ eV/atom ;

$$\frac{\Delta E_{1\to3}}{\Delta E_{1\to2}} = \frac{\frac{1}{1^2}-\frac{1}{3^2}}{\frac{1}{1^2}-\frac{1}{2^2}} = \frac{.32}{27}$$

28. $$Y_A = \frac{P_A^0X_A}{P_A^0X_A + P_B^0X_B} = \frac{100.\frac{1}{2}}{100.\frac{1}{2}+500.\frac{1}{2}}$$

$$= \frac{50}{50+250} \quad \left| \because Ps = (p_A^0 - p_B^0)X_A + p_B^0 \right.$$

$$= \frac{50}{300} \qquad 300 = (-400)X_A + 500$$

$$= 1/6 \qquad X_A = \frac{1}{2}$$

$$\therefore X_B = \frac{1}{2}$$

29. Total droplets = 2.4×10^{18}

total area = total droplets × area of one drop

$= 2.4 \times 10^{18} \times 12.5 \times 10^{-16}$

$= 12.5 \times 2.4 \times 10^2 \text{ m}^2$

Energy consumption = $0.03 \times 12.5 \times 2.4 \times 10^2$

= 90 Joule

31. $\frac{3}{2}KT = 1.6\times10^{-19}$

$\frac{3}{2} \times 1.38 \times 10^{-23}\, T = 1.6 \times 10^{-19}$

$T = 10^4$ K

32. $t_{1/2} \propto \frac{1}{a^{n-1}}$

for zero order reaction n = 0

so $t_{1/2} \propto a$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{a_1}{a_2}$$

so $$\frac{.2}{(t_{1/2})_2} = \frac{[A_0]}{2[A]_0}$$

$t_{1/2} = .4$ sec

33. $Mn^{+2} = 23\ e^-$

$Fe^{+3} = 23\ e^-$

35. $$pH = pKa + \log\frac{[\text{Conjugate base}]}{[\text{Acid}]}$$

$\because$ [Conjugate base] = [Acid]

pH = pKa = 5.85

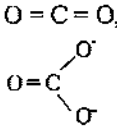
36. $K = Ae^{-\frac{E_a}{RT}}$

$\ln K = \ln A - \frac{E_a}{RT}$

37. $$\text{Bond Length} \propto \frac{1}{\text{Bond Order}}$$

CO, B.O. = 3

O = C = O, B.O. = 2



B.O. = 1.33

38 $2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g) \quad K_1$

$2PCl_3(g) + 2Cl_2(g) \rightleftharpoons 2PCl_5(g) \quad K_2^2$

Net reaction : $2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g)$

$K = K_1K_2^2$

40. $\frac{A}{C} = 0.7$

$\frac{35\times10^4}{C} = 0.7$

$C = \frac{35\times10^4}{0.7} = 5\times10^5$

Moles of cytosine = moles of guanine

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41. $E = \phi + K.E.$

$\because E = \frac{hC}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{660\times10^{-9}}$

$= 3\times10^{-19}\ J$

$\phi = 1ev = 1.6\times10^{-19}\ J$

$K.E. = 3\times10^{-19} - 1.6\times10^{-19} = 1.4\times10^{-19}\ J$

for wave length of emitted electron

$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.4\times10^{-19}}}$

$= \frac{6.6\times10^{-34}}{5\times10^{-25}} = 1.32\times10^{-9}$ meter

42. $d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$

$= \frac{450}{\sqrt{4+4+1}} = \frac{450}{\sqrt{9}} = 150$ pm

43. $\Delta H = \Delta E + \Delta n_g RT$

$20 = \Delta E + 8.314 + 10^{-3} \times 333$

$\Rightarrow \Delta E = 17.2$ kJ/mol

44. $W = -2.303nRT\log\frac{V_2}{V_1}$

$= -2.303\times3\times8.314\times10^{-3}\times300\log\frac{20}{2}$

$= -17.2$kJ / mol

45. $r = K[X]^x[Y]^y$

Total order = $n = x + y$

By exp.(1) & (2)

$\frac{r_1}{r_2} = \frac{K[.25]^x[.25]^y}{K[.50]^x[.25]^y} = \frac{1.0\times10^{-6}}{4.0\times10^{-6}}$

$\frac{1}{(2)^x} = \frac{1}{4}, x = 2$

By exp.(1) & (3)

$\frac{r_1}{r_3} = \frac{K[.25]^x[.25]^y}{K[.25]^x[.50]^y} = \frac{1\times10^{-6}}{8\times10^{-6}}$

$\frac{1}{(2)^y} = \frac{1}{8}, y = 3$

So Total order = 2 + 3 = 5

46. $-\frac{dC}{dt} = k$ (for zero order)

$\int_{C_0}^{C_t} dC = -\int_0^t k\,dt$

$C_t - C_0 = -kt$

$C_t = -kt + C_0$

slope = $-k$

47. According to Henry's law

$P_{gas} = K_H X_{gas}$ (constant temperature)

$\therefore$ for same pressure

$K_H X_{gas}$ = constant

$\therefore$ greater the value of K_H, smaller is solubility and vice-versa.

48.

	A	$\rightleftharpoons$	nB
t = 0	0.06		0
eq^m	0.06 − x		nx

$\therefore 0.06 - x = 0.03$

$x = 0.03$

& $nx = 0.06$

$n = \frac{0.06}{0.03} = 2$

& $K_c = \frac{[B]^2_{eq^m}}{[A]_{eq^m}} = \frac{(0.06)^2}{(0.03)^1} = 0.12$

49. $\Delta S = nR\,\ell n\left(\frac{V_2}{V_1}\right)$

$\Delta S = 2\times8.314\times\ell n\left(\frac{100}{10}\right)$

$= 2\times8.314\times2.303$ J/k

$= 38.3$ J/K

50. $n\lambda = 2d\sin\theta$

$2\times3 = 2\times\frac{6}{\sqrt{1}}\times\sin30^\circ$

$1\times3 = 2\times\frac{6}{\sqrt{4}}\times\sin\theta$

$\therefore 6\times\sin30^\circ = 2\times\frac{6}{\sqrt{4}}\times\sin\theta$

$\theta = 30^\circ$

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52. $[Co(NH_3)_4Cl]Cl \longrightarrow \underset{\text{2 Ions}}{[Co(NH_3)Cl_2]^+} + Cl^{\ominus}$

co-ordination no. 6

53. $[Fe(NH_3)_6]^{3+}$ so unpaired e^- 1

Inner sphere complex

(Low spin complex) $\therefore \mu = \sqrt{1(1+2)} = 1.73$ BM

$[FeF_6]^{3-}$ so unpaired e^- 5

Outer sphere complex

(High spin complex)

$\therefore \mu = \sqrt{5(5+2)} = 5.92$ BM

54.

M	Y
$4\times\frac{1}{2}=2$	4

$\therefore$ Formulae = MY_2

55. $\dfrac{760-750}{760} = \dfrac{\left(\frac{18}{MM}\right)}{\left(\frac{108}{18}\right)}$

$MM = 228$

56. $E_{cell} = \{0.34 - (-0.76)\}$ volt

$= 1.1$ volt

& $\Delta G = -nF\,E_{cell}$

$\Delta G = -2 \times 96500 \times 1.1 \times 10^{-3}$ kJ

$= -213$ kJ

57. $K_2 = \dfrac{1}{K_1^2}$ or $K_1^2 = \dfrac{1}{K_2}$

58. $W = \dfrac{eq-wt \times i \times t}{96500}$

$\left(\frac{0.005}{10}\right) \times 80 \times 10.5 = \dfrac{100\times 3\times t(sec)}{96500}$

$t = 135$ s

59. $\underset{\text{valance factor = 2}}{I_2} + \underset{\text{valency-factor = 1}}{Na_2S_2O_3.5H_2O} \longrightarrow Na_2S_4O_6 + I^-$

$\therefore$ eq . of $I_2 = 0.25 \times \dfrac{100}{1000} = 0.025$

$\therefore$ eq of $Na_2S_2O_3.5H_2O = 0.025$

or mol of $Na_2S_2O_3.5H_2O = \left(\frac{0.025}{1}\right)$

$\therefore$ mass of $Na_2S_2O_3.5H_2O = (0.025 \times 248)$ g $= 6.2$ g

60. $K_{sp(AgBr)} = [Ag^+]\,[Br^-]$

$= (5 \times 10^{-10})\,(10^{-3})$

$= 5 \times 10^{-13}$

Now

$5 \times 10^{-13} = (10^{-2})\,[Br^-]$

$[Br^-] = 5 \times 10^{-11}$ M

61. $K_2CrO_4 \Rightarrow Cr^{+6}$ (highest oxidation state)

$NbCl_5 \Rightarrow Nb^{+5}$

$MnO_2 \Rightarrow Mn^{+4}$

62. For plot between ln k v/s 1/T y-intercept is ln A

& slope is $\dfrac{-E_a}{R}$

therefore; $E_{II} < E_I$ and $A_I > A_{II}$

63. Corner $\Rightarrow \frac{1}{8} \Rightarrow$ Face $\Rightarrow \frac{1}{2}$

64. $Q = \dfrac{[B][C]}{[A^2]}$ & $K_C = 0.5$

For option (A)

$Q = \dfrac{(10^{-2})\times(10^{-2})}{[10^{-3}]^2} = 100$

& $Q > K_C$ i.e. reaction proceed in backward direction.

65.

$126 = \lambda^{\infty}_{Na^+} + \lambda^{\infty}_{Cl^-}$

$150 = \lambda^{\infty}_{K^+} + \lambda^{\infty}_{Cl^-}$

$250 = \lambda^{\infty}_{Na^+} + \lambda^{\infty}_{OH^-}$

$(150 + 250 - 126) = \lambda^{\infty}_{K^+} + \lambda^{\infty}_{OH^-}$

or $\lambda^{\infty}_{KOH} = 274$

66. Atomic no. 33 and 17 belongs to 15th & 17th group respectively therefore co-valent bond form between both elements

X^{+3} Y^{-1} → XY_3

Atomic no. 33 = As

Atomic no. 17 = Cl → $AsCl_3$

67. $KMnO_4 + KI + H_2SO_4 \longrightarrow MnSO_4 + I_2 + K_2SO_4 + H_2O$

v.f = 5 v.f = 1

$\therefore (eq)_{KMnO_4} = (eq)_{KI} = 1$

Eq. = V.F. × mole

1 = 5 × mole

Mole = 1/5

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68. The correct way of reporting the average value should have exactly the same number of digit after decimal which has least digit after decimal among the data given.

69. $\Delta S = \frac{\Delta H_{Melting}}{T_{F.P}} = \frac{6\times1000}{273}\ \frac{J}{K}$

$= 21.978 \approx 22 J/k$

70. $PM = \rho RT$

$M = \left(\frac{5\times0.0821\times300}{1}\right) = 123.15$

$\therefore$ Number of acetic acid molecule

$= \frac{123.15}{60} \approx 2$

71. $W = -nRT = -(1\times8.314\times10^{-3}\times373)kJ$

$= -3.10\ kJ$

$q = \Delta H = 41\ kJ$

& $\Delta E = q + w = (41 - 3.1) \cong 37.9\ kJ$

72. $H_2 + I_2 \rightleftharpoons 2HI$; $K_c = 50$

$N_2 + 3H_2 \rightleftharpoons 2NH_3$; $K_c = 1000$

$N_2 + 6HI \rightleftharpoons 2NH_3 + 3I_2$; $K_c = \frac{1000}{(50)^3}$

$K_c = 0.008$

73. $N_2 + 2H_2 \rightarrow 1\ N_2H_4$; ΔH_f

$\Delta H_f = 1\times E_{N\equiv N} + 2\ E_{H-H} - 4\ E_{N-H} - 1\ E_{N-N}$

$= [(1\times946) + (2\times435) - 4\times(389) - 1\times(159)]\ kJ$

$= 101\ kJ/mol$

74. $r_{K^+} + r_{Cl^-} = \frac{a}{2}$

$133 + 181 = \frac{a}{2}$

$a = 2(133 + 181)$

$a = 628\ pm$

or $a = 628\times10^{-10}\ cm$

and

volume $= a^3 = (6.28\times10^{-8})^3\ cm^3$

$= 2.4767\times10^{-22}\ cm^3$

$\approx 2.48\times10^{-22}\ cm^3$

$= -(6\times96500\times0.3)\ J$

$= -173{,}700\ J$

77. $K = Ae^{-Ea/RT}$

$T \rightarrow \infty$ $k = A$

78. SRP↓ Reducing power↑

79. Since conductance of H^+ is highest so molar conductivity of HCl will be highest and after that conductance of CH_3COOH will come

$\therefore$ order

$HCl > CH_3COOH > NaCl > CH_3COONa$

80. $[ZCl_4]^{-2}$ is 3.87

$x + (-1)\ 4 = 2$

$u = \sqrt{n(n+2)}$

$x = +4 - 2$

$x = +2$

$Mn^{+2} \longrightarrow 4s^0 3d^5$ ↑ ↑ ↑ ↑ ↑

$Ni^{+2} \longrightarrow 4s^0 3d^8$ ↑↓ ↑↓ ↑↓ ↑ ↑

$Co^{+2} \longrightarrow 4s^0 3d^7$ ↑↓ ↑ ↑ ↑ ↑

$Cu^{+2} \longrightarrow 4s^0 3d^9$ ↑↓ ↑↓ ↑↓ ↑↓ ↑

81. $pH = -\log[H^+]$

$pH \propto \frac{1}{[H^+]}$

Order of pH

$CH_3COOH > HCOOH > HCl$

Acidic strength order

$CH_3-\underset{\underset{O}{\|}}{C}-OH < H-\underset{\underset{O}{\|}}{C}-OH < HCl$

Acidic strength ∝ stability of Anion

$Cl^{\ominus} > H-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O} > \overset{+I}{CH_3}-\underset{\underset{O}{\|}}{C}-\overset{\ominus}{O}$

Size large

82. By definition

83. $N_2 + 3X_2 \longrightarrow 2NX_3$

$N_2 + 3F_2 \longrightarrow 2NF_3$

$\Delta H_{NF_3} = 941 + 3(155) - 6(272) = -226$

$\Delta H_{NCl_3} = 941 + 3(242) - 6(200) = +467$

84.

	$X \rightleftharpoons 2Y$	k_1	$Z \rightleftharpoons P + Q$	k_2
	1 0		1 0 0	
	$1-\alpha$ 2α		$1-\alpha$ α α	

$\frac{k_1}{k_2} = \frac{\frac{(2\alpha)^2}{(1-\alpha)}}{\frac{\alpha^2}{(1-\alpha)}} = \frac{4\alpha^2}{\alpha^2} = 4$

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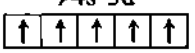
85. $[Mn\,Br_4]^{-2}$

$x + (-1)4 = -2$

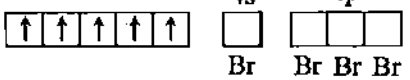
$x = +2$

$Mn \longrightarrow 4s^2 3d^5$

$Mn^{+2} \longrightarrow 4s^0 3d^5$



We know that Br is weak field ligand so Hund's Rule is applicable

then 

Hybridization is $\longrightarrow sp^3$

and No. of unpaired electron. = 5

86. $$0 = 1.1 - \frac{0.0591}{2}\log\frac{Zn^{+2}}{Cu^{+2}}$$

$$-1.1 = \frac{0.0591}{2}\log\frac{Zn^{+2}}{Cu^{+2}}$$

$$\log\frac{Zn^{+2}}{Cu^{+2}} = 37.3$$

87. At X $V = 50L$ $T = 200\,K$

$n = 1$

$$P_X = \frac{nRT}{V} = \frac{1\times0.0821\times200}{50}$$

$$= 0.328$$

At Z $$P_Z = \frac{1\times0.0821\times200}{20} = 0.821$$

At Y $$P_Y = \frac{1\times0.0821\times500}{50} = 0.821$$

88. Acidic strength of acid is

$HClO_4 > HClO_3 > HClO_2 > HClO$

$[H^+]$ ↑ ka ↑ p^{ka} ↓

$\therefore$ p^{k_a} order : $HClO_4 < HClO_3 < HClO_2 < HClO$

89.

	FCC	BCC	SC
η	74%	68%	52.4%

order FCC > BCC > SC

90. $$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(V_{rms})_{H_2}}{(V_{rms})_{O_2}} = \frac{\sqrt{\frac{3\times R\times50}{2}}}{\sqrt{\frac{3\times R\times500}{28}}} = 1.18$$

91. $t_{1/2} = 1$ hr

$$t_{87.5} = \frac{2.303}{k}\log\frac{a}{a-x}$$

$$= \frac{2.303}{\frac{0.693}{1}}\log\frac{a}{a-.875}$$

$$= \frac{2.303}{0.693}\log 8 = \frac{2.303}{0.693}\times3\times.3010$$

$$= 3$$

92. No. of F^- will be equal to eight since for one Ca^{+2} there should be two F^- ion.

93. $\Delta G^\circ = -2.303 \times 8.314 \times 298 \log (3.8 \times 10^{-3})J$

$= 13809.3876J = 13.809\,Kg$

$\Delta G^\circ = -nFE^\circ$

$13809.387 = -2 \times 96500 \times E^\circ$

$E^\circ_{cell} = .071$

94. no. of α-particle $= \frac{232-208}{4} = 6$

no. of β-particle = 4

95. $Z = \frac{(V_M)_r}{(V_M)_i} < 1$ at $p < 200$ bar

$\therefore (V_M)_r < (V_M)_i$

96. $$d = \frac{N\times M}{N_A \times a^3}$$

$$8.93 = \frac{4\times63.5}{6.023\times10^{23}\times a^3}$$

$a^3 = 4.72 \times 10^{-23}$

$a = (47.2\times10^{-24})^{1/3}$

$= 3.61 \times 10^{-8}$ cm

$= 3.61 \times 10^{-10}$ m

$= 361$ pm

force $a = 2\sqrt{2}\,r$

$$r = \frac{a}{2\sqrt{2}} = 127.8$$

97. $Cu^{+2} + 2e^- \longrightarrow Cu \quad \Delta G_1^\circ = -n_1E_1^\circ F$

$Cu^{+2} + e^- \longrightarrow Cu \quad \Delta G_2^\circ = -n_2E_2^\circ F$

$Cu^{+2} + e^- \longrightarrow Cu^+ \quad \Delta G_3^\circ = -n_3E_3^\circ F$

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$\Delta G_3^\circ = \Delta G_1^\circ - \Delta G_2^\circ$

$-n_3E_3^\circ F = -n_1E_1^\circ F + n_2E_2^\circ F$

$E_3^\circ = \dfrac{n_1E_1^\circ \quad n_2E_2^\circ}{n_3}$

$= \dfrac{2\times 0.34 - .522}{1}$

$= .158$ V

98. No. of mole $= \dfrac{i\times t}{96500\times v.f}$,

$n\times v.f = \dfrac{i\times t}{96500}\times n = \dfrac{1.5\times 250}{96500\times 2}\times 0.85$

$n = .00165$ (deposited)

$n_i = \dfrac{250\times .15}{1000} = .0375$ (initial mole)

$n_{left} = .03585$ (mole left)

$M_{left} = \dfrac{.03585}{.25} = 0.143$

99. M.wt $= 59 + 12 + 14\times 4 + 35.5\times 3 = 233.5$

$C_0H_{12}N_4Cl_3 \xrightarrow{AgNO_3} 1.435$g AgCl

$\dfrac{2.33}{233.5}$g $= 0.01$ mole (0.01 mole)

(i) 0.01 mole molecule produce 0.01 mole AgCl

$\therefore$ one replaceable $Cl^{\ominus}$ ion

so formula of complex is

$[CO(NH_3)_4Cl_2]Cl$

(ii) Oxidation no. of Co is +3 so primary valency is 3.

(iii) Coordination no. is 6 so sec. valency is 6 so ans. is 3,6

100. $\lambda_M = \dfrac{1000\times K}{M}$

$= \dfrac{1000\times 1.65\times 10^{-4}}{0.2} = 8.25$

$\lambda_H^\infty = \lambda_M^\infty(H^+) + \lambda_M^\infty(CH_3COO^-)$

$= 349.1 + 40.9$

$= 390$

$\alpha = \dfrac{8.25}{390} = .0211$

101. According to their SRP value.

102. $Mav = \dfrac{M_1n_1 + M_2n_2}{n_1 + n_2}$

$35.45 = \dfrac{35n_1 + 37n_2}{n_1 + n_2}$

$\therefore \quad n_1 : n_2 = 3 : 1$

103. According to Lechatelier principal on increasing volume of closed vessel equilibrium will shift towards right.

104. For exothermic reaction

$\Delta H = E_P - E_R < 0$

$E_B - E_A < 0$

$E_B < E_A$

Catalyst does not changes the initial and final position of the reaction so correct answer is (A)

105. $K_{sp}\ Mg(OH)_2 = [Mg^{+2}]\ [OH^-]^2$

$5.6\times 10^{-12} = [10^{-10}][OH^-]^2$

$[OH^-] = \sqrt{5.6\times 10^{-2}} = 0.24$ M

106.

	Ag^1	:	Cu^{+2}	:	Al^{+3}
no. of eq. deposit	x	:	x	:	x
no. of mol deposit	$\frac{x}{1}$	:	$\frac{x}{2}$	:	$\frac{x}{3}$
	6x	:	3x	:	2x
no. of mole deposit	6	:	3	:	2
mass deposit	$6M_{Ag}$	:	$3M_{Cu}$	:	$2M_{Al}$

107. $\log_{10} = \dfrac{209\times 10^3}{2.303\times 8.314}\left[\dfrac{1}{300} - \dfrac{1}{T}\right]$

$9.16\times 10^{-5} = 3.33\times 10^{-3} - \dfrac{1}{T}$

$T = 308.4$ K

or 35° C $= X$

108. No of O^{-2} per unit cell $= 8\times\dfrac{1}{8} + 6\times\dfrac{1}{2} = 4$

No of Al^{+3} per unit cell $= 4\times\dfrac{1}{2} = 2$

No of Mn^{+2} per unit cell $= 8\times\dfrac{1}{8} = 1$

$MnAl_2O_4$

109. no. of radial node $= n - \ell - 1 = 4 - 1 - 1 = 2$

no. of angular node $= \ell = 1$

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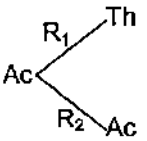
110.



$$\% \text{ Th} = \frac{R_1}{R_T} = \frac{2}{100}$$

$$\% \text{ Ac} = \frac{R_2}{R_T} = \frac{98}{100}$$

$$\therefore \frac{R_1}{R_2} = \frac{2}{98}$$

$$R_T = R_1 + R_2$$

$$\frac{0.693}{22} = R_1 + \frac{98}{2} R_1$$

$$\therefore \mathbf{R_1 = 6.3 \times 10^{-4}}$$

111. For process A → C, W = 0, ΔE = 0, q = 0
For process A → B
W = –1 (1.5 – 1) = –0.5 lit. atm
ΔE = q + W
0 = q + W
q = –W = – (– 0.5) = 0.5 lit. atm
For process B → C, W = 1 atm
ΔE = q + W
0 = q – W
–q = W = –1 atm
∴ Total heat exchanged = 1.5

112. $1.013 = 0.742\ X_t + 1.8\ X_b$
$1.013 = 0.742(1 - X_b) + 1.8\ X_b$
$X_b = 0.256$
$\therefore X_t = 1 - X_b = 0.744$

113. The unit cell of the above pattern will consist of 8 white square and 1 black square i.e. it will form centre unit cell.
∴ No. of white square Y = 8
No. of black square X = 1
∴ **Formula XY_8**

114. It is decided by SRP value

115. $1.1 \times 10^{-12} = [Ag^+]^2 [5 \times 10^{-13}]$
$\therefore [Ag^+] = 1.5 \times 10^{-5}$

116. $$\eta = \frac{2 \times 4/3\pi R^3}{\left(\frac{4R}{\sqrt{3}}\right)^3} \times 100 \cong 68\%$$

117.

	X	⟶ Y	⟶ Z
t = 0	A_0	0	0
t = t	A_0–x	y	z

So total moles of constituents will be more at any time 't' as compared to t = 0

118. no of α particle $= \frac{238 - 206}{4} = 8$

no of β-particle = 6

119.

n = 2	ℓ = 0	m = 0	$m_s = \pm 1/2$
	1	m = –1, 0, + 1	$m_s = \pm 1/2$

120. $\frac{x}{m} \propto P^{1/n}$

$\frac{x}{m} = k P^{1/n}$

$\log \frac{x}{m} = \log k + \frac{1}{n} \log P$

$y = c + mx$

121. $[H^+] = \sqrt{K_a . c} = \sqrt{1.8 \times 10^{-5} \times 0.1} = \sqrt{1.8} \times 10^{-3}$
pH = 3–log (1.34) = 2.87

122. $\lambda^\circ_{KCl} = 73.5 + 76.3 = 149.8$
$\lambda^\circ_{CaCl_2} = 119 + 2 \times 76.3 = 271.6$
$\lambda^\circ_{K_2SO_4} = 2 \times 73.5 + 160 = 307$

123. $Co(g) + \frac{1}{2} O_2(g) \longrightarrow CO_2(g)\ \Delta N = -283.5$

$\Delta S = 213.7 - 197.7 - \frac{20501}{2} = -86.5$

$\Delta G = -283 - \frac{298 \times (-86.5)}{1000} = -2570$ kJ

$W_{max} = -\Delta G = -(-257\text{ kJ}) = 257$ kJ

124. $\Delta T_b = K_b \times m$

$= 0.52 \times \frac{18/180}{1}$

$= 0.052$

$\therefore B \cdot P = 373.15 + 0.052$

$= 373.2$ K

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125. $d = \dfrac{N \times M}{N_A \times a^3}$

$$9.32 = \frac{1 \times 209}{6.023 \times 10^{23} \times a^3}$$

$\therefore a^3 = 37.2 \times 10^{-24}$ S

$a = 3.33 \times 10^{-8}$ cm

$\cong 334$ pm

126. $E_{cell} = E^\circ_{cell} - \dfrac{.0591}{2} \log \dfrac{2 \times 10^{-3}}{4 \times 10^{-3}}$

$1.091 = E^\circ_{cell} - \dfrac{.0591}{2} \log (.5)$

$E^\circ_{cell} = 1.099$

$E^\circ_{cell} = -\dfrac{.0591}{2} \log k$

$\log k = -\dfrac{1.099 \times 2}{.0591} = -37.22$

$\therefore k = 2 \times 10^{37}$

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Chapter

2 Inorganic Chemistry

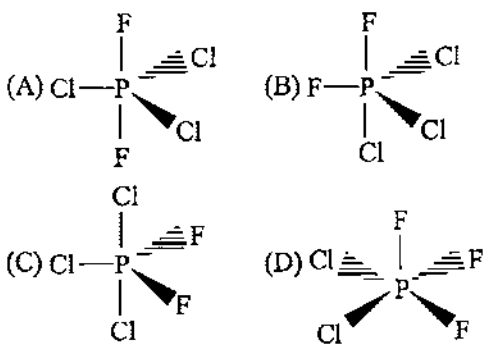
1. The number of isomers of Co (diethylene triamine) Cl_3 is - [2010]
(A) 2 (B) 3 (C) 4 (D) 5

2. Among the following, the π-acid ligand is - [2010]
(A) F (B) NH_3 (C) CN^- (D) I^-

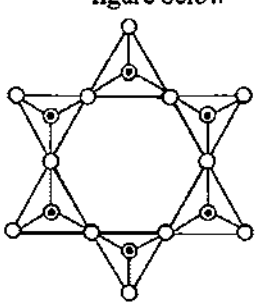
3. The bond order in O_2^{2-} is- [2010]
(A) 2 (B) 3 (C) 1.5 (D) 1

4. The shape of the molecule ClF_3 is- [2010]
(A) trigonal planar (B) pyramidal
(C) T-shaped (D) Y-shaped

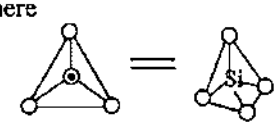
5. The correct structure of PCl_3F_2 is- [2010]



6. Identify the cyclic silicate ion given in the figure below [2010]



where



• = Si
○ = O

(A) $[Si_4O_{25}]^{24-}$ (B) $[Si_6O_{18}]^{18-}$
(C) $[Si_4O_{12}]^{12}$ (D) $[Si_6O_{24}]^{12-}$

7. Diborane is formed the elements as shown in equation (1) [2010]

$2B(s) + 3H_2(g) \rightarrow B_2H_6(g)$ ... (1)

Given that

$H_2O(l) \rightarrow H_2O(g)$
$\Delta H_1^\circ = 44$ kJ

$2B(s) + 3/2O_2(g) \rightarrow B_2O_3(s)$
$\Delta H_2^\circ = -1273$ kJ

$B_2H_6(g) + 3O_2(g) \rightarrow B_2O_3(s) + 3H_2O(g)$
$\Delta H_3^\circ = -2035$ kJ

$H_2(g) + 1/2\ O_2(g) \rightarrow H_2O(l)$
$\Delta H_4^\circ = -286$ kJ

the ΔH° for the reaction (1) is-
(A) 36 kJ (B) 509 kJ
(C) 520 kJ (D) –3550 kJ

8. The Crystal Field Stabilization Energy (CPSE) and the spin-only magnetic moment in Bohr Magneton (BM) for the complex $K_3[Fe(CN)_6]$ are, respectively- [2010]
(A) 0.0 Δ_s and $\sqrt{35}$ BM
(B) –2.0 Δ_s and $\sqrt{3}$ BM
(C) –0.4 Δ_s and $\sqrt{24}$ BM
(D) –2.4 Δ_s and 0 BM

9. The hybridizations of $Ni(CO)_4$ and $Cr(H_2O)_6^{2+}$, respectively, are [2011]
(A) sp^3 and d^3sp^2 (B) dsp^2 and d^2sp^3
(C) sp^3 and d^2sp^3 (D) dsp^2 and sp^3d^2

10. Extraction of silver is achieved by initial complexation of the ore (Argentite) with X followed by reduction with Y. X and Y respectively are [2011]
(A) CN^- and Zn (B) CN^- and Cu
(C) Cl^- and Zn (D) Br^- and Zn

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11. All the products formed in the oxidation of $NaBH_4$ by I_2, are [2011]
(A) B_2H_6 and NaI (B) B_2H_6, H_2 and NaI
(C) BI_3 and NaH (D) $NaBL_4$ and HI

12. The spin-only magnetic moments of $[Mn(CN)_6]^{4-}$ and $[MnBr_4]^{2-}$ in Bohr Magnetons, respectively, are [2011]
(A) 5.92 and 5.92 (B) 4.89 and 1.73
(C) 1.73 and 5.92 (D) 1.73 and 1.73

13. XeF_6 hydrolyses to give an oxide. The structure of XeF_6 and the oxide, respectively, are- [2011]
(A) octaltedral and tetrahedral
(B) distorted octahedral and pyramidal
(C) octahedral and pyramidal
(D) distorted octahedral and tetrahedral

14. Among the following, the species with the highest bond order is – [2012]
(A) O_2 (B) F_2 (C) O_2^+ (D) F_2

15. The molecule with **non-zero** dipole moment is [2012]
(A) BCl_3 (B) $BeCl_2$
(C) CCl_4 (D) NCl_3

16. For a tetrahedral complex $[MCl_4]^{2-}$, the spin-only magnetic moment is 3.83 B.M. The element **M** is – [2012]
(A) Co (B) Cu (C) Mn (D) Fe

17. The C–O bond length in CO, CO_2 and CO_3^{2-} follows the order – [2012]
(A) $CO < CO_2 < CO_3^{2-}$
(B) $CO_2 < CO_3^{2-} < CO$
(C) $CO > CO_2 > CO_3^{2-}$
(D) $CO_3^{2-} < CO_2 < CO$

18. Among the following, the species that is both tetrahedral and diamagnetic is – [2012]
(A) $[NiCl_4]^{2-}$ (B) $[Ni(CN)_4]^{2-}$
(C) $Ni(CO)_4$ (D) $[Ni(H_2O)_6]^{2+}$

19. Among the following, the set of isoelectronic ions is - [2013]
(A) Na^+, Mg^{2+}, F^-, Cl^-
(B) Na^+, Ca^{2+}, F^-, O^-
(C) Na^+, Mg^{2+}, F^-, O^{2-}
(D) Na^+, K^+, S^{2-}, Cl^-

20. Hydrolysis of BCl_3 gives X which on treatment with sodium carbonate produces Y. X and Y, respectively, are - [2013]
(A) $H_3BO_3$and $NaBO_2$
(B) $H_3BO_3$and $Na_2B_4O_7$
(C) B_2O_3 and $NaBO_2$
(D) B_2O_3 and $Na_2B_4O_7$

21. The numbers of lone pair(s) on Xe in XeF_2 and XeF_4 are, respectively, - [2013]
(A) 2 and 3 (B) 4 and 1
(C) 3 and 2 (D) 4 and 2

22. In the structure of borax, the numbers of boron atoms and B-O-B units, respectively are- [2013]
(A) 4 and 5 (B) 4 and 3
(C) 5 and 4 (D) 5 and 3

23. Aqueous solution of a metallic nitrate X reacts with NH_4OH to form Y which dissolves in excess NH_4OH. The resulting complex is reduced by actaldehyde to deposit the metal. X and Y, respectively, are - [2013]
(A) Cs (NO_3) and CsOH
(B) $Zn(NO_3)_2$ and ZnO
(C) $AgNO_3$and Ag_2O
(D) $Mg(NO_3)_2$ and $Mg(OH)_2$

24. In aqueous solution, $[Co(H_2O)_6]^{2+}$ (X) reacts with molecular oxygen in the presence of excess liquor NH_3 to give a new complex Y. The number of unpaired electrons in X and Y are, respectively - [2013]
(A) 3, 1 (B) 3, 0
(C) 3, 3 (D) 7, 0

25. The number of geometrical isomers of $[CrCl_2(en)(NH_3)_2]$, where en = ethylenediamine, is - [2014]
(A) 2 (B) 3 (C) 4 (D) 1

26. The element that combines with oxygen to give an amphoteric oxide is - [2014]
(A) N (B) P
(C) Al (D) Na

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27. $Ni(CO)_4$ is [2014]
(A) tetrahedral and paramagnetic
(B) square planar and diamagnetic
(C) tetrahedral and diamagnetic
(D) square planar and paramagnetic

28. The energies of d_{xy} and d_z^2 orbitals in octahedral and tetrahedral transition metal complexes are such that – [2014]
(A) E (d_{xy}) > E (d_z^2) in both tetrahedral and octahedral complexes
(B) E (d_{xy}) < E (d_z^2) in both tetrahedral and octahedral complexes
(C) E (d_{xy}) > E (d_z^2) in tetrahedral but E (d_{xy}) < E (d_z^2) in octahedral complexes
(D) E (d_{xy}) < E (d_z^2) in tetrahedral but E (d_{xy}) > E (d_z^2) in octahedral complexes

29. XeF_6 on complete hydrolysis yields 'X'. The molecular formula of X and its geometry, respectively, are : [2014]
(A) XeO_2 and linear
(B) XeO_3 and trigonal planar
(C) XeO_3 and pyramidal
(D) XeO_4 and tetrahedral

30. The Lewis acid strength of BBr_3, BCl_3 and BF_3 is in the order [2015]
(A) $BBr_3 < BCl_3 < BF_3$
(B) $BCl_3 < BF_3 < BBr_3$
(C) $BF_3 < BCl_3 < BBr_3$
(D) $BBr_3 < BF_3 < BCl_3$

31. O^{2-} is isoelectronic with [2015]
(A) Zn^{2+} (B) Mg^{2+}
(C) K^+ (D) Ni^{2+}

32. The H-C-H, H-N-H, and H-O-H bond angles (in degrees) in methane, ammonia and water are respectively, closest to [2015]
(A) 109.5, 104.5, 107.1
(B) 109.5, 107.1, 104.5
(C) 104.5, 107.1, 109.5
(D) 107.1, 104.5, 109.5

33. In alkaline medium, the reaction of hydrogen peroxide with potassium permanganate produces a compound in which the oxidation state of Mn is [2015]
(A) 0 (B) +2 (C) +3 (D) +4

34. When H_2S gas is passed through a hot acidic aqueous solution containing Al^{3+}, Cu^{2+}, Pb^{2+} and Ni^{2+}, a precipitate is formed which consists of [2015]
(A) CuS and Al_2S_3 (B) PbS and NiS
(C) CuS and NiS (D) PbS and CuS

35. The electronic configuration of an element with the largest difference between the 1st and 2nd ionization energies is [2015]
(A) $1s^2\ 2s^2\ 2p^6$ (B) $1s^2\ 2s^2\ 2p^6\ 3s^1$
(C) $1s^2\ 2s^2\ 2p^6\ 3s^2$ (D) $1s^2\ 2s^2\ 2p^1$

36. The order of electronegativity of carbon in sp, sp^2 and sp^3 hybridized states follows [2015]
(A) $sp > sp^2 > sp^3$ (B) $sp^3 > sp^2 > sp$
(C) $sp > sp^3 > sp^2$ (D) $sp^2 > sp > sp^3$

37. The most abundant transition metal in human body is [2015]
(A) copper (B) iron
(C) zinc (D) manganese

38. MnO_2 when fused with KOH and oxidized in air gives a dark green compound X. In acidic solution, X undergoes disproportionation to give an intense purple compound Y and MnO_2. The compounds X and Y, respectively, are [2015]
(A) K_2MnO_4 and $KMnO_4$
(B) Mn_2O_7 and $KMnO_4$
(C) K_2MnO_4 and Mn_2O_7
(D) $KMnO_4$ and K_2MnO_4

39. A metal (X) dissolves both in dilute HCl and dilute NaOH to liberate H_2. Addition of NH_4Cl and excess NH_4OH to an HCl solution of X produces Y as a precipitate. Y is also produced by adding NH_4Cl to the NaOH solution of X. The species X and Y, respectively, are [2015]
(A) Zn and $Zn(OH)_2$ (B) Al and $Al(OH)_3$
(C) Zn and Na_2ZnO_2 (D) Al and $NaAlO_2$

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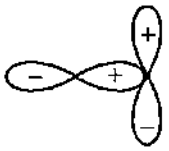
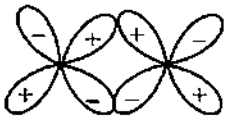
40. The shape of SCl_4 is best described as a **[2016]**

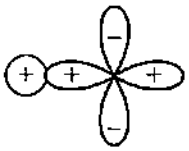
(A) square (B) tetrahedron

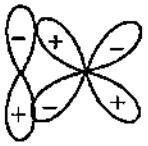
(C) square pyramid (D) see-saw

41. Among the following atomic orbital overlap, the non-bonding overlap is **[2016]**

(A) 

(B) 

(C) 

(D) 

42. Among the following complexes, the one that can exhibit optical activity is **[2016]**

(A) $[CoCl_6]^{3-}$

(B) $[Co(en)Cl_4]^-$

(C) cis-$[Co(en)_2Cl_2]^{3+}$

(D) trans-$[Co(en)_2Cl_2]^+$

43. The molecule with the highest dipole moment among the following is **[2016]**

(A) NH_3 (B) NF_3

(C) CO (D) HF

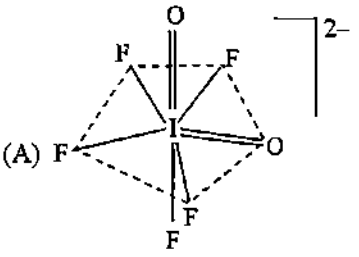
44. The most stable Lewis acid-base adduct among the following is **[2016]**

(A) $H_2O \rightarrow BCl_3$ (B) $H_2S \rightarrow BCl_3$

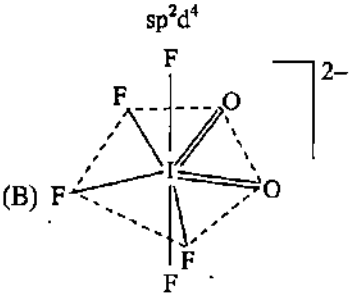
(C) $H_3N \rightarrow BCl_3$ (D) $H_3P \rightarrow BCl_3$

45. The hybridization of the central atom and the shape of $[IO_2F_5]^{2-}$ ion respectively, are - **[2016]**

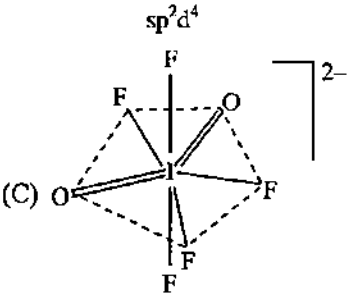
(A) sp^3d^3



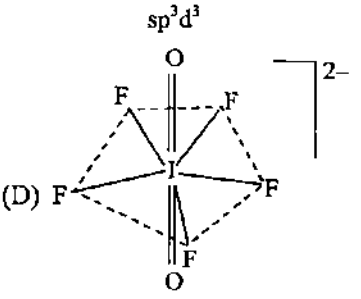
(B) sp^2d^4



(C) sp^2d^4



(D) sp^3d^3



46. Which of the following is **NOT** an oxidation-reduction reaction ? **[5 Nov, 2017]**

(A) $H_2 + Br_2 \rightarrow 2HBr$

(B) $NaCl + AgNO_3 \rightarrow NaNO_3 + AgCl$

(C) $2\,Na_2S_2O_3 + I_2 \rightarrow Na_2S_4O_6 + 2NaI$

(D) $Cl_2 + H_2O \rightarrow HCl + HOCl$

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47. The thermal stability of alkaline earth metal carbonates-$MgCO_3$, $CaCO_3$, $SrCO_3$ and $BaCO_3$, follows the order **[5 Nov, 2017]**
(A) $BaCO_3 > SrCO_3 > CaCO_3 > MgCO_3$
(B) $CaCO_3 > SrCO_3 > BaCO_3 > MgCO_3$
(C) $MgCO_3 > CaCO_3 > SrCO_3 > BaCO_3$
(D) $SrCO_3 > CaCO_3 > MgCO_3 > BaCO_3$

48. When a mixture of diborane and ammonia is heated, the final product is **[5 Nov, 2017]**
(A) BH_3 (B) NH_4BH_4
(C) NH_2NH_2 (D) $B_3N_3H_6$

49. The molecule which is **NOT** hydrolysed by water at 25°C is **[5 Nov, 2017]**
(A) $AlCl_3$ (B) $SiCl_4$
(C) BF_3 (D) SF_6

50. Among the following compounds, the one which does **NOT** produce nitrogen gas upon heating is **[5 Nov, 2017]**
(A) $(NH_4)_2 Cr_2O_7$ (B) NaN_3
(C) NH_4NO_2 (D) $(NH_4)_2 (C_2O_4)$

51. When heated in air, brown copper powder turns black. This black powder would turn brown again when heated with **[5 Nov, 2017]**
(A) CO (B) O_2
(C) H_2 (D) NH_3

52. The geometry and magnetic property of $[NiCl_4]^{2-}$, respectively, are **[5 Nov, 2017]**
(A) tetrahedral, paramagnetic
(B) tetrahedral, diamagnetic
(C) square planar, paramagnetic
(D) square planar, diamagnetic

53. Among (i) $[Cr(en)_3]^{3+}$, (ii) trans-$[Cr(en)_2 Cl_2]^+$, (iii) Cis-$[Cr(en)_2 Cl_2]^+$ (iv) $[Co(NH_3)_4Cl_2]^+$ the optically active complexes are **[5 Nov, 2017]**
(A) i and ii (B) i and iii
(C) ii and iii (D) ii and iv

54. Phosphorous reacts with chlorine gas to give a colourless liquid, which fumes in moist air to produce HCl and **[19 Nov, 2017]]**
(A) $POCl_3$ (B) H_3PO_3
(C) PH_3 (D) H_3PO_4

55. The complete hydrolysis of XeF_6 results in the formation of **[19 Nov, 2017]**
(A) XeO_2F_2 (B) $XeOF_4$
(C) XeO_3 (D) XeO_2

56. The reactivity of the following compounds towards water is in the order **[19 Nov, 2017]**
(A) $Cl_2O_7 < P_2O_5 < B_2O_3$
(B) $B_2O_3 < P_2O_5 < Cl_2O_7$
(C) $P_2O_5 < B_2O_3 < Cl_2O_7$
(D) $B_2O_3 < Cl_2O_7 < P_2O_5$

57. Among the following complexes, the one that can exist as facial (*fac*) and meridional (*mer*) isomers is **[19 Nov, 2017]**
(A) $[Co(NO_2)_3(NH_3)_3]$
(B) $K_3[Fe(CN)_6]$
(C) $[Co(H_2O)_2(NH_3)_4]Cl_3$
(D) $[CoCl(NH_3)_5]Cl_2$

58. The fusion of chromite ore ($FeCr_2O_4$) with Na_2CO_3 in air gives a yellow solution upon addition of water. Subsequent treatment with H_2SO_4 produces an orange solution. The yellow and orange colours, respectively, are due to the formation of **[19 Nov, 2017]**
(A) Na_2CrO_4 and $Na_2Cr_2O_7$
(B) $Cr(OH)_3$ and $Na_2Cr_2O_7$
(C) $Cr_2(CO_3)_3$ and $Fe_2(SO_4)_3$
(D) $Cr(OH)_3$ and Na_2CrO_4

59. Hybridization and geometry of $[Ni(CN)_4]^{2-}$ are **[19 Nov, 2017]**
(A) sp^2d and tetrahedral
(B) sd^3 and square planar
(C) sp^3 and tetrahedral
(D) dsp^2 and square planar

60. The total number of geometrical isomers possible for an octahedral complex of the type $[MA_2B_2C_2]$ is
(M = transition metal ; A, B and C are monodentate ligands) **[19 Nov, 2017]**
(A) 3 (B) 4
(C) 5 (D) 6

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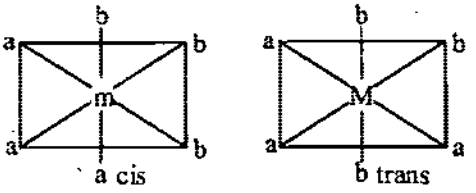
ANSWERS

1. (A)	2. (C)	3. (D)	4. (C)	5. (A)	6. (B)	7. (A)
8. (D)	9. (C)	10. (A)	11. (B)	12. (C)	13. (B)	14. (C)
15. (D)	16. (A)	17. (A)	18. (C)	19. (C)	20. (B)	21. (C)
22. (A)	23. (C)	24. (B)	25. (B)	26. (C)	27. (C)	28. (C)
29. (C)	30. (C)	31. (B)	32. (B)	33. (D)	34. (D)	35. (B)
36. (A)	37. (B)	38. (A)	39. (B)	40. (D)	41. (A)	42. (C)
43. (D)	44. (C)	45. (D)	46. (B)	47. (A)	48. (D)	49. (D)
50. (D)	51. (C)	52. (A)	53. (B)	54. (B)	55. (C)	56. (B)
57. (A)	58. (A)	59. (D)	60. (C)			

SOLUTIONS

1. Isomers of [Co (dien) Cl_3] is ma_3b_3 type complex therefore it shows two cis & trans isomers



2. CN^- accept electrons from metal ion in its vacant π^* ABMO.

3. Bond order of O_2^{2-}

Total electron = 18

Configuration = KK $\sigma(2s)^2\ \sigma^*(2s)^2\ \sigma_x(2p_z)^2$

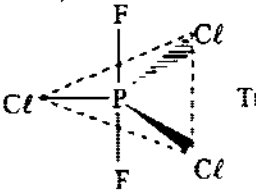
$\pi(2p_x)^2\ \pi(2p_y)^2\ \pi^*(2p_x)^2\ \pi^*(2p_y)^2$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{8-6}{2} = 1.0$$

4. Three bond pair & two lone pair present in ClF_3 molecule.

T-shape

5. Correct structure of PCl_3F_2 is



Trigonal biprymidal

for minimum repulsion between atoms.

6. Cyclic or ring silicates have general formula

$(Si{\cdot}O_3^{2-})_n$ or $(Si{\cdot}O_3)_n^{2n-}$

↓ ↓

$Si_3O_9^{6-}$ $Si_6O_{18}^{12-}$

7. (1) $H_2O_{(\ell)} \longrightarrow H_2O_{(g)}$ $\Delta H = 44$ kJ

(2) $2B + \frac{1}{3}O_2 \rightarrow B_2O_3$ $\Delta H = -1273$ kJ

(3) $B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O_{(g)}$

$\Delta H = -2035$ kJ

(4) $H_2 + 1/2\ O_2 \rightarrow H_2O_{(\ell)}$ $\Delta H = -286$ kJ

equation (2) + (4) × 3 + (1) × 3 – (3)

$\Delta H = (-1273) + (-286) \times 3 + (44) \times 3 - (-2035) = 36$ kJ

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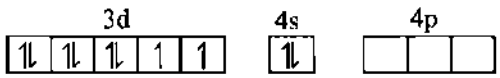
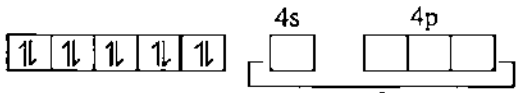
8. $Ni(\overset{\circ}{C}O)_4$

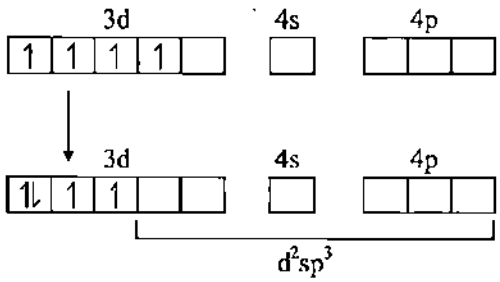
$Ni^0 = 3d^8 4s^2$



By effect of S.F.L. CO.



$[Cr(H_2O)_6]^{+2}$ $Cr^{+2} = 3d^4 4s^0 4p^0$



9. Complexation step

$Ag_2S + CN^{\ominus} \rightarrow [Ag(CN)_2]^- + S^{-2}$

$\downarrow$ + Zn Reduction step

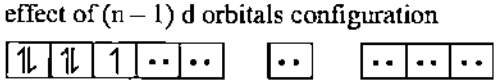
$[Zn(CN)_4]^{-2}$ + Ag

10. $NaBH_4 + I_2 \rightarrow NaI + B_2H_6 + H_2$

11. $[Mn^{+2}(CN)_6]^{-4}$

$Mn^{+2} \rightarrow 3d^5 4s^0 4p$

CN^- is strong ligands so creates back paring effect of (n – 1) d orbitals configuration



So, unpaired $e^- = 1$

$\mu = \sqrt{n(n+2)}$ B.M

$\mu = 1.73$ B.M

And in $[MnBr_4]^{-2}$

Br is a weak ligands so no back pairing effect on (n – 1) d orbital so, unpaired e^- is = 5

$\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.92$ B.M

12. $\underline{XeF_6} + 3H_2O \rightarrow \underline{XeO_3} + 6HF$

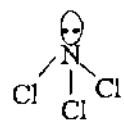
$\downarrow$ $\downarrow$

sp^3d^3 Hybridisation (distorted octahedral) — sp^3 Hybridisation (Pyramidal)

13. (A) O_2, B.O = 2 (B) F_2, B.O = 1

(C) O_2^+ B.O = 2.5 (D) F_2^- B.O = 0.5

14.



$\mu_R \neq 0$

15. $[MCl_4]^{2-}$ Tetrahedral = sp^3 hybridisation

M^{+2}

$\because \mu = \sqrt{n(n+2)}$ B.M. = 3.83

n = 3

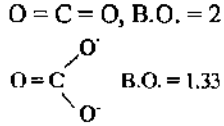
Means configuration of $M^{+2} = 3d^7$

so, $M = 3d^7 4s^2 = {}_{27}Co$

16. $\text{Bond Length} \propto \frac{1}{\text{Bond Order}}$

CO, B.O. = 3

O = C = O, B.O. = 2

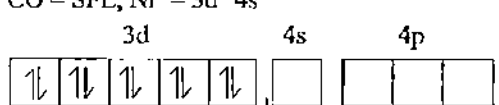


B.O. = 1.33

17. $[Ni(CO)_4]$

$Ni^0 = 3d^8 4s^2$

CO = SFL, $Ni^0 = 3d^{10} 4s^0$



Tetrahedral

Unpaired electron = 0, Diamagnetic

18.

Na^+	Mg^{2+}	F^-	O^{2-}
$10e^-$	$10e^-$	$10e^-$	$10e^-$

19. $BCl_3 \xrightarrow{H_2O} H_3BO_3 \xrightarrow{Na_2CO_3} Na_2B_4O_7$

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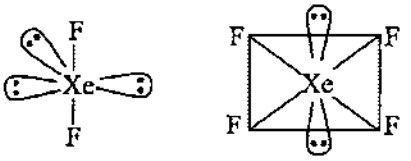
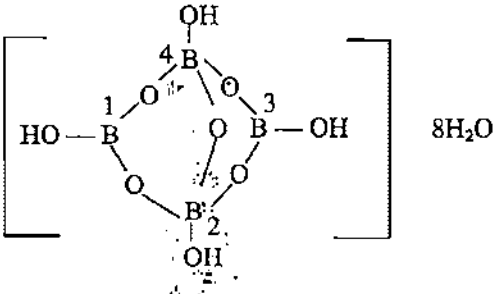
20.



3 ℓ.p. 2 ℓ.p.

21. $Na_2[B_4O_5(OH)_4].8H_2O$



$8H_2O$

In Border no. of boron atom is 4.

In B–O–B unit is 5

22. Reagent which is reduced by aldehyde is Tollens reagent (Ag_2O) so metallic nitrite must be $AgNO_3$.

24. $[Co(H_2O)_6]^{2+} + O_2 + NH_3 \longrightarrow [Co(NH_3)_6]^{3+}$

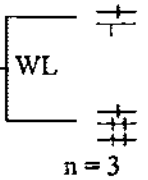
X Y

$Co^{2+} \rightarrow 3d^7\ 4s^2$

$Co^{3+} \rightarrow 3d^6\ 4s^0$

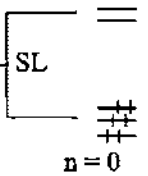
Acc. to CEFT

In complex X $Co^{+2} \rightarrow 3d^7$ — WL



n = 3

In complex Y $Co^{+3} \rightarrow 3d^6$ — SL

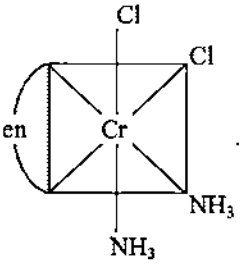


n = 0

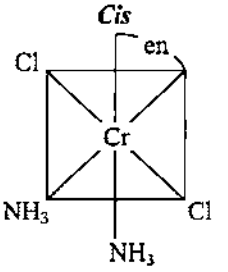
n = 3 (X) n = 0 (Y)

25. Total 3 geometrical isomers are possible -

(1)

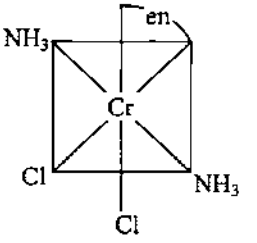


(2)



Cis

(3)



Trans

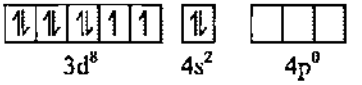
Trans

26. Aluminium form amphoteric oxide with oxygen (Al_2O_3)

27. Ni exist in zero oxidation state so its configuration is -

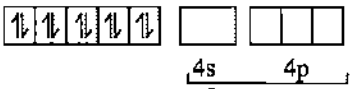
its configuration is -

$_{28}Ni = [Ar]\ 3d^8\ 4s^2$



$3d^8$ $4s^2$ $4p^0$

– CO is strong ligand so pairing of electron possible and configuration will be



4s 4p

sp^3/Tetrahedral

– Number of unpaired electron are zero hence it is diamagnetic in nature.

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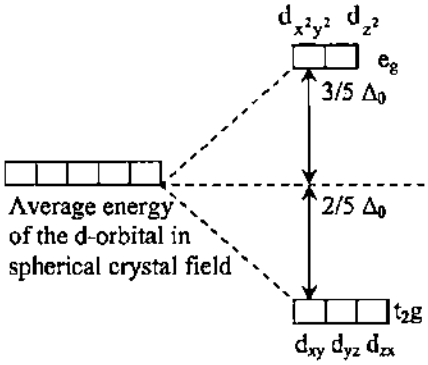
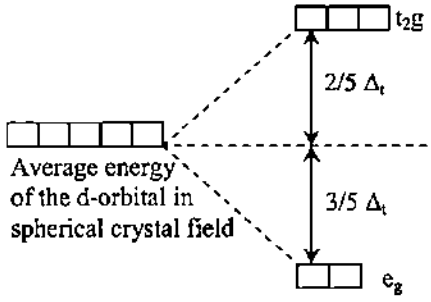
28. Energy of d_{z^2} is greater than d_{xy} in case of octahedral crystal field while energy of d_{z^2} is less than d_{xy} in case of tetrahedral splitting



[Spliting in octahedral crystal field]



[Spliting in tetrahedral crystal field]

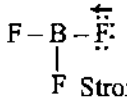
29. $O{=}Xe(=O){=}O$ (sp^3/Pyramidal geometry)

$XeF_6 + 3H{-}OH \longrightarrow 6HF + XeO_3$

30. Lewis acid strength of BBr_3, BCl_3 and BF_3 is in the order of

$$BF_3 < BCl_3 < BBr_3$$

Due to back bonding



Strong 2p – 2p Back bonding

31. O^{-2} is isoelectronic with Mg^{+2}

$O^{-2} \longrightarrow 8 + 2 = 10\ e^-$

$Mg^{+2} \longrightarrow 12 - 2 = 10\ e$

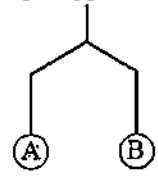
32.

	H–C–H (CH_4)	H–N–H (NH_3)	H–O–H (H_2O)
	B.P. = 4	B.P. = 3	B.P. = 2
	L.P. = 0	L.P. = 1	L.P. = 1
Hybridization	sp^3	sp^3	sp^3
Bond angle	109°, 28'	107°.1	104.5°

33. $KMnO_4 + H_2O_2 \xrightarrow{OH^-} MnO_2 + H_2O$

34. H_2S gas is passed through a hot acidic aqueous solution containing Al^{+3}, Cu^{+2}, Pb^{+2} and Ni^{+2}

II group elements give ppt $\longrightarrow$ CuS, PbS



Pb^{+2} Cu^{+2} Cd^{+2} Bi^{+2} Hg^{+2} As^{+3} Sb^{+3} Sn^{+2} Sn^{+4}

35. $Na = 1s^2\ 2s^2\ 2p^6\ 3s^1$

$Na^+ \rightarrow 1s^2\ 2s^2 2p^6$ [Ne]

[Ne] is inert gas, so, electron removal is very difficult so I.P. is very high.

$$Na^{+2} \longrightarrow 1s^2\ 2s^2\ 2p^5$$

36. Electronegativity $\propto$ % s character

$CH_3 - CH_3$	<	$CH_2 = CH_2$	<	$CH \equiv CH$
sp^3		sp^2		sp
25%		33.3%		50%
s character		s character		s character

38

$MnO_2 + KO_2 \xrightarrow[\text{In Air}]{\text{Oxidized}}$ dark green compound (X)

MnO_4^{-2} & K_2MnO_4

Disproportionation ↓

Purple colour (Y) + $\overset{+4}{MnO_2}$

$\overset{+7}{KMnO_4}$

(X) $\longrightarrow K_2MnO_4$

(Y) $\longrightarrow KMnO_4$

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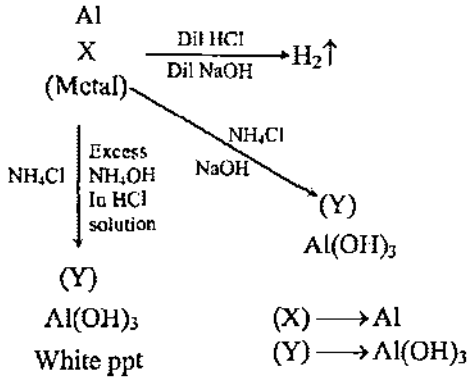
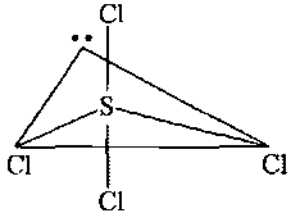
39.



(X) $\longrightarrow$ Al

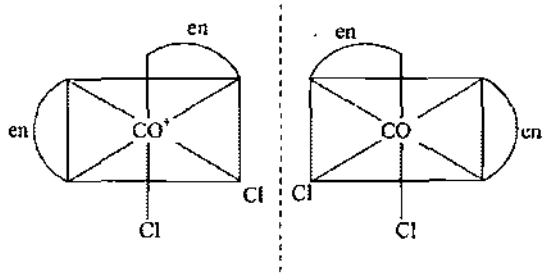
(Y) $\longrightarrow$ $Al(OH)_3$

40. $SCl_4 \Rightarrow 4b.p + 1\ \ell.p.$



41. It has +ve & –ve overlap both simultaneous. So it leads to non-bonding overlap.

42.



43. $\mu \propto \Delta E{\cdot}N$

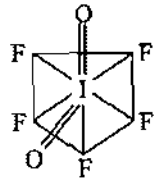
So HF has highest value of dipole moment

44. Greater is the tendency to donate ℓ.p more stable will be the lewis. acid-acid-base adduct.

45. $[IO_2F_5^-]^{2-}$ ion

Hybridisation is sp^3d^3 shape is pentagonal bipyramidal

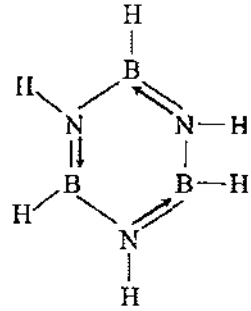
Double bond cause more repulsion so they would be on Axial position 180° angle to each other so shape is



46. $NaCl + AgNO_3 \longrightarrow NaNO_3 + AgCl$ is not a oxidation-reduction reaction because there is no change in oxidation state of any element.

47. Thermal stability of polyvalent anion salt like CO_3^{2-} increase down the group due to increase in ionic character

48. $B_2H_6 + 2NH_3 \xrightarrow{\Delta} B_3N_3H_6$



Borazole or Inorganic benzene

49. Due to steric hinderance SF_6 is not hydrolysed by H_2O at 25°C

50. $(NH_4)_2\ Cr_2O_7 \longrightarrow N_2 + Cr_2O_3 + H_2O$

$2NaN_3 \longrightarrow 2Na + 3N_2$

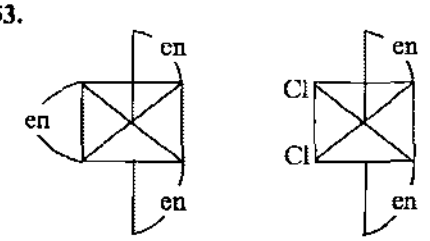
$NH_4NO_2 \longrightarrow N_2 + H_2O$

$(NH_4)_2C_2O_4 \longrightarrow 2NH_3 + H_2C_2O_4$

51. H_2 is reducing agent

52. $[NiCl_4]^2$ sp^3 hybrid, Tetrahedral

53.



optically active optically active

54. $P_4 + Cl_2 \longrightarrow PCl_3$

colourless liquid

H_2O moisture

$H_3PO_3 + HCl$

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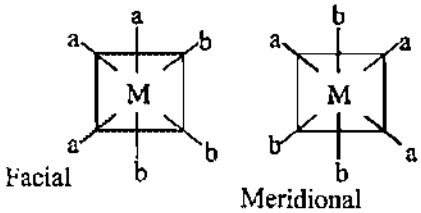
55. $XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$
white
explosive compound

56. These dissolve in water to form hydroxy acid. Stronger acidic oxide react more faster Acidic strength increase with increase in EN

57. Ma_3b_3 Type exist in facial and meridional (mer)

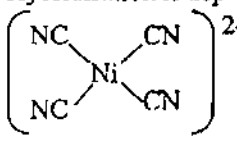


Example $[Co(NO_2)_3(NH_3)_3]$

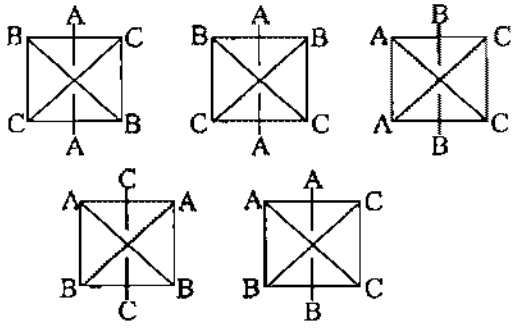
58. $8Na_2CO_3 + 4FeCr_2O_4 + 7O_2 \longrightarrow Na_2CrO_4 + 2Fe_2O_3 + 8O_2$
yellow colour

$2Na_2CrO_4 \xrightarrow{2H^+} Na_2Cr_2O_7 + 2Na^+ + H_2O$
yellow orange

59. Hybridisation is dsp^2 and shape is square planar



60.



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Chapter

3 Organic Chemistry

1. Friedel-Crafts acylation is- **[2010]**

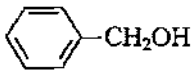
(A) α-acylation of a carbonyl compound

(B) acylation of phenols to generate esters

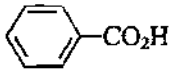
(C) acylation of aliphatic olefins

(D) acylation of aromatic nucleus

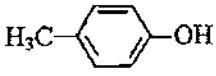
2. The order of acidity of compounds I-IV, is- **[2010]**



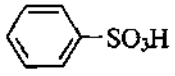
(I)



(II)



(III)

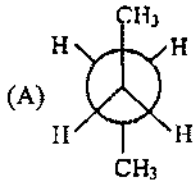


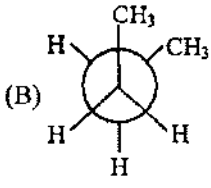
(IV)

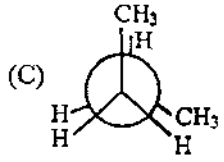
(A) I < III < II < IV (B) IV < I < II < III

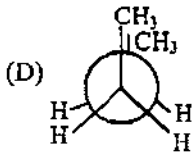
(C) III < I < II < IV (D) II < IV < III < I

3. The most stable conformation for n-butane is- **[2010]**

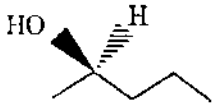
(A) 

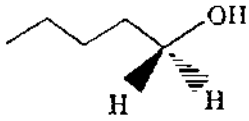
(B) 

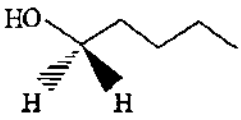
(C) 

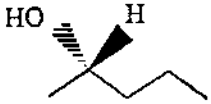
(D) 

4. The enantiomeric pair among the following four structures- **[2010]**

I. 

II. 

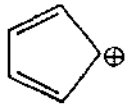
III. 

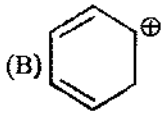
IV. 

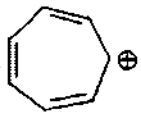
(A) I & II (B) I & IV

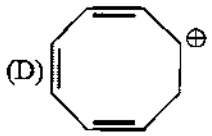
(C) II & III (D) II & IV

5. The aromatic carbocation among the following is- **[2010]**

(A) 

(B) 

(C) 

(D) 

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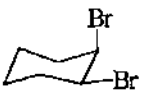
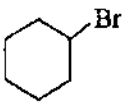
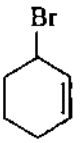
6. Cyclohexene is reacted with bromine in CCl_4 in the dark. The product of the reaction is- **[2010]**

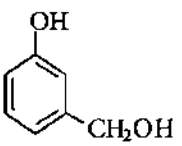
(A)

(B) 

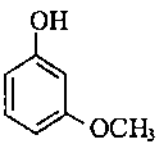
(C) 

(D) 

7. Two isomeric compounds I and II are heated with HBr – **[2010]**

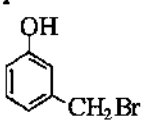


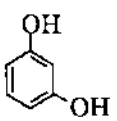
(I)

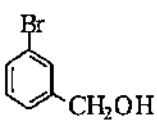


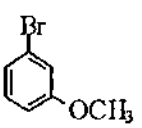
(II)

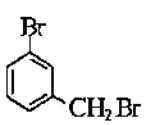
The products obtained are-

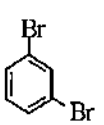
(A) 

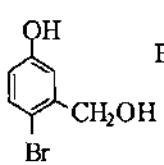


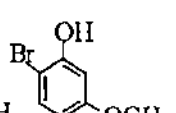
(B) 



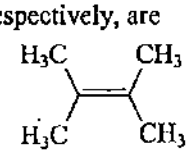
(C) 



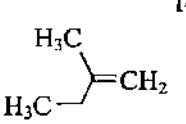
(D) 



8. The number of possible enatiomeric pair(s) produced from the bromination of I and II, respectively, are **[2010]**



(I)

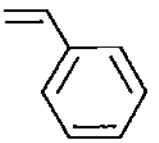


(II)

(A) 0, 1 (B) 1, 0

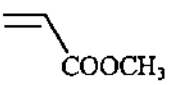
(C) 0, 2 (D) 1, 1

9. Among i-iv

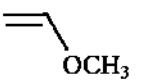


(i)

(ii)



(iii)

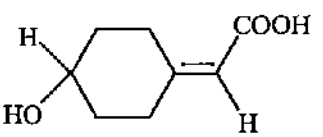


(iv)

the compound that does not undergo polymerization under radical initiation, is **[2011]**

(A) i (B) ii (C) iii (D) iv

10. Two possible stereoisomers for

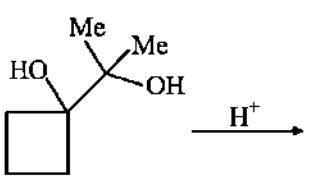


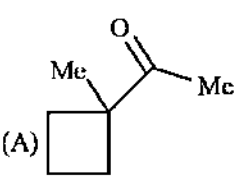
Are **[2011]**

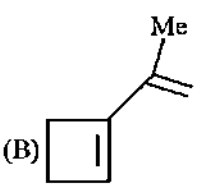
(A) enantiomers (B) diastereomers

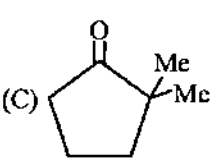
(C) conformers (D) rotamers

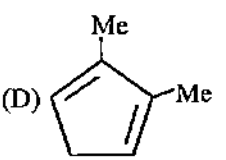
11. The major product of the following reaction is : **[2011]**



(A) 

(B) 

(C) 

(D) 

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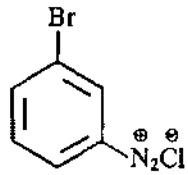
12. For the transformation **[2011]**

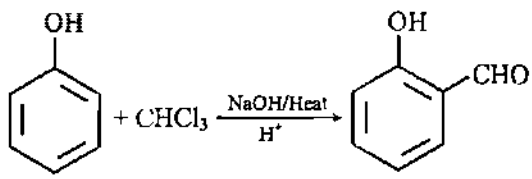


the reagent used is

(A) $LiAlH_4$ (B) H_3PO_2

(C) H_3O^+ (D) H_2/Pt

13. The reaction



is known as **[2011]**

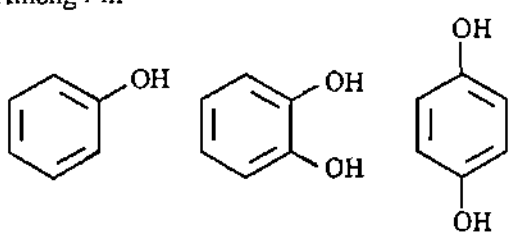
(A) Perkin reaction

(B) Sandmeyer reaction

(C) Reimer-Tiemann reaction

(D) Cannizzaro reaction

14. Among i-iii



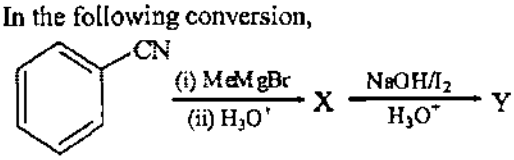
(i) (ii) (iii)

the boiling point follows the order **[2011]**

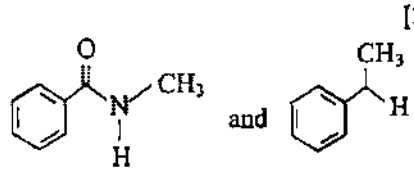
(A) ii < i < iii (B) iii < ii < i

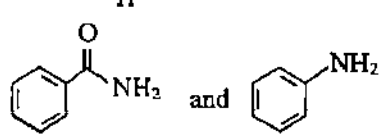
(C) i < ii < iii (D) ii < iii < i

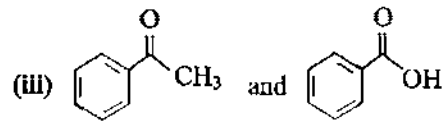
15. In the following conversion,

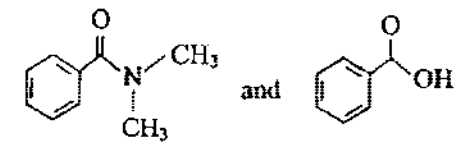


the major products X and Y, respectively, are- **[2011]**

(i) 

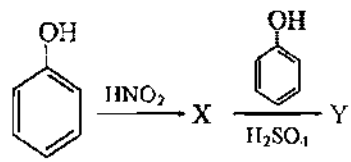
(ii) 

(iii) 

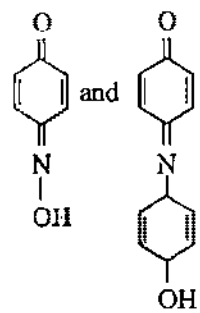
(iv) 

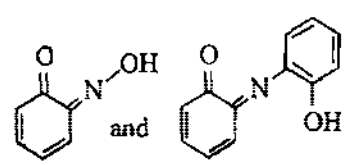
(A) i (B) ii (C) iii (D) iv

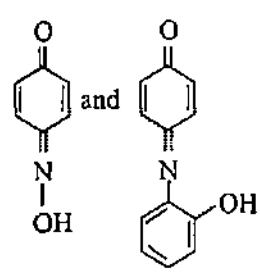
16. In the reaction sequence,

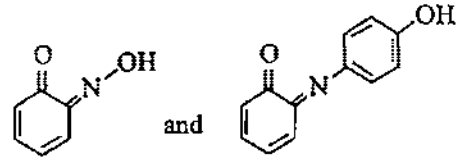


the major products X and Y, respectively, are- **[2011]**

(i) 

(ii) 

(iii) 

(iv) 

(A) i (B) ii (C) iii (D) iv

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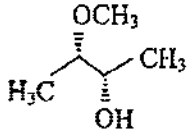
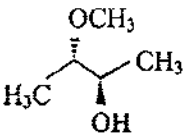
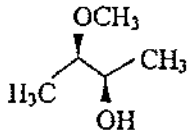
17. Optically active (S)-α-methoxyacetaldehyde on reaction with MeMgX gave a mixture of alcohols. The major diastereomer 'P' on treatment with MeI/K_2CO_3 gave an optically inactive compound. P is- **[2011]**



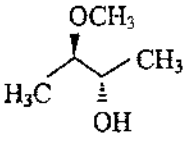
(i)



(ii)



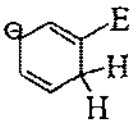
(iii)

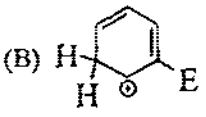


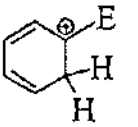
(iv)

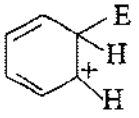
(A) i (B) ii (C) iii (D) iv

18. In the reaction benzene with an electrophile E^+, the structure of the intermediate σ– complex can be represented as **[2012]**

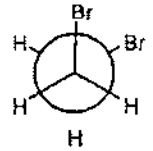
(A) 

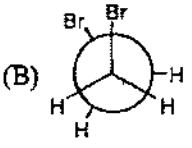
(B) 

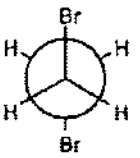
(C) 

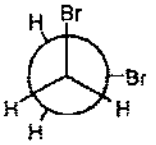
(D) 

19. The most stable conformation of 2, 3-dibromobutane is – **[2012]**

(A) 

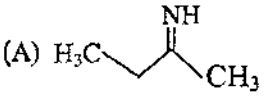
(B) 

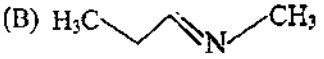
(C) 

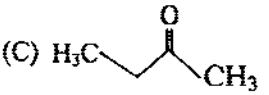
(D) 

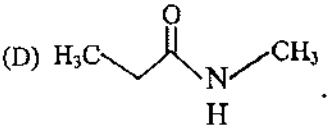
20. The major final product in the following reaction is – **[2012]**

$CH_3CH_2CN \xrightarrow[(2)\ H_3O^+]{(1)\ CH_3MgBr}$

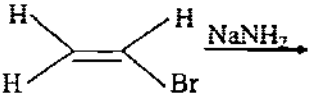
(A) 

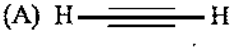
(B) 

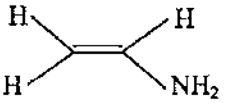
(C) 

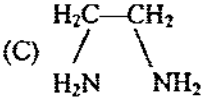
(D) 

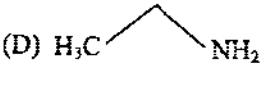
21. The major product in the following reaction is – **[2012]**



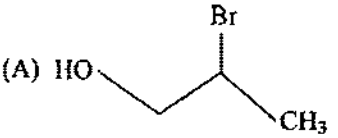
(A) 

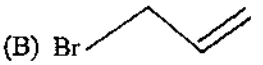
(B) 

(C) 

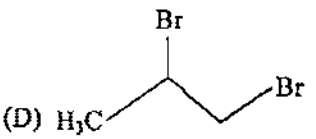
(D) 

22. The major product of the following reaction is – **[2012]**

(A) 

(B) 

(C)

(D) 

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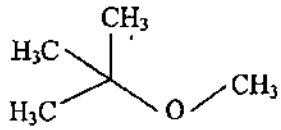
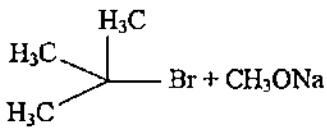
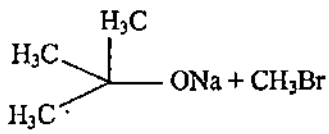
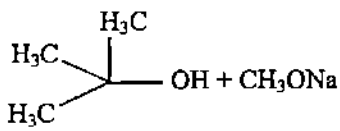
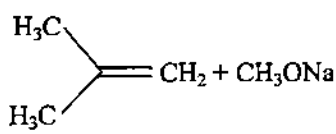
23. The reaction that gives the following molecule as the major product is – **[2012]**



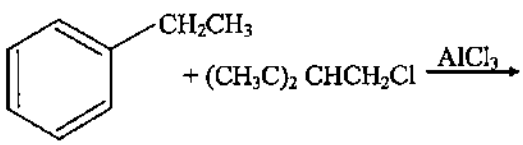
(A) 

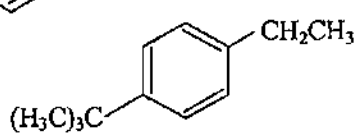
(B) 

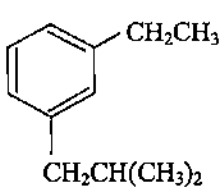
(C) 

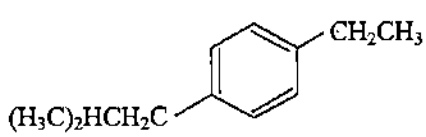
(D) 

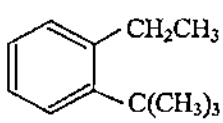
24. The major product of the following reaction is – **[2012]**



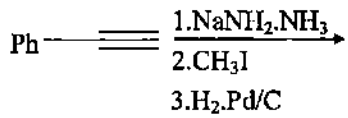
(A) 

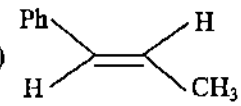
(B) 

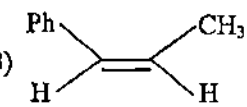
(C) 

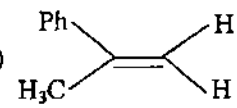
(D) 

25. The final major product obtained in the following sequence of reactions is – **[2012]**



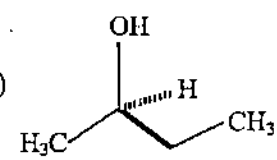
(A) 

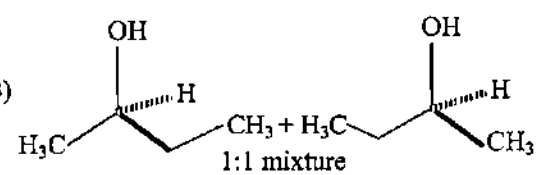
(B) 

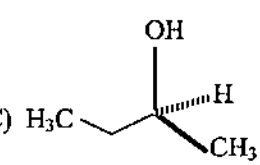
(C) 

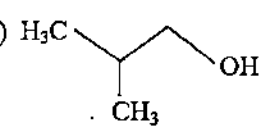
(D)

26. (R)-2-bromobutane upon treatment with aq. NaOH gives – **[2012]**

(A) 

(B) 

(C) 

(D) 

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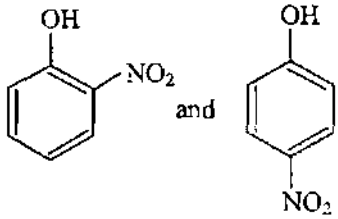
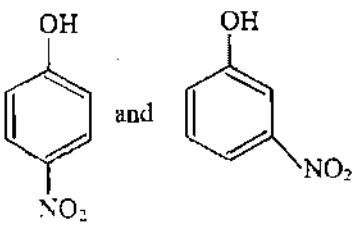
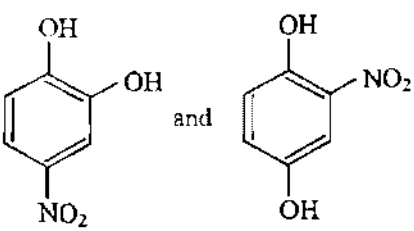
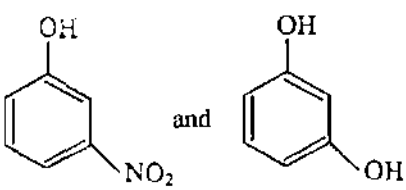
27. Phenol on treatment with dil. HNO_3 gives two products **P** and **Q**. **P** is steam volatile but **Q** is not. **P** and **Q** are, respectively – **[2012]**

(A)  and

(B) 

(C) 

(D) 

28. The compound which reacts with excess bromine to produce 2,4,6-tribromophenol, is - **[2013]**

(A) 1, 3-cyclohexadiene
(B) 1,3-cyclohexanedione
(C) salicylic acid
(D) cyclohexanone

29. Ethyl acetate reacts with $NH_2NHCONH_2$ to form – **[2013]**

(A) $CH_3CONHCONHNH_2$
(B) $CH_3CON(NH_2)CONH_2$
(C) $CH_3CONHNHCONH_2$
(D) $CH_3CH_2NHNHCONH_2$

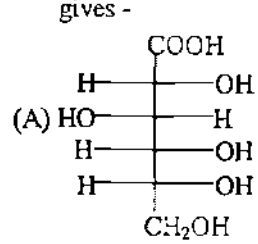
30. The reaction of ethyl methyl ketone with Cl_2/excess OH⁻ gives the following major product - **[2013]**

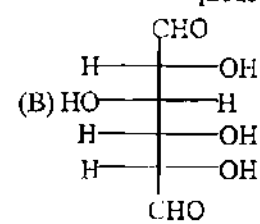
(A) $ClCH_2CH_2COCH_3$
(B) $CH_3CH_2COCCl_3$
(C) $ClCH_2CH_2COCH_2Cl$
(D) $CH_3CCl_2COCH_2Cl$

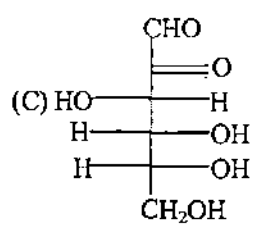
31. The compound that readily tautomerizes is – **[2013]**

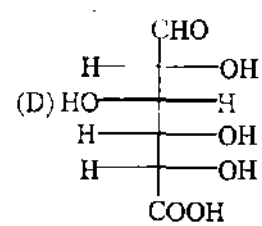
(A) $CH_3COCH_2CO_2C_2H_5$
(B) $CH_3COCH_2CHCH_3$
(C) $CH_3COCH_2CH_2CH_3$
(D) $(CH_3)_3CCOC(CH_3)_3$

32. D-Glucose upon treatment with bromine-water gives - **[2013]**

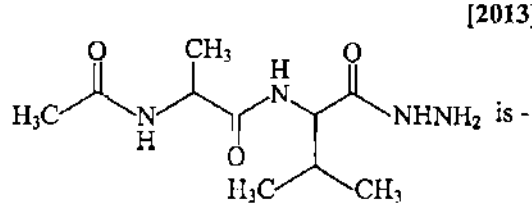
(A) 

(B) 

(C) 

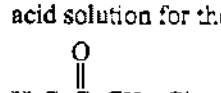
(D) 

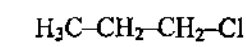
33. The number of peptide bonds in the compound – **[2013]**

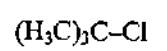
 is -

(A) 1 (B) 2 (C) 3 (D) 4

34. The order of S_N1 reactivity in aqueous acetic acid solution for the compounds **[2013]**


1


2


3

is -

(A) 1 > 2 > 3 (B) 1 > 3 > 2
(C) 3 > 2 > 1 (D) 3 > 1 > 2

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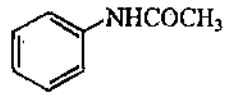
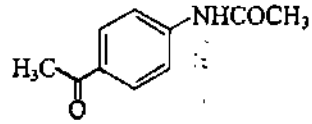
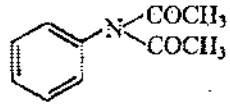
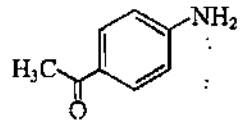
35. The major product obtained in the reaction of aniline with acetic anhydride is - **[2013]**

(A) 

(B) 

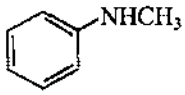
(C) 

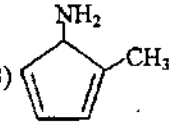
(D) 

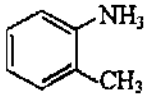
36. The maximum number of isomers that can result from monobromination of 2-methyl-2-pentene with N-bromosuccinimide in boiling CCl_4 is - **[2013]**

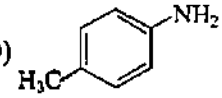
(A) 1 (B) 2 (C) 3 (D) 4

37. The compound X (C_7H_9N) reacts with benzenesulfonyl chloride to give Y ($C_{13}H_{13}NO_2S$) which is insolube in alkali. The compound X is - **[2013]**

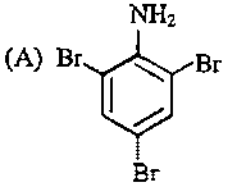
(A) 

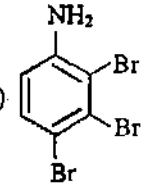
(B) 

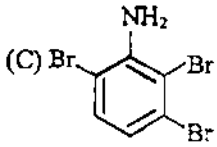
(C) 

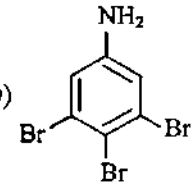
(D) 

38. Aniline reacts with excess Br_2/H_2O to give the major product **[2014]**

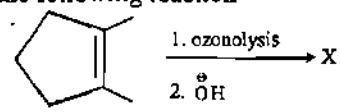
(A) 

(B) 

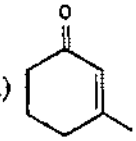
(C) 

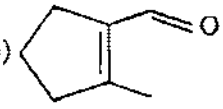
(D) 

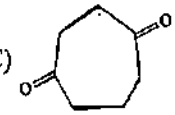
39. In the following reaction - **[2014]**

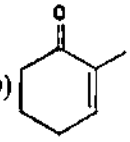


the major product X is-

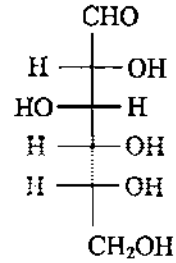
(A) 

(B) 

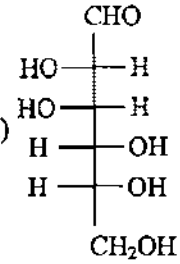
(C) 

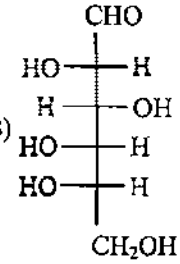
(D) 

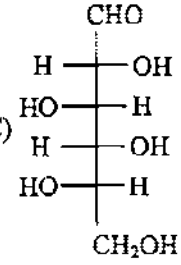
40. Given the structure of D-(+)-glucose as **[2014]**

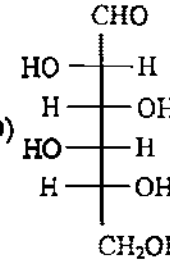


The structure of L-(–)-glucose is

(A) 

(B) 

(C) 

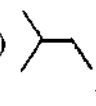
(D) 

41. Major products formed in the reaction of t-butyl methyl ether with HI are - **[2014]**

(A) H_3C-I and

(B) and H_3C-OH

(C) H_3C-OH and

(D)  and H_3C-OH

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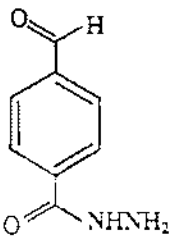
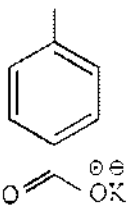
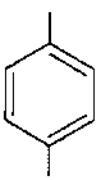
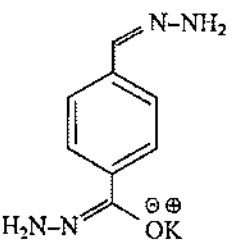
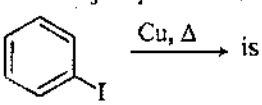
42. 4-Formylbenzoic acid on treatment with one equivalent of hydrazine followed by heating with alcoholic KOH gives the major product - **[2014]**

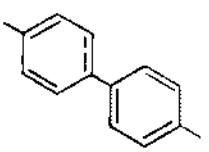
(A) 

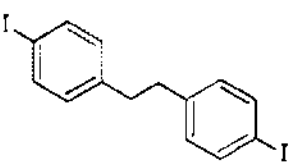
(B) 

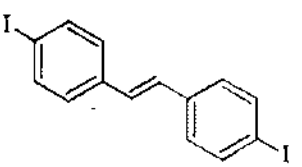
(C) 

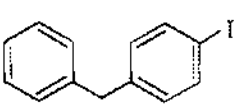
(D) 

43. The major product of the following reaction  is **[2014]**

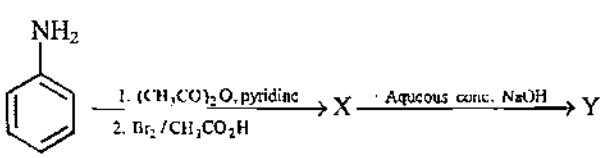
(A) 

(B) 

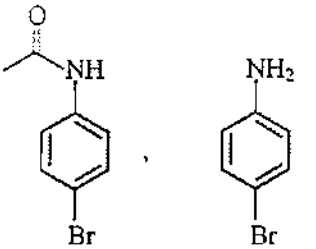
(C) 

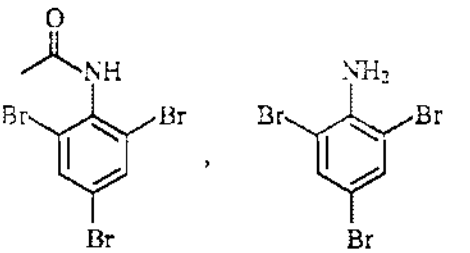
(D) 

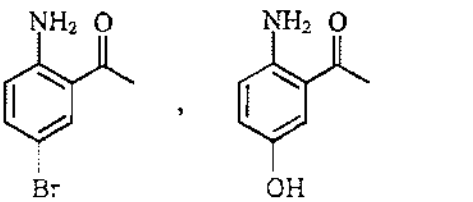
44. In the reaction sequence

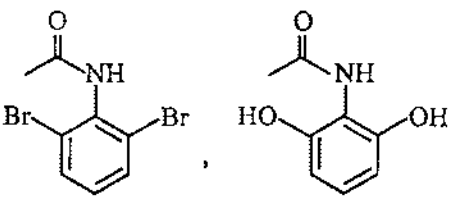


X and Y are, respectively, **[2014]**

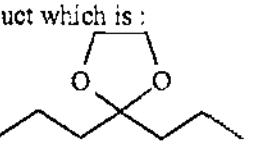
(A) 

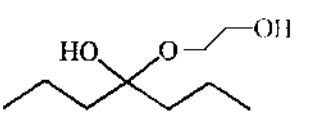
(B) 

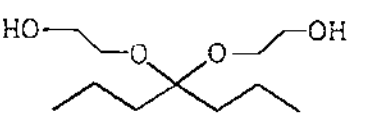
(C) 

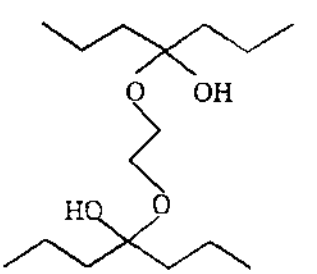
(D) 

45. Calcium butanoate on heating followed by treatment with 1,2-ethanediol in the presence of catalytic amount of an acid, produces a major product which is : **[2014]**

(A) 

(B) 

(C) 

(D) 

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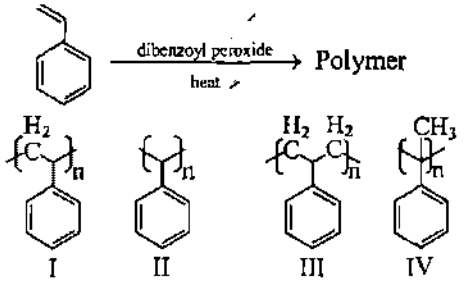
46. Which of the following molecules can exhibit optical activity? **[2015]**

(A) 1-bromopropane (B) 2-bromobutane
(C) 3-bromopentane (D) bromocyclohexane

47. The structure of the polymer obtained by the following reaction is **[2015]**



I II III IV

(A) I (B) II (C) III (D) IV

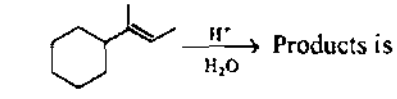
48. The major product of the reaction between CH_3CH_2ONa and $(CH_3)_3CCl$ in ethanol is **[2015]**

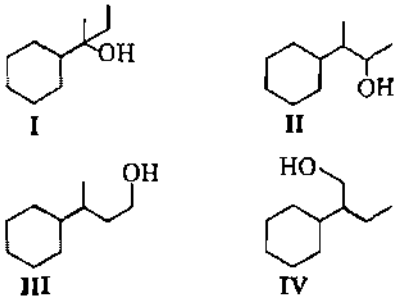
(A) $CH_3CH_2OC(CH_3)_3$ (B) $CH_2 = C(CH_3)_2$
(C) $CH_3CH_2C(CH_3)_3$ (D) $CH_3CH{=}CHCH_3$

49. If α–D–glucose is dissolved in water and kept for a few hours, the major constituent(s) present in the solution is (are) **[2015]**

(A) α–D-glucose
(B) mixture of β–D-glucose and open chain D-glucose
(C) open chain D-glucose
(D) mixture of α–D-glucose and β–D-glucose

50. The major product of the reaction **[2015]**

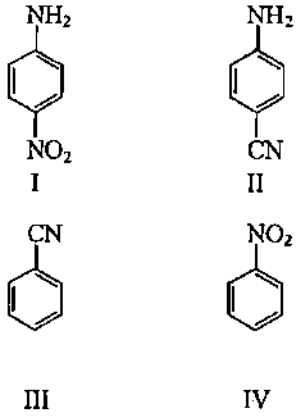




I II III IV

(A) I (B) II (C) III (D) IV

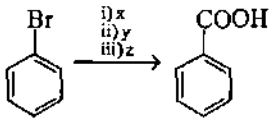
51. Reaction of aniline with $NaNO_2$ + dil. HCl at 0 °C followed by reaction with CuCN yields **[2015]**



I II III IV

(A) I (B) II
(C) III (D) IV

52. In the reaction



x, y and z are **[2015]**

(A) x = Mg, dry ether; y = CH_3Cl; z = H_2O
(B) x = Mg, dry methanol; y = CO_2; z = dil.HCl
(C) x = Mg, dry ether; y = CO_2; z = dil. HCl
(D) x = Mg, dry methanol; y = CH_3Cl; z = H_2O

53. An organic compound having molecular formula C_2H_6O undergoes oxidation with $K_2Cr_2O_7/H_2SO_4$ to produce X which contains 40% carbon, 6.7% hydrogen and 53.3% oxygen. The molecular formula of the compound X is **[2015]**

(A) CH_2O (B) $C_2H_4O_2$
(C) C_2H_4O (D) $C_2H_6O_2$

54. The maximum number of cyclic isomers (positional and optical) of a compound having molecular formula $C_3H_2Cl_2$ is **[2015]**

(A) 2 (B) 3
(C) 4 (D) 5

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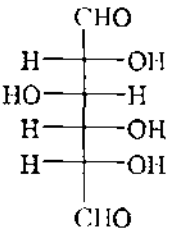
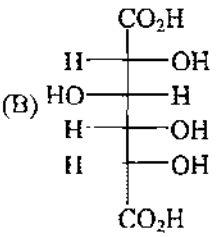
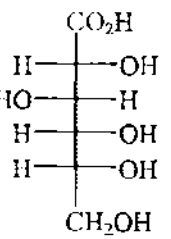
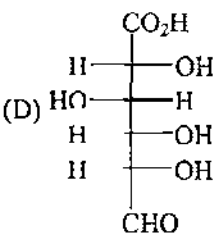
55. The reaction of D-glucose with ammoniacal $AgNO_3$ produces [2016]

(A) 

(B) 

(C) 

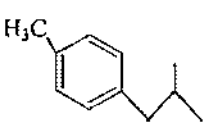
(D) 

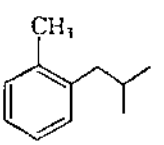
56. The reagent (s) used for the conversion of benzene diazonium hydrogensulfate to benzene is / are - [2016]

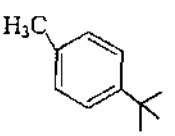
(A) H_2O (B) $H_3PO_2 + H_2O$

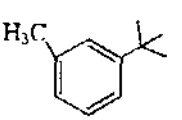
(C) $H_2SO_4 + H_2O$ (D) $CuCl / HCl$

57. The major product obtained in the reaction of toluene with 1-bromo-2-methyl propane in the presence of anhydrous $AlCl_3$ is [2016]

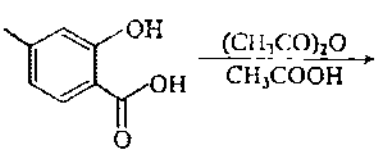
(A) 

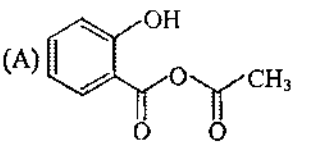
(B) 

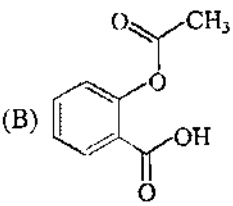
(C) 

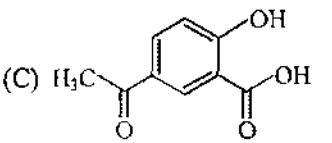
(D) 

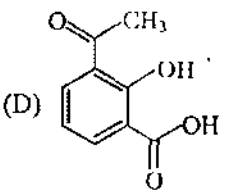
58. The major product in the following reaction is [2016]



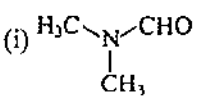
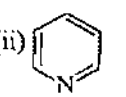
(A) 

(B) 

(C) 

(D) 

59. The compounds contaning sp hybridized carbon atom are [2016]

(i)  (ii) 

(iii) H_3C CN (iv) $H_2C{=}C{=}CHCH_3$

(A) (i) and (ii) (B) (iii) and (iv)

(C) (ii) and (iii) (D) (i) and (iv)

60. Upon heating with acidic $KMnO_4$ an organic compound produces hexan-1,6-dioic acid as the major product the starting compound is [2016]

(A) benzene

(B) cyclohexene

(C) 1-methylcyclohexene

(D) 2-methylcyclohexene

61. The number of stereoisomer possible for the following compound is [2016]

$CH_3{-}CH{=}CH{-}CH(Br){-}CH_2{-}CH_3$

(A) 2 (B) 3

(C) 4 (D) 8

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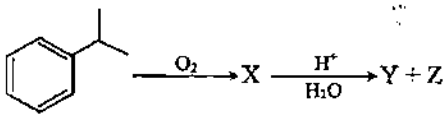
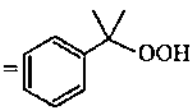
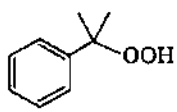
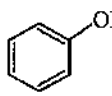
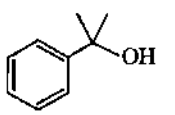
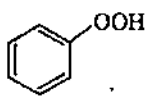
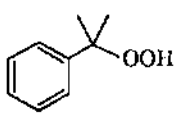
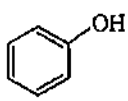
62. X, Y and Z in the following reaction sequence are **[2016]**



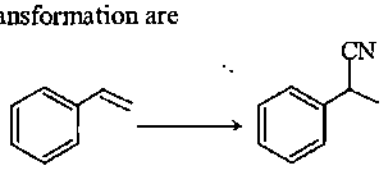
(A) X =  Y = Z =

(B) X =  Y =  Z =

(C) X =  Y =  Z =

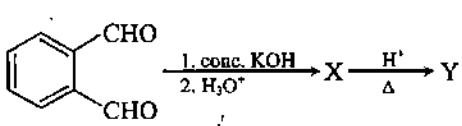
(D) X =  Y =  Z =

63. The reagent required for the following two step transformation are **[2016]**

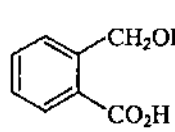
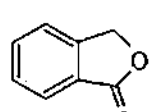


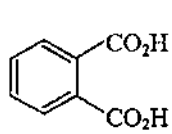
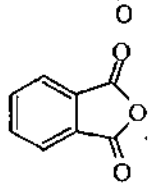
(A) (i) HBr, benzoyl peroxide; (ii) CH_3CN (B) (i) HBr, (ii) NaCN
(C) (i) Br_2, (ii) NaCN
(D) (i) NaBr, (ii) NaCN

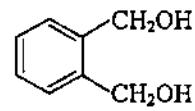
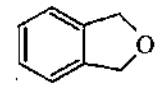
64. In the reaction sequence **[2016]**

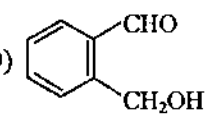
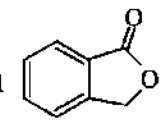


The major product X and Y, respectively, are

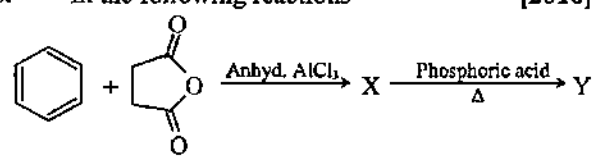
(A)  and 

(B)  and 

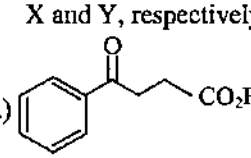
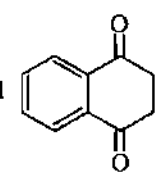
(C)  and 

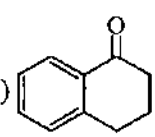
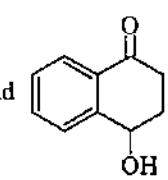
(D)  and 

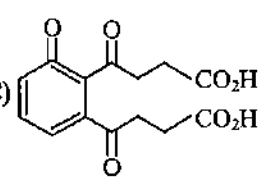
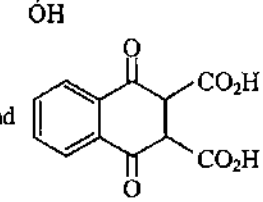
65. In the following reactions **[2016]**

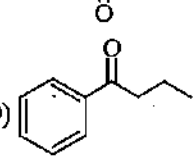
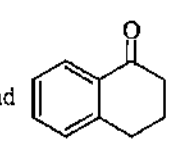


X and Y, respectively, are

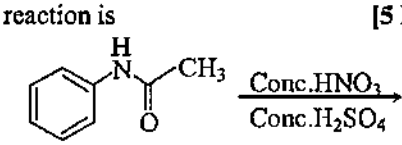
(A)  and 

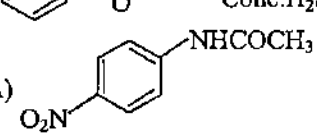
(B)  and 

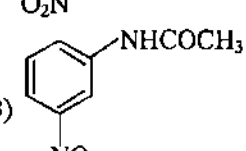
(C)  and 

(D)  and 

66. The major product formed in the following reaction is **[5 Nov, 2017]**



(A) 

(B) 

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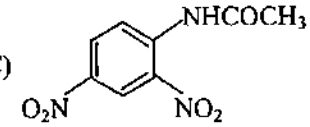
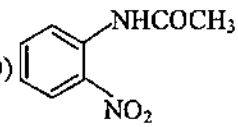
(C) 

(D) 

67. Among the α-amino acids - threonine, tyrosine, methionine, arginine and tryptophan, those which contain an aromatic group in their side chain are **[5 Nov, 2017]**

(A) threonine and arginine
(B) tyrosine and tryptophan
(C) methionine and tyrosine
(D) arginine and tryptophan

68. The number of stereoisomers possible for the following compound is

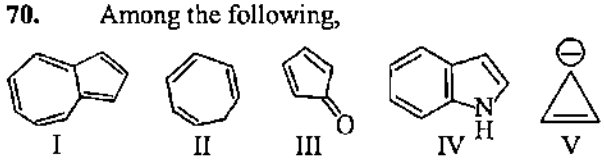
CH_3-CH=CH-CH(OH)-CH_3 **[5 Nov, 2017]**

(A) 1 (B) 2
(C) 3 (D) 4

69. In electrophilic aromatic substitution reactions of chlorobenzene, the ortho/para-directing ability of chlorine is due to its **[5 Nov, 2017]**

(A) positive inductive effects (+I)
(B) negative inductive effect (–I)
(C) positive resonance effect (+R)
(D) negative resonance effect (–R)

70. Among the following,



I II III IV V

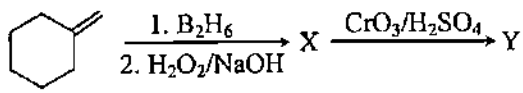
the antiaromatic compounds are **[5 Nov, 2017]**

(A) I and IV (B) III and V
(C) II and V (D) I and III

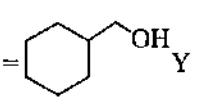
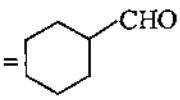
71. Upon reaction with CH_3MgBr followed by protonation, the compound that produces ethanol is **[5 Nov, 2017]**

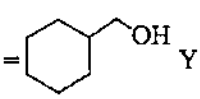
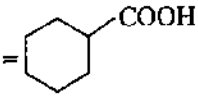
(A) CH_3CHO (B) HCOOH
(C) HCHO (D) $(CHO)_2$

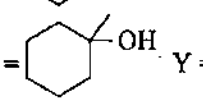
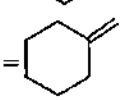
72. In the following reaction sequence

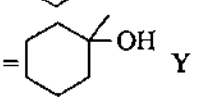
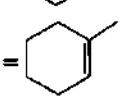


X and Y are **[5 Nov, 2017]**

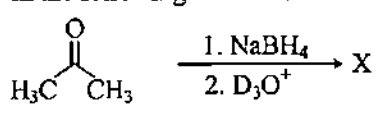
(A) X =  Y = 

(B) X =  Y = 

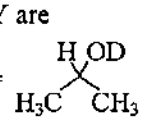
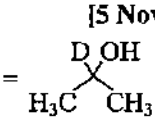
(C) X =  Y = 

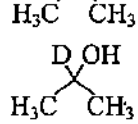
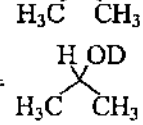
(D) X =  Y = 

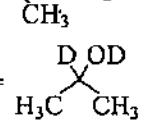
73. In the following reactions

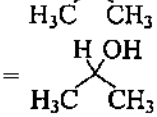


X and Y are **[5 Nov, 2017]**

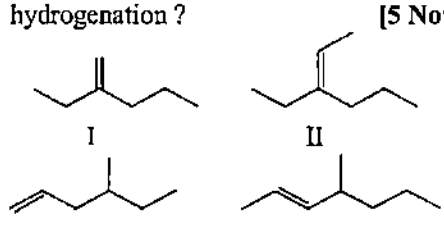
(A) X =  Y = 

(B) X =  Y = 

(C) X = Y = 

(D) X = Y = 

74. Which of the following alkenes can generate optically active compounds upon hydrogenation ? **[5 Nov, 2017]**



I II III IV

(A) I, III and IV (B) II and III
(C) I and III (D) II and IV

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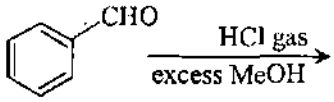
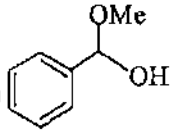
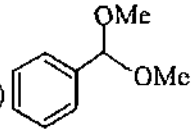
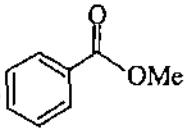
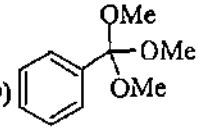
75. The major product formed in the following reaction is **[19 Nov, 2017]**



(A) 

(B) 

(C) 

(D) 

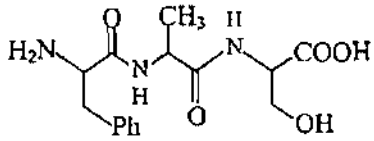
76. Which among the following is a non-benzenoid aromatic compound ? **[19 Nov, 2017]**

(A) o-Xylene (B) Phenanthrene
(C) Indole (D) Thiophene

77. Natural rubber is a polymer of **[19 Nov, 2017]**

(A) Neoprene (B) Chloroprene
(C) Isoprene (D) Styrene

78. The following tripeptide **[19 Nov, 2017]**



can be represented as

(A) Tyr-Val-Thr (B) Phe-Ala-Ser
(C) Phe-Leu-Cys (D) Lys-Ala-Ser

79. The sugar units present in natural DNA and RNA, respectively, are **[19 Nov, 2017]**

(A) D-2-deoxyribose and L-ribose
(B) L-2-deoxyribose and D-ribose
(C) D-2-deoxyribose and D-ribose
(D) L-2-deoxyribose and L-ribose

80. The major product formed in the following reaction is **[19 Nov, 2017]**

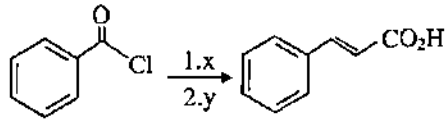
$CH_3Br + CH_3CH_2ONa \rightarrow$

(A) $CH_3CH_2CH_2OH$
(B) CH_3OCH_3
(C) $CH_3CH_2OCH_3$
(D) $CH_3CH_2OCH_2Br$

81. The most abundant metal ion present in the human body is **[19 Nov, 2017]**

(A) Zn^{2+} (B) Ca^{2+}
(C) Na^{+} (D) Fe^{2+}

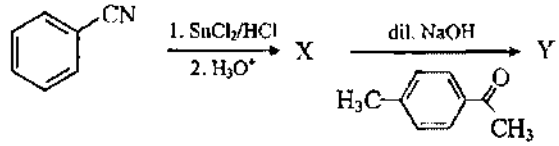
82. In the reaction



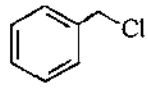
x and y are **[19 Nov, 2017]**

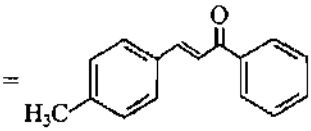
(A) x = H_2, Pd/$BaSO_4$; y = NaOAc, Ac_2O
(B) x = $LiAlH_4$; y = NaOAc, Ac_2O
(C) x = H_2, Pd/C ; y = NaOH, Ac_2O
(D) x = $LiAlH_4$; y = NaOH, Ac_2O

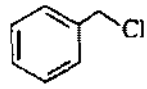
83. In the following reaction

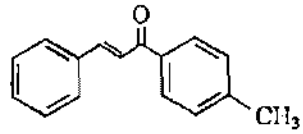


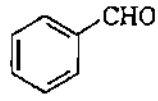
X and Y are **[19 Nov, 2017]**

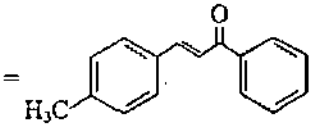
(A) X = 

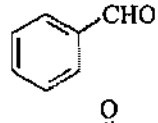
Y = 

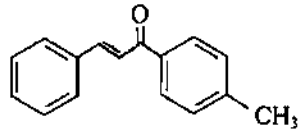
(B) X = 

Y = 

(C) X = 

Y = 

(D) X = 

Y = 

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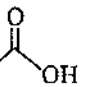
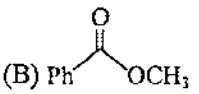
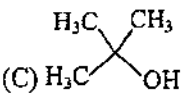
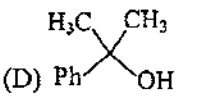
84. Acetophenone ($PhCOCH_3$) reacts with perbenzoic acid to produce a compound X. Reaction of X with excess CH_3MgBr followed by treatment with aqueous acid predominantly produces **[19 Nov, 2017]**

(A) 

(B) 

(C) 

(D) 

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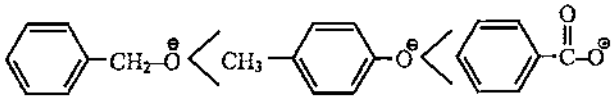
ANSWERS

1. (D)	2. (A)	3. (A)	4. (B)	5. (C)	6. (B)	7. (A)
8. (A)	9. (D)	10. (A)	11. (C)	12. (B)	13. (C)	14. (C)
15. (C)	16. (C)	17. (*)	18. (D)	19. (C)	20. (C)	21. (A)
22. (B)	23. (B)	24. (A)	25. (D)	26. (C)	27. (A)	28. (C)
29. (C)	30. (B)	31. (A)	32. (A)	33. (A)	34. (C)	35. (A)
36. (D)	37. (A)	38. (A)	39. (A)	40. (B)	41. (C)	42. (A)
43. (A)	44. (A)	45. (A)	46. (B)	47. (A)	48. (B)	49. (D)
50. (A)	51. (C)	52. (C)	53. (B)	54. (C)	55. (C)	56. (B)
57. (C)	58. (B)	59. (B)	60. (B)	61. (C)	62. (D)	63. (B)
64. (A)	65. (A)	66. (A)	67. (B)	68. (D)	69. (C)	70. (B)
71. (C)	72. (B)	73. (A)	74. (C)	75. (B)	76. (D)	77. (C)
78. (B)	79. (C)	80. (C)	81. (B)	82. (A)	83. (D)	84. (C)

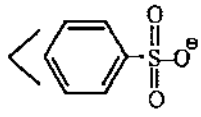
SOLUTIONS

1. Friedel craft reaction used for introducing an alkyl or acyl group in benzene nucleus by an alkylating or acylating agent in presence of a suitable catalyst.

2.

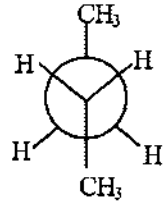


No. resonance
less stable
less acidic nature



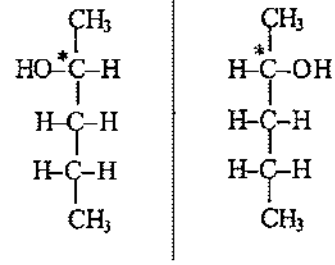
More-stable
more acidity

3. Most stable conformer of n-butane is



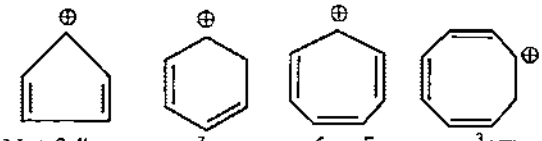
because dihedral angle between CH_3 group is 180°

4.



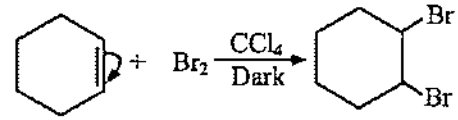
Mirror image & not superimposable on each other. While chiral center absent in II and III.

5.



Not follow Huckel rule | sp^3 'C' present (nonplaner) | 6 π e⁻ & planer molecule (Aromatic) | sp^3 'C' present (non-planer)

6.



Br_2 molecule ionizes on interaction with π bond

$Br_2 \longrightarrow Br^+ + :Br^-$

Electrophile Nucleophile

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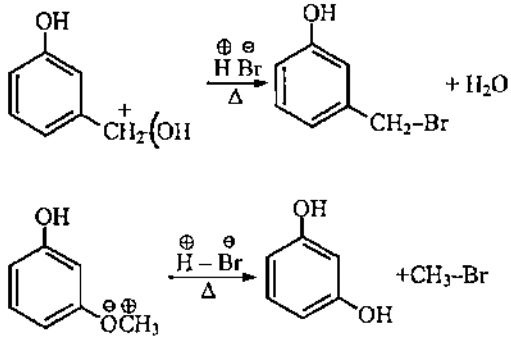
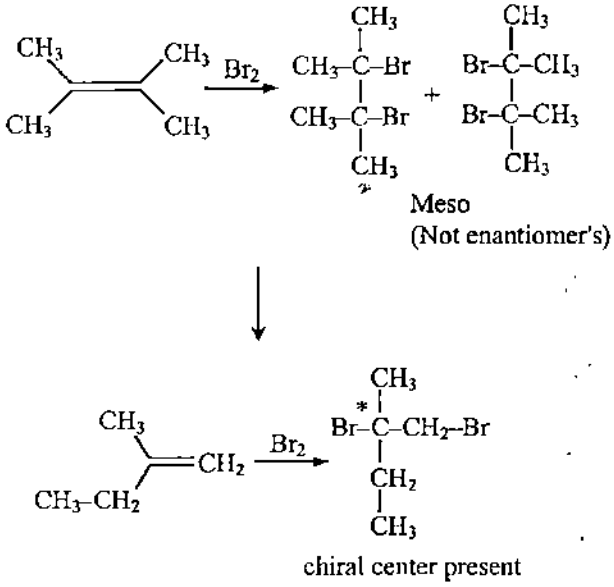
7.



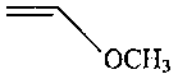
8.



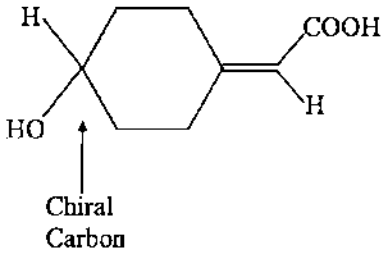
Meso
(Not enantiomer's)

chiral center present

9.



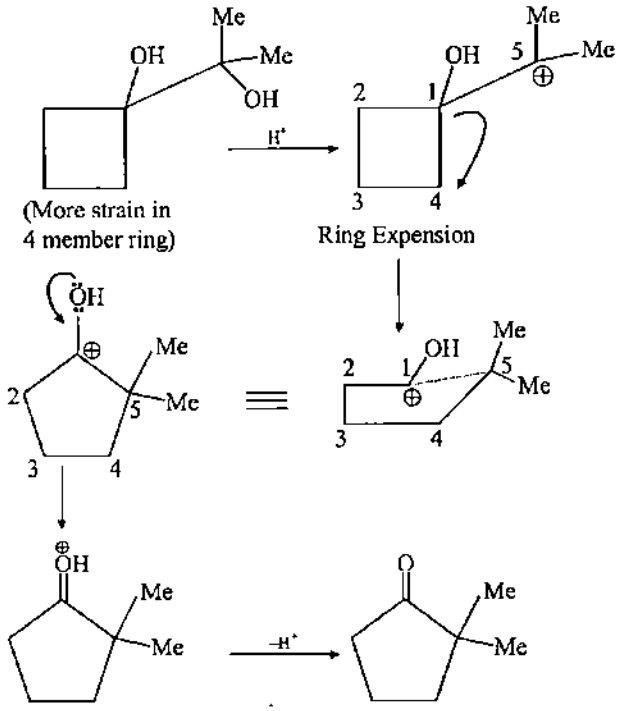
10.



Chiral
Carbon

Enantionmers Formed because cyclohexa diene shows optical activity.

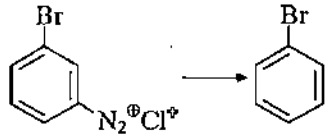
11.



(More strain in
4 member ring)

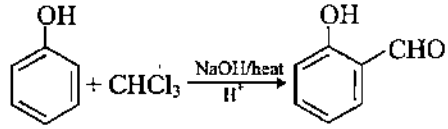
Ring Expension

12.



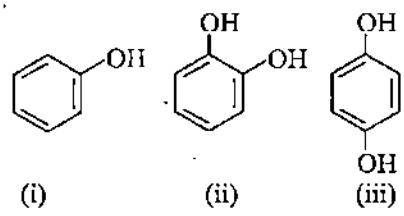
H_3PO_2 As a reagent used because other are strong reducing agent

13.



Reimer – Tiemann Reaction – Phenol react with $CHCl_3$ in presence of NaOH given product

14.



(i) (ii) (iii)

(iii) > (ii) > (i)

In (ii) Intra H-Bonding is not formed because ring formation is not stable

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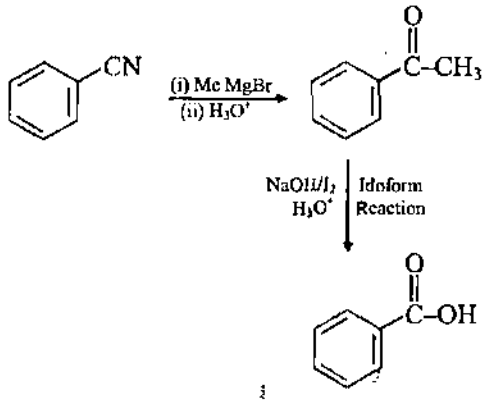
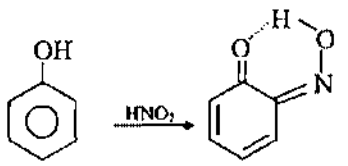
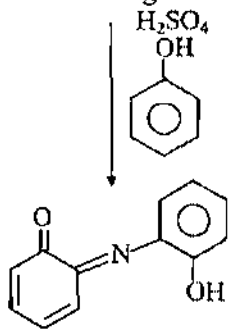
15.



16.



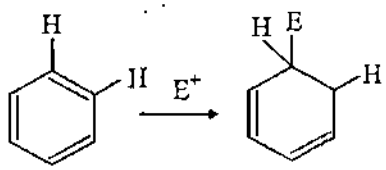
intramolecular H-Bonding



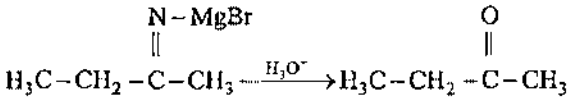
17. Incorrect question

The statement optically active (s) – α – methoxy acetaldehyde is incorrect.

18.

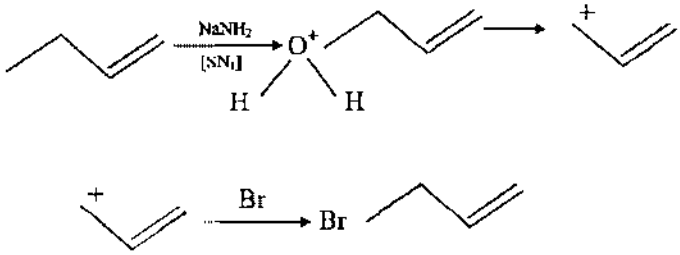


20.

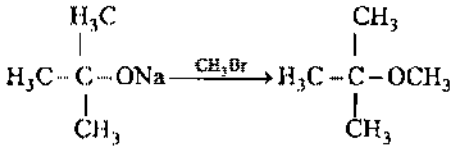


21. $CH_2 = CHBr \xrightarrow[-HBr]{NaNH_2} CH \equiv CH$

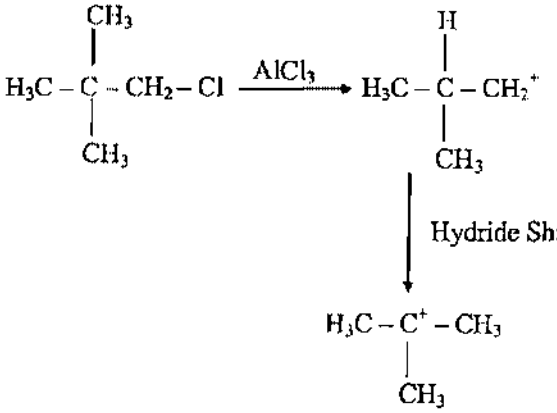
22.

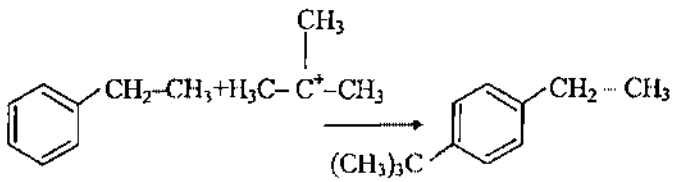


23.



24.





25. $PhC \equiv CH \xrightarrow{NaNH_2/NH_3} PhC \equiv C^- \xrightarrow{CH_3I} PhC \equiv C - CH_3 \xrightarrow{H_2, Pd/C}$

$Ph - CH_2 - CH_2 - CH_3$

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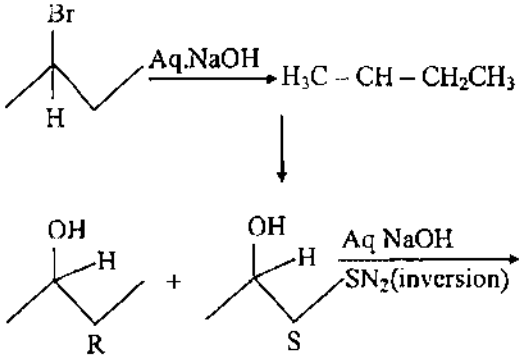
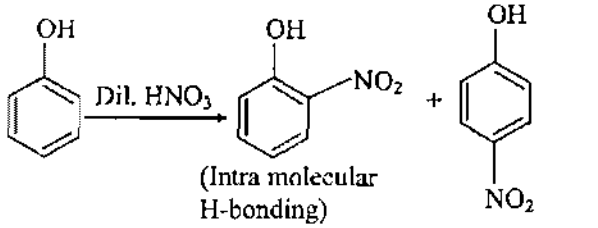
26.

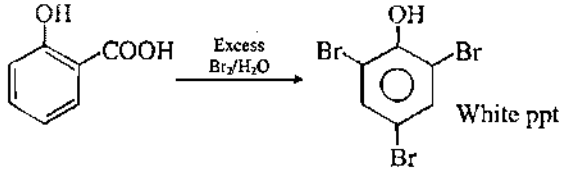


(S) -2 bromo butane

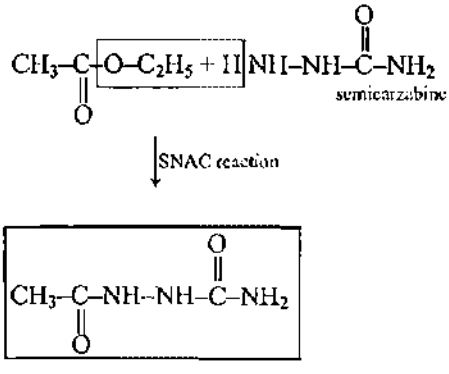
27.



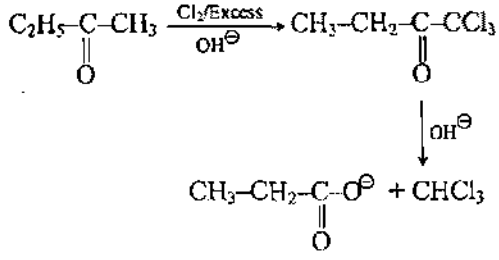
28.



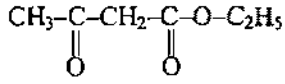
29.



30. Example of iodoform reaction

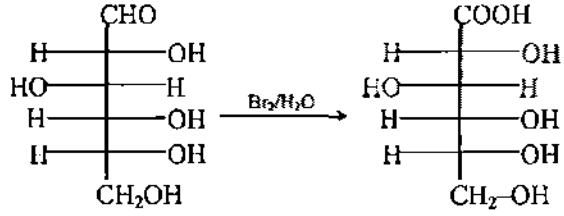


31.



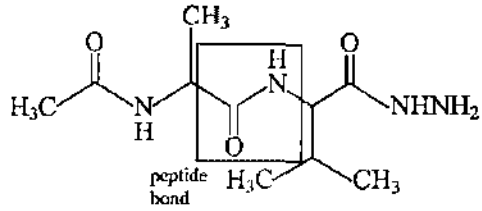
Aceto Acetic ester

32.



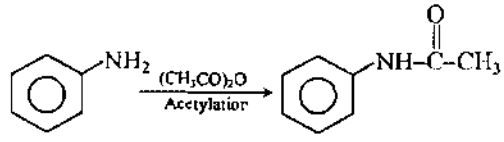
Gluconic acid

33. Peptide bond is formed when two amino acid combines it is amide linkage



34. Rate of S_N1 reaction ∝ stability carbocation.

36.



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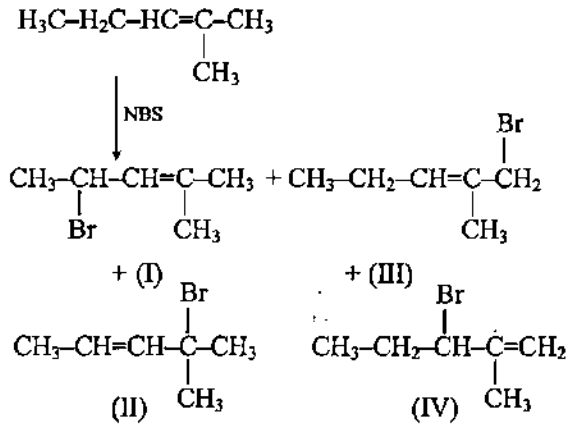
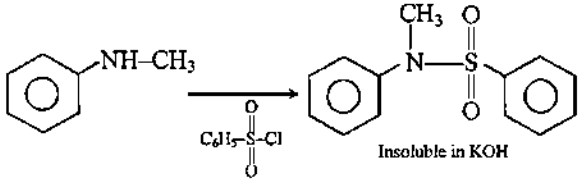
37.

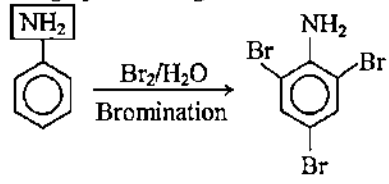


38. C_7H_9N [X] $\xrightarrow[\text{Sulfonyl chloride}]{\text{Benzene}}$ [Y] Insoluble in KOH

It must be 2° amine

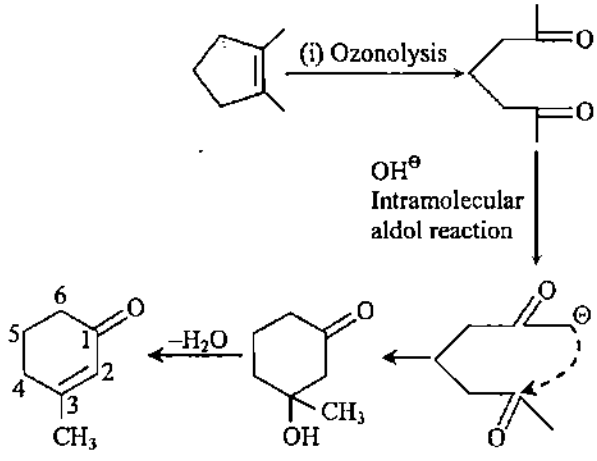


39. Highly activating

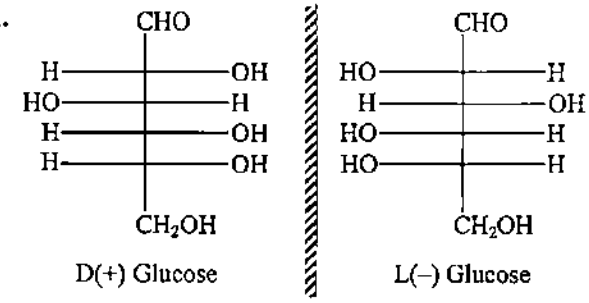


2,4,6-Tribromoaniline

40.

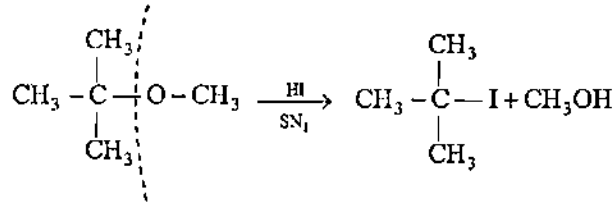


41.



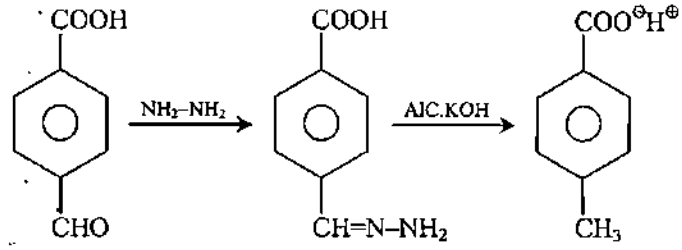
Enantiomer

42.



If one of the alkyl group is 3°. Then mechanism is SN_1 and nucleophile attach to the carbon where carbocation more stable.

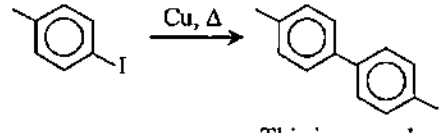
43.



This is example of wolfkishner reduction which converts.

$>C=O$ in $>CH_2$ Group But do not reduce –COOH group.

44.



This is example of Ulman reaction which gives product like Wurtz reaction.

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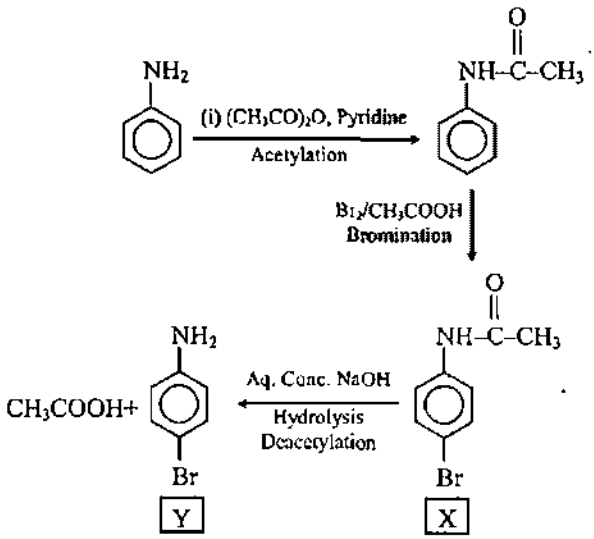
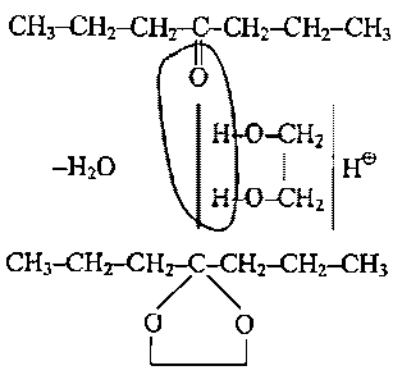
45.

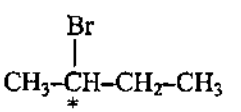


46.

$(CH_3CH_2CH_2COO)_2Ca \xrightarrow{\Delta}$

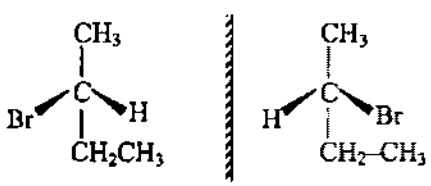


47.



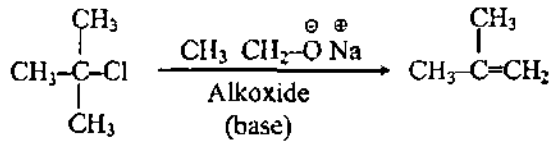
2 Bromo butane

This molecule contain 1 chiral centre and molecule having one chiral carbon do not have any type of symmetry so it is optically active



Non superimposable on mirror image

48.



3° (halide)

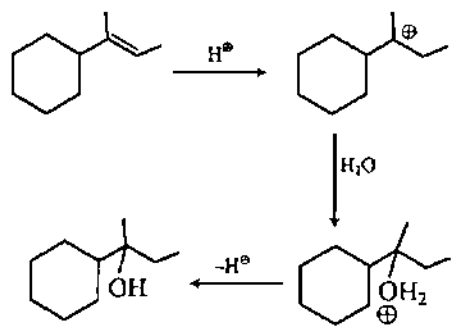
Alkoxide ion is strong nucleophile and strong base & with 3° Alkyl halide Alkenes is the major product [E_2 Elimination]

49.

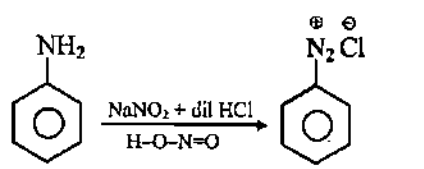
α-D Glucose ⇌ Open chain Structure Glucose ⇌ β-D Glucose

35% 65%

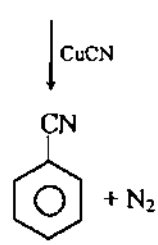
50.



51.



Benzene diazonium chloride



Cynobenzene

(III)

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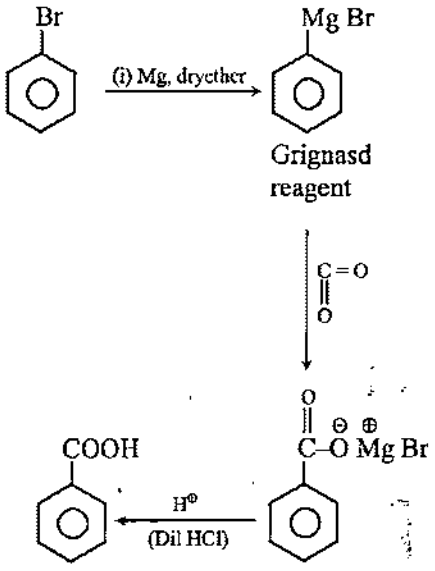
52.



53.

$CH_3–CH_2–OH \xrightarrow{K_2Cr_2O_7/H_2SO_4} CH_3–\overset{O}{\overset{\|}{C}}–O–H$

(C_2H_{60})

Alcohol

$(C_nH_{2n+2}O)$

Carboxylic acid

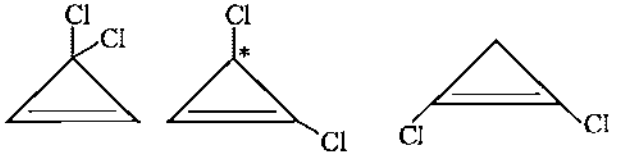
Total wt = 60

C = 24 (40% carbon)

O = 32 (53.3%)

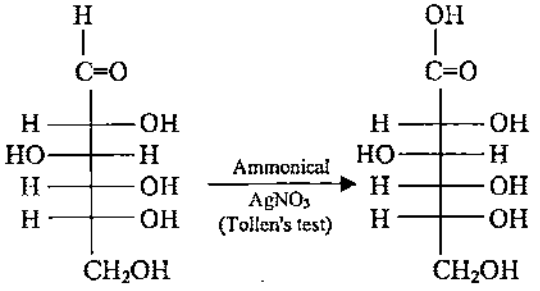
H = 4 (6.7%)

54.



Racemic mixture (Enantiomer) pair

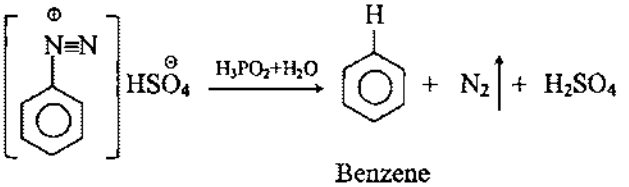
55.



D-Glucose

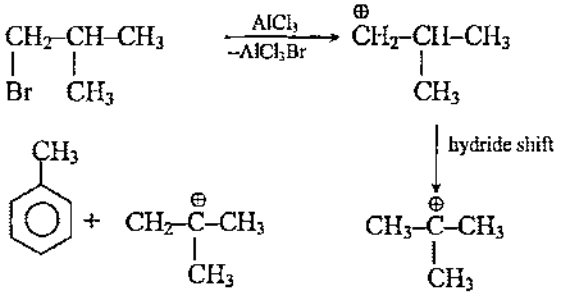
* Tollen's reagent oxidise aldehyde group $\left(-\overset{O}{\overset{\|}{C}}-H\right)$ in to carboxylic acid $\left(-\overset{O}{\overset{\|}{C}}-OH\right)$.

56.



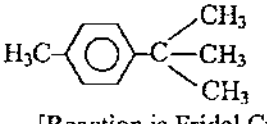
Benzene

57.



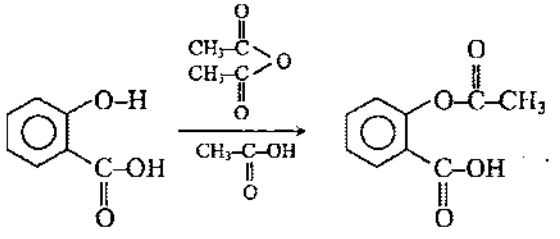
Toluene [Act As a $E^{\oplus}$]

[subs.] [$E^{\oplus}$]



[Reaction is Fridel Craft Alkylation]

58.



Salicylic acid [Acetylation] Acetyl Salicylic Acid [Aspirin]

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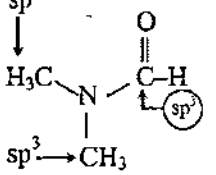
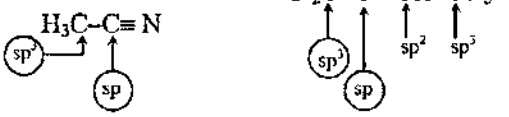
59. (I) sp^3 (II) sp^2

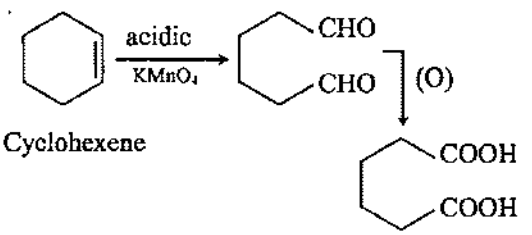


(III) (IV) $H_2C=C=CH-CH_3$



III & IV compound contain sp hybridised carbon

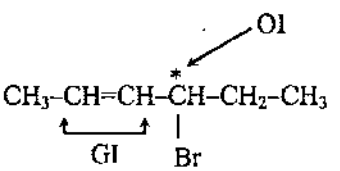
60.



Cyclohexene

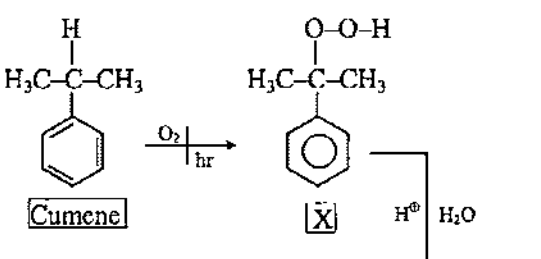
Hexane-1,6-dioic acid

61.

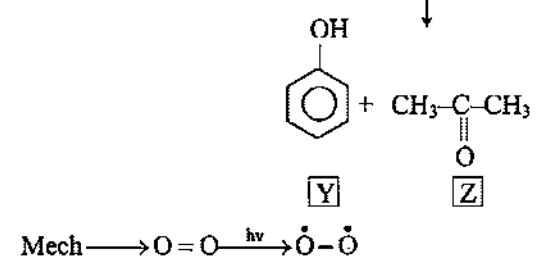


n = 2 [No. of stereogenic area]
Total stereoisomer = 2^n
[When symm. is/are absent]
Total stereo isomer = 2^2 = 4

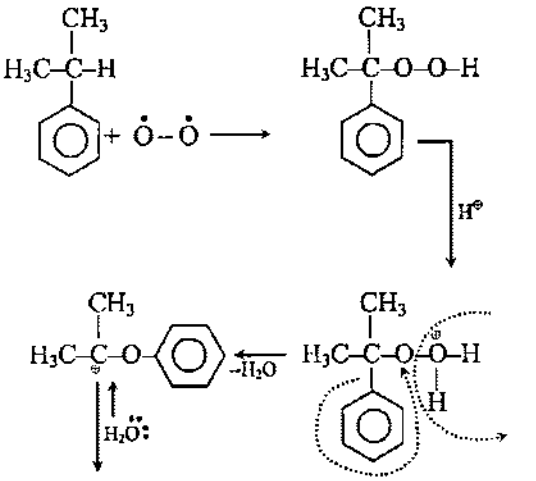
62.



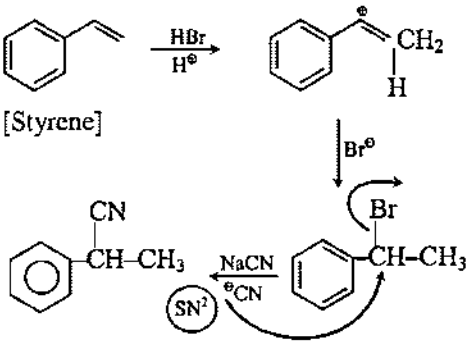
Cumene



Mech $\longrightarrow O=O \xrightarrow{h\nu} \dot{O}-\dot{O}$

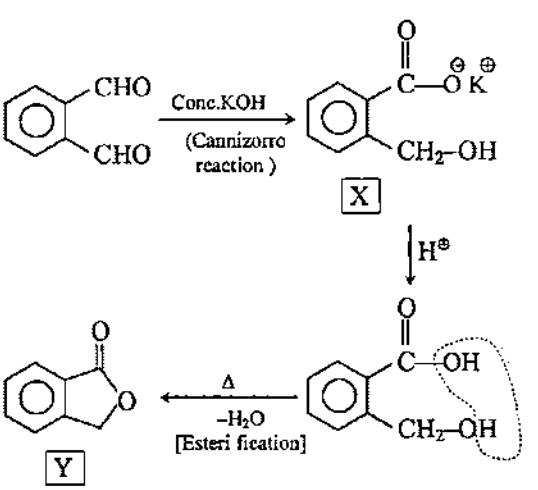


63.



[Styrene]

64.



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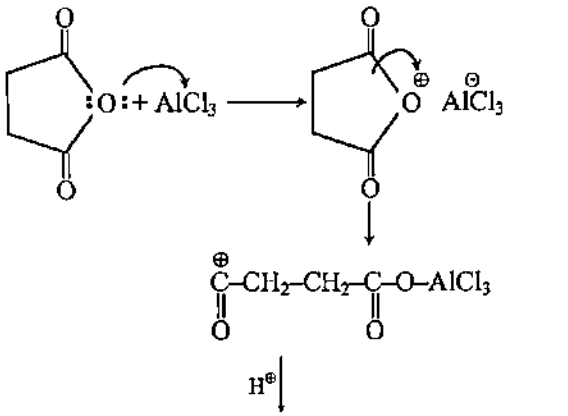
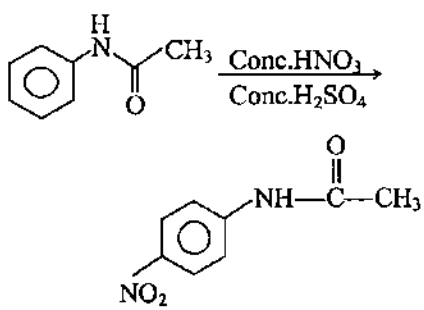
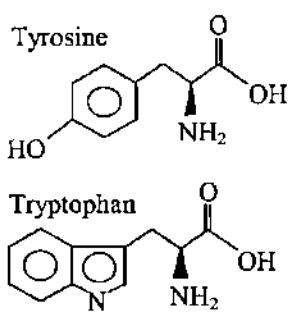
65.



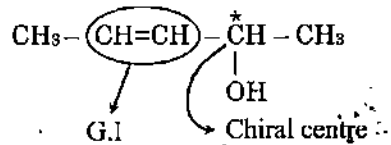
66.



67.



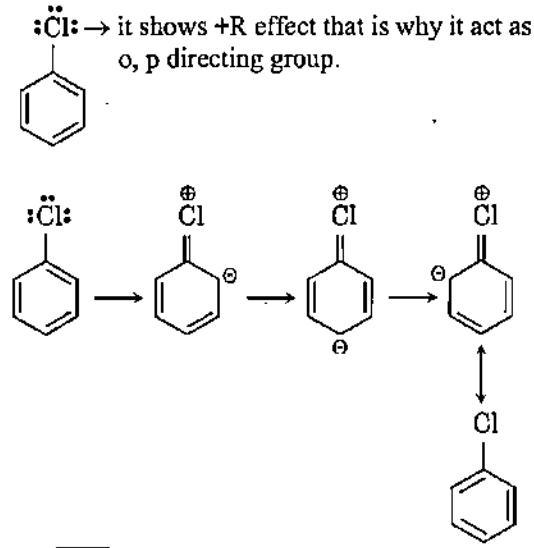
68.



No. of S. I $= 2^n = 2^2 = 4$

Cis	R
Trans	R
Cis	S
Trans	S

69. :Cl: → it shows +R effect that is why it act as o, p directing group.



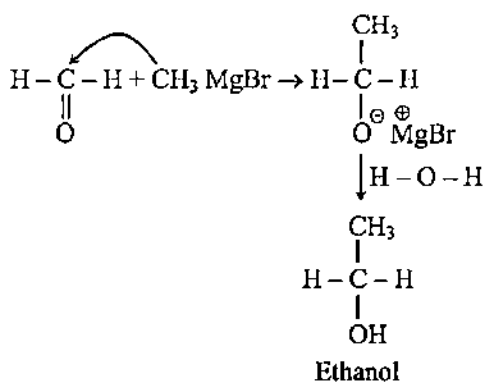
70. III. $4\pi\ e^-$

All C, sp^2 hybrid anti aromatic

V. $4\pi\ e^-$

All C, sp^2 hybrid anti aromatic

71.



Ethanol

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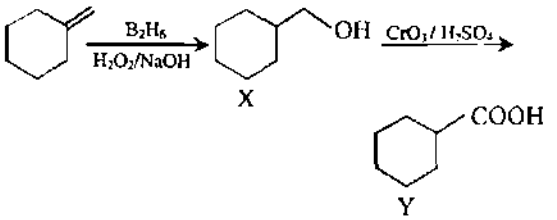
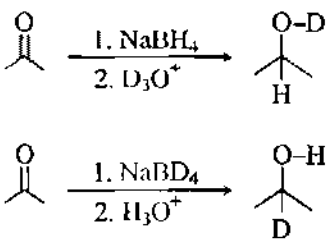
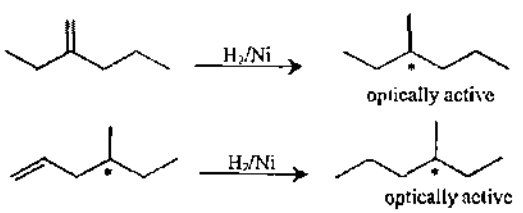
72.



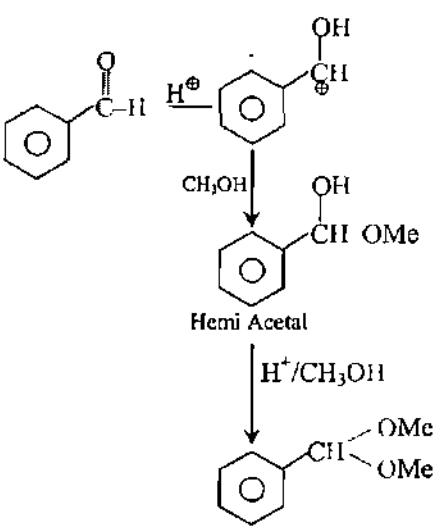
73.



74.



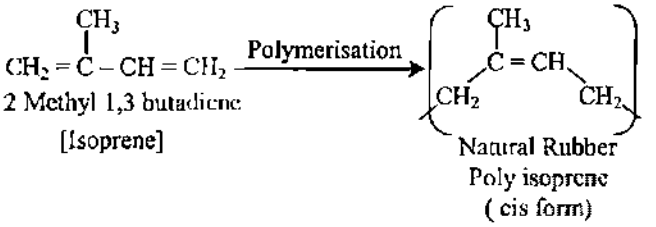
75. It is example of Nucleophilic addition in which Alcohol attack as Nucleophile and final product is Acetal



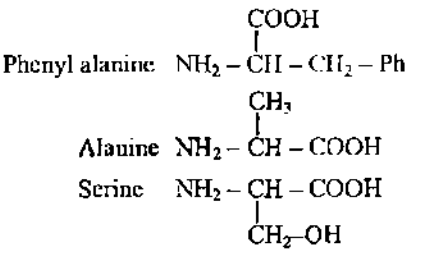
76.

It is Non-Benzenoid compound

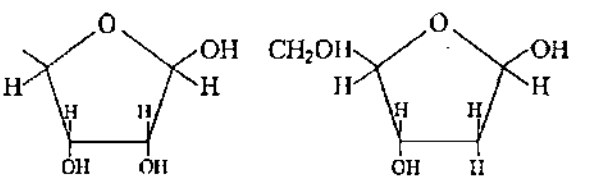
77.



78.

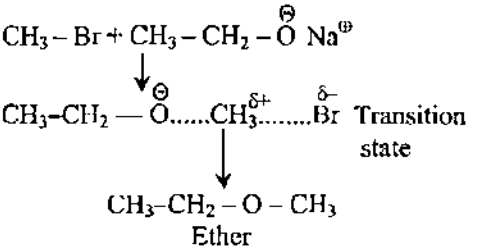


79.



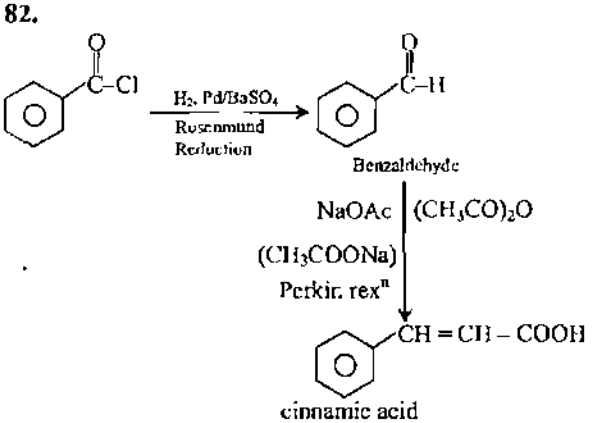
β-D Ribose (in RNA) β-2 Deoxy Ribose

80. It is Williamson synthesis Reaction (SN^2 rex^n)


$CH_3 - Br + CH_3 - CH_2 - \overset{\ominus}{O}\ Na^\oplus$
$CH_3 - CH_2 - \overset{\ominus}{O}.....\overset{\delta+}{CH_3}.....\overset{\delta-}{Br}$
Transition state
$CH_3 - CH_2 - O - CH_3$
Ether

81. Calcium Mainly present in Bones all other are required in lower amount

82.



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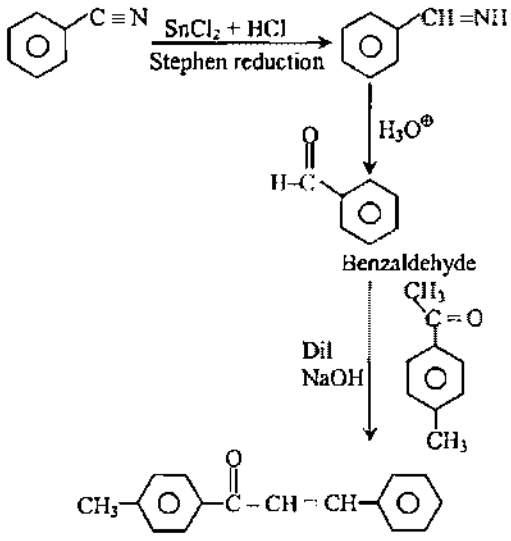
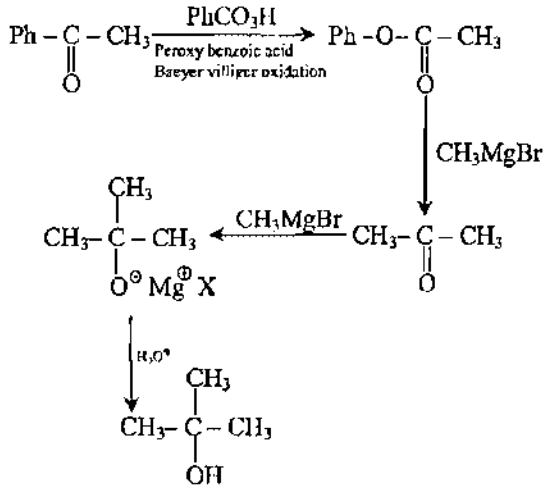
83.



84.



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Mathematics

[Topic- wise KVPY [STREAM-SX] Questions with Solutions]

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Chapter 1

Trigonometry

1. Let ABC be an equilateral triangle, let KLMN be a rectangle with K, L on BC, M on AC and N on AB. Suppose AN/NB = 2 and the area of triangle BKN is 6. The area of the triangle ABC is- **[2010]**
(A) 54
(B) 108
(C) 48
(D) not determinable with the above data

2. The number of roots of the equation $\cos^7\theta - \sin^4\theta = 1$ that lie in the interval $[0, 2\pi]$ is- **[2010]**
(A) 2 (B) 3
(C) 4 (D) 8

3. The product $(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \ldots (1 + \tan 45^\circ)$ equals- **[2010]**
(A) 2^{21} (B) 2^{22}
(C) 2^{23} (D) 2^{25}

4. In triangle ABC, we are given that $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Then the measure of the angle C is - **[2011]**
(A) 30° (B) 150°
(C) 60° (D) 75°

5. Let $A = \{\theta \in R \mid \cos^2(\sin\theta) + \sin^2(\cos\theta) = 1\}$ and $B = \{\theta \in R \mid \cos(\sin\theta)\sin(\cos\theta) = 0\}$. Then $A \cap B$ **[2011]**
(A) is the empty set
(B) has exactly one element
(C) has more than one but finitely many elements
(D) has infinitely many elements

6. The value of $\tan 81^\circ - \tan 63^\circ - \tan 27^\circ + \tan 9^\circ$ is **[2012]**
(A) 0 (B) 2 (C) 3 (D) 4

7. The sum of all $x \in [0, \pi]$ which satisfy the equation $\sin x + \frac{1}{2}\cos x = \sin^2\left(x + \frac{\pi}{4}\right)$ is **[2012]**
(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) π (D) 2π

8. Let ABC be an acute-angled triangle and let D be the midpoint of BC. If AB = AD, then $\frac{\tan B}{\tan C}$ equal- **[2013]**
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

9. The angles α, β, γ of a triangle satisfy the equations $2\sin\alpha + 3\cos\beta = 3\sqrt{2}$ and $3\sin\beta + 2\cos\alpha = 1$. Then angle γ equals- **[2013]**
(A) 150° (B) 120° (C) 60° (D) 30°

10. Let P be a closed polygon with 10 sides and 10 vertices (assume that the sides do not intersect except at the vertices). Let k be the number of interior angles of P that are greater than 180°. The maximum possible value of k is- **[2013]**
(A) 3 (B) 5 (C) 7 (D) 9

11. Let XY be the diameter of a semicircle with centre O. Let A be a variable point on the semicircle and B another point on the semicircle such that AB is parallel to XY. The value of $\angle BOY$ for which the inradius of triangle AOB is maximum, is **[2013]**
(A) $\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{5}$

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12. Suppose that the earth is a sphere of radius 6400 kilometers. The height from the earth's surface from where exactly a fourth of the earth's surface is visible, is [2013]

(A) 3200 km (B) $3200\sqrt{2}$ km

(C) $3200\sqrt{3}$ km (D) 6400 km

13. Let ABC be a triangle such that AB = BC. Let F be the midpoint of AB and X be a point on BC such that FX is perpendicular to AB. If BX = 3XC then the ratio BC/AC equals [2014]

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) $\sqrt{\frac{3}{2}}$ (D) 1

14. The number of solutions to the equation $\cos^4 x + \frac{1}{\cos^2 x} = \sin^4 x + \frac{1}{\sin^2 x}$ in the interval $[0, 2\pi]$ is [2014]

(A) 6 (B) 4 (C) 2 (D) 0

15. The number of real numbers λ for which the equality $\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$, holds for all real α which are not integral multiples of $\pi/2$ is - [2015]

(A) 1 (B) 2

(C) 3 (D) Infinite

16. Suppose ABCDEF is a hexagon such that AB = BC = CD = 1 and DE = EF = FA = 2. If the vertices A, B, C, D, E, F are concyclic, the radius of the circle passing through them is - [2015]

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{7}{3}}$ (C) $\sqrt{\frac{11}{5}}$ (D) $\sqrt{2}$

17. The sides of a right-angled triangle are integers. The length of one of the sides is 12. The largest possible radius of the incircle of such a triangle is [2015]

(A) 2 (B) 3 (C) 4 (D) 5

18. Let $C(\theta) = \sum_{n=0}^{\infty} \frac{\cos(n\theta)}{n!}$

Which of the following statements is FALSE ? [2015]

(A) $C(0).C(\pi) = 1$

(B) $C(0) + C(\pi) > 2$

(C) $C(0) > 0$ for all $\theta \in R$

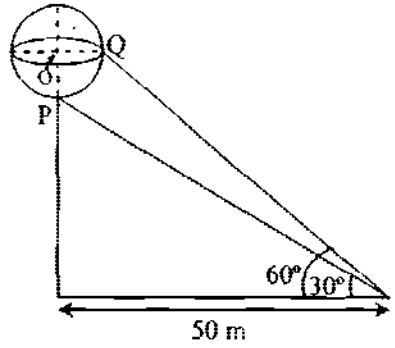
(D) $C'(\theta) \neq 0$ for all $\theta \in R$

19. $X = \{x \in R : \cos(\sin x) = \sin(\cos x)\}$. The number of elements in X is - [2016]

(A) 0 (B) 2

(C) 4 (D) not finite

20. A sphere with centre O sits atop a pole as shown in the figure. An observer on the ground is at a distance 50m from the foot of the pole. She notes that the angles of elevation from the observer to points P and Q on the sphere are 30° and 60°, respectively. Then, the radius of the sphere in meters is - [2016]



(A) $100\left(1-\frac{1}{\sqrt{3}}\right)$ (B) $\frac{50\sqrt{6}}{3}$

(C) $50\left(1-\frac{1}{\sqrt{3}}\right)$ (D) $\frac{100\sqrt{6}}{3}$

21. Let AB be a sector of a circle with centre O and radius d, $\angle AOB = \theta\left(<\frac{\pi}{2}\right)$, and D be a point on OA such that BD is perpendicular OA. Let E be the midpoint of BD and F be a point on the arc AB such that EF is parallel to OA. Then the ratio of length of the arc AF to the length of the arc AB is - [2016]

(A) $\frac{1}{2}$ (B) $\frac{0}{2}$

(C) $\frac{1}{2}\sin\theta$ (D) $\frac{\sin^{-1}\left(\frac{1}{2}\sin\theta\right)}{\theta}$

22. The number of real solutions of the equation $2\sin 3x + \sin 7x - 3 = 0$ which lie in the interval $[-2\pi, 2\pi]$ is [5 Nov, 2017]

(A) 1 (B) 2

(C) 3 (D) 4

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23. One of the solutions of the equation $8\sin^3\theta - 7\sin\theta + \sqrt{3}\cos\theta = 0$ lies in the interval **[5 Nov, 2017]**

(A) $(0, 10°]$ (B) $(10°, 20°]$

(C) $(20°, 30°]$ (D) $(30°, 40°]$

24. The integer part of the number $\sum_{k=0}^{44} \frac{1}{\cos k° \cos(k+1)°}$ is **[19 Nov, 2017]**

(A) 50 (B) 52

(C) 57 (D) 59

25. The number of solutions of the equation $\sin\theta + \cos\theta = \sin 2\theta$ in the interval $[-\pi, \pi]$ is **[19 Nov, 2017]**

(A) 1 (B) 2

(C) 3 (D) 4

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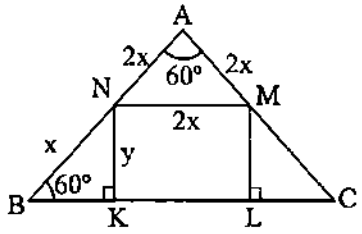
ANSWERS

1. (B)	2. (A)	3. (C)	4. (A)	5. (A)	6. (D)	7. (C)
8. (D)	9. (D)	10. (C)	11. (A)	12. (D)	13. (C)	14. (B)
15. (B)	16. (B)	17. (D)	18. (D)	19. (A)	20. (C)	21. (D)
22. (B)	23. (B)	24. (C)	25. (B)			

SOLUTIONS

1.



$y = \frac{\sqrt{3}x}{2}$

$z = \frac{x}{2}$

$\frac{1}{2}yz = 6 \Rightarrow x^2 = \frac{48}{\sqrt{3}}$

Area of ΔABC

$= 6 + 6 + 2xy + \frac{1}{2}(2x)(2x)\sin 60°$

$= 12 + 2x\frac{\sqrt{3}x}{2} + 2x^2\frac{\sqrt{3}}{2}$

$= 12 + 2\sqrt{3} \times \frac{48}{\sqrt{3}} = \boxed{108}$

2. $\cos^7\theta = 1 + \sin^4\theta \geq 1$ but $\cos^7\theta \leq 1$

so $\cos\theta = 1$; $\sin\theta = 0$

$\boxed{\theta = 0, 2\pi}$

3. $(1 + \tan\theta)(1 + \tan(45 - \theta))$

$= (1 + \tan\theta)\left(1 + \frac{1 - \tan\theta}{1 + \tan\theta}\right) = 2$

$(1 + \tan 1°)(1 + \tan 44°) = 2$ etc

so product $= 2^{22}(1 + \tan 45°) = 2^{23}$

4. Square & add both equations

$9 + 16 + 24\sin(A + B) = 37$

$\Rightarrow \sin(A + B) = \frac{1}{2}$

$\Rightarrow A + B = \frac{\pi}{6}$

$\Rightarrow C = \frac{5\pi}{6}$ (wrong)

$\Rightarrow A + B = \frac{5\pi}{6} \Rightarrow C = \frac{\pi}{6}$

because $C = \frac{5\pi}{6}$ does not follow equation

$3\sin A + 4\cos B = 6$

5. For $A \cap B$

$\cos(\sin\theta) = 1$ or -1 & $\sin(\cos\theta) = 0$

which is not possible

or $\cos(\sin\theta) = 0$ & $\sin(\cos\theta) = 1$ or -1

also not possible

so $A \cap B$ is an empty set

6. $\tan 81° - \tan 63° - \tan 27° + \tan 9°$

$= \tan(90° - 9°) - \tan(90° - 27°) - \tan 27° + \tan 9°$

$= \cot 9° - \cot 27° - \tan 27° + \tan 9°$

By solving we get

$= 4$

7. $\sin x + \frac{1}{2}\cos x = \sin^2\left(x + \frac{\pi}{4}\right)$

$\Rightarrow \sin x + \frac{1}{2}\cos x = \frac{1}{2}\left(1 - \cos\left(\frac{\pi}{2} + 2x\right)\right)$

$\Rightarrow \sin x + \frac{1}{2}\cos x = \frac{1}{2}(1 + \sin 2x)$

$\Rightarrow 2\sin x + \cos x = 1 + \sin x\cos x$

$\Rightarrow 2\sin x\cos x - 2\sin x + 1 - \cos x = 0$

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$\Rightarrow (1-\cos x)-2\sin x(1-\cos x)=0$

$\Rightarrow (1-\cos x)(1-2\sin x)=0$

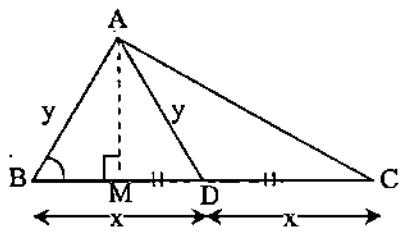
$\Rightarrow 1-\cos x=0$ or $1-2\sin x=0$

$\Rightarrow \cos x=1$ or $\sin x=\frac{1}{2}$

$\Rightarrow x=0,$ or $x=\frac{\pi}{6},\frac{5\pi}{6}$

$\text{Sum}=0+\frac{\pi}{6}+\frac{5\pi}{6}=\pi$

8.



Draw $AM \perp BD$

$BM=MD=\frac{x}{2}$

$\tan B=\frac{AM}{x/2}=\frac{2AM}{x}$

$\tan C=\frac{AM}{\frac{3x}{2}}=\frac{2AM}{3x}$

$\therefore \frac{\tan B}{\tan C}=3$

9. Square and add given equations, we get

$4+9+12\sin(\alpha+\beta)=19$

$\sin(\alpha+\beta)=\frac{1}{2}$

$\alpha+\beta=30^\circ, 150^\circ$

But $\alpha+\beta$ cannot be 30°

as if $\alpha \le 30^\circ$ or $\beta \le 30^\circ$

then eq.(i) is not satisfied

so $\alpha+\beta=150^\circ$

$\gamma=30^\circ$

10. Total sum of angles $=8\pi$

$\because [(n-2)\pi]$

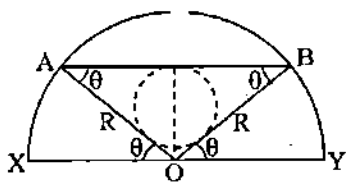
Maximum can be $7\pi^+$

$(7\pi^+)+x=8\pi$

sum of remaining three angles can be less than π

For example

11.



Let radius of semicircle is 'R'

Radius of incircle, $r=\frac{\Delta}{s}$

where $AB=2R\cos\theta$

$s=\frac{2R+2R\cos\theta}{2}$

$=\frac{2R\left(2\cos^2\frac{\theta}{2}\right)}{2}$

$=2R\cos^2\frac{\theta}{2}$

$\Delta=\frac{1}{2}R^2\sin 2\theta=R^2\sin\theta\cos\theta$

$r=\frac{R\sin\theta\cos\theta}{2\cos^2\frac{\theta}{2}}$

$=\frac{2R\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$

$=R\tan\frac{\theta}{2}\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$

Let $\tan\frac{\theta}{2}=x$

$r=R\left(\frac{x-x^3}{1+x^2}\right)$

$\frac{dr}{dx}=R\left(\frac{(1+x^2)(1-3x^2)-(x-x^3)(2x)}{(1+x^2)^2}\right)$

$=R\left(\frac{1-2x^2-3x^4-2x^2+2x^4}{(1+x^2)^2}\right)$

$=R\left(\frac{-x^4-4x^2+1}{(1+x^2)^2}\right)$

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$\frac{dr}{dx} = 0$ at $x^2 = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$

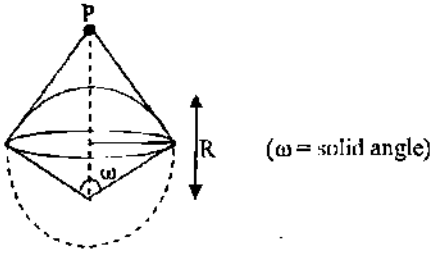
at $x^2 = 2 + \sqrt{5}$, r is maximum

$\therefore \cos\theta = \frac{1-x^2}{1+x^2} = \frac{\sqrt{5}-1}{2}$

12. For a cone solid angles.

Total solid angle full round = 4π

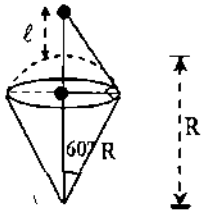
for $\pi r^2 \to \pi$ (Because one fourth of area is covered)



(ω = solid angle)

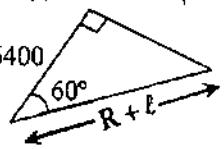
$\omega = \pi$ = solid angle

$\omega = 2\pi(1 - \cos\theta)$



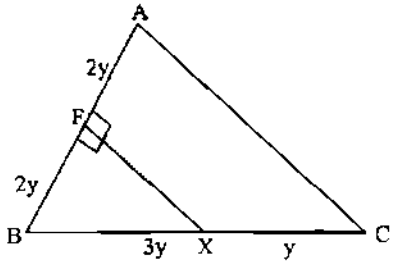
$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60°$



$\cos 60° = \frac{R}{R+\ell} \Rightarrow \ell = 6400$

13.



$\because \cos B = \frac{2}{3}$

Also $\cos B = \frac{16y^2 + 16y^2 - AC^2}{2.4y.4y}$

$\frac{2}{3} = \frac{32y^2 - AC^2}{32y^2}$

$64y^2 = 96y^2 - 3AC^2$

$3AC^2 = 32y^2$

$AC = \frac{4\sqrt{2}}{\sqrt{3}}y$

$\therefore \frac{BC}{AC} = \frac{4y}{\left(\frac{4\sqrt{2}y}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{\sqrt{2}}$

14. $\cos^4 x - \sin^4 x = \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}$

$(\cos^2 x \quad \sin^2 x) = \frac{(\cos^2 x - \sin^2 x)}{\sin^2 x \cos^2 x}$

$\cos 2x = \frac{4\cos 2x}{\sin^2 2x}$

$\cos 2x\,(1 - 4\,\mathrm{cosec}^2 2x) = 0$

$\cos 2x = 0$

$2x = 2n\pi + \frac{\pi}{2}$

$x = n\pi \pm \frac{\pi}{4}$

At $n = 0, x = \frac{\pi}{4}$

$n = 1;\ x = \frac{5\pi}{4}, \frac{3\pi}{4}$

$n = 2, x = \frac{7\pi}{4}$

15. $\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$

By observation

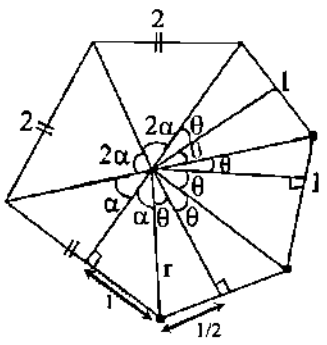
$\sin(\lambda\alpha)\cos\alpha - \cos(\lambda\alpha)\sin\alpha$

$= (\lambda - 1)\sin\alpha\cos\alpha$

$\sin(\lambda - 1)\alpha = (\lambda - 1)\sin\alpha\cos\alpha$

Clearly $\lambda = 1, \lambda = 3$ is solution

16.



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From the figure:

$\sin\theta = \frac{1}{2r}$ & $\sin\alpha = \frac{1}{r}$

$3 \times (2\theta) + (2\alpha) \times 3 = 360°$

$\theta + \alpha = 60°$

Now, $\cos(\theta + \alpha) = \frac{1}{2}$

$\Rightarrow \cos\theta.\cos\alpha - \sin\theta.\sin\alpha = \frac{1}{2}$

$\Rightarrow \sqrt{1-\frac{1}{4r^2}}\sqrt{1-\frac{1}{r^2}} - \frac{1}{2r}\frac{1}{r} = \frac{1}{2}$

$\Rightarrow \sqrt{4r^2-1}\ \sqrt{r^2-1} - 1 = r^2$

$\Rightarrow (4r^2-1)(r^2-1) = (r^2+1)^2$

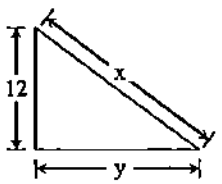
$\Rightarrow 4r^4 - 5r^2 + 1 = r^4 + 2r^2 + 1$

$\Rightarrow 3r^4 = 7r^2$

$\Rightarrow r^2 = \frac{7}{3}$

$\Rightarrow r = \sqrt{\frac{7}{3}}$

17.



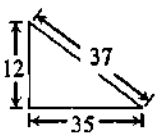
Clearly

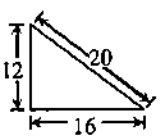
$x^2 - y^2 = 144$

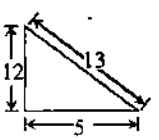
$(x-y)(x+y) = 144,\quad x, y \in I$

So factorsize 144 into two even factors

$x+y=72$	$x+y=36$	$x+y=18$
$x-y=2$	$x-y=9$	$x-y=8$
$x=37$	$x=20$	$x=13$
$y=35$	$y=16$	$y=5$







$r = \Delta/s$ $\qquad$ $r = \Delta/s$ $\qquad$ $r = \Delta/s$

$r = \frac{1/2 \times 12 \times 35}{\left(\frac{12+35+37}{2}\right)}$ $\qquad$ $r = \frac{1/2 \times 12 \times 16}{\left(\frac{16+20+12}{2}\right)}$ $\qquad$ $r = 2$

$r = \frac{12 \times 35}{84}$ $\qquad$ $r = \frac{12 \times 16}{48}$

$r = 5$ $\qquad$ $r = 4$

18. $C(\theta) = \sum_{n=0}^{\infty} \frac{\cos n\theta}{n!}$

$= \lim_{n\to\infty}\left(1 + \frac{\cos\theta}{1!} + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots + \frac{\cos n\theta}{n!}\right)$

$C(0) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$ up to ∞ term

$= e$

$C(\pi) = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots$ up to ∞ term

$= e^{-1}$

Clearly $C(0) \cdot C(\pi) = 1$

$C(0) \cdot C(\pi) = e + \frac{1}{e} > 2$

$C'(\theta) = -\frac{\sin\theta}{1!} - 2\frac{\sin 2\theta}{2!} - \frac{3\sin 3\theta}{3!} + \ldots$

...up to ∞ term

And that value is equal to zero at $\theta = 0$

19. $\cos(\sin x) = \sin(\cos x)$

$\Rightarrow \sin\left(\frac{\pi}{2} \pm \sin x\right) = \sin(\cos x)$

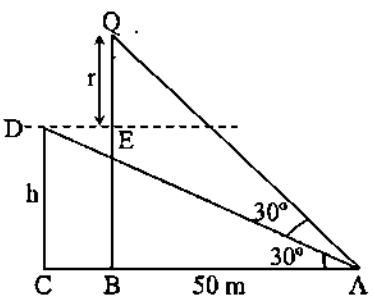
$\Rightarrow \cos x = n\pi + (-1)^n\left(\frac{\pi}{2} + \sin x\right), n \in I$

$\Rightarrow \cos x \pm \sin x = n\pi + (-1)^n\frac{\pi}{2}, n \in I$

As LHS $\in [-\sqrt{2}, \sqrt{2}]$, and it does not satisfies RHS

So No solution possible

20.



$DE = BC = r$

$\tan 30° = \frac{h}{50}$

$\boxed{h = \frac{50}{\sqrt{3}}}$

$\tan 60° = \frac{h+r}{50-r}$

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$\Rightarrow \sqrt{3}(50-r) = h + r$

$\Rightarrow \sqrt{3}(50-r) = \frac{50}{\sqrt{3}} + r$

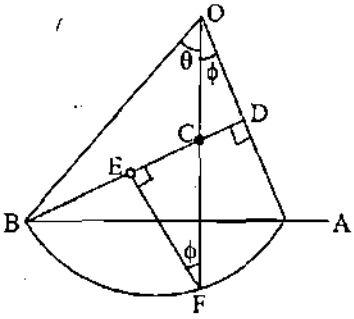
$\Rightarrow 3(50-r) = 50 + \sqrt{3}\, r$

$\Rightarrow 100 = (3+\sqrt{3})r$

$\Rightarrow r = \frac{100}{3+\sqrt{3}}$

$\Rightarrow r = \frac{100(3-\sqrt{3})}{6} = 50\left(1-\frac{1}{\sqrt{3}}\right)$

21.



$CF = r(1-\cos\theta \sec\phi)$

$EC = r(\sin\phi - \cos\theta \tan\phi)$

$CD = r\cos\theta \tan\phi$

$EC + CD = ED$

$r\sin\phi = \frac{r\sin\theta}{2}$

$\phi = \sin^{-1}\left(\frac{\sin\theta}{2}\right)$

22. only possible when $\sin 3x = 1$ & $\sin 7x = 1$

$\sin 3x = 1$

$\sin 3x = \sin(4n+1)\frac{\pi}{2}$, $n \in I$

$3x = (4n+1)\frac{\pi}{2} \Rightarrow x = (4n+1)\frac{\pi}{6}$

$\sin 7x = \sin(4m+1)\frac{\pi}{2}$, $m \in I$

$x = (4m+1)\frac{\pi}{14}$

for common solution

$(4n+1)\frac{\pi}{6} = (4m+1)\frac{\pi}{14}$

Solving these $1 = 3m - 7n$

First solution is $m = 5, n = 2$

Second solution is $m = 12, n = 5$

So two solutions are possible

23. $6\sin\theta - 2\sin 3\theta - 7\sin\theta + \sqrt{3}\cos\theta = 0$

$\sqrt{3}\cos\theta - \sin\theta = 2\sin 3\theta$

It can be written as

$2(\sin(60^\circ - \theta)) = 2\sin 3\theta$

$\sin(60^\circ - \theta) = \sin 3\theta$

$60^\circ = 4\theta$

$\Rightarrow \theta = 15^\circ$ is one of the value

24. $\frac{1}{\cos 0^\circ \cos 1^\circ} + \frac{1}{\cos 1^\circ \cos 2^\circ} + \frac{1}{\cos 2^\circ \cos 3^\circ} + \ldots$

$\ldots \frac{1}{\cos 44^\circ \cos 45^\circ}$

multiply & divided by $\sin 1^\circ$

$\frac{1}{\sin 1^\circ}\left[\frac{\sin 1^\circ}{\cos 0^\circ \cos 1^\circ} + \frac{\sin 1^\circ}{\cos 1^\circ \cos 2^\circ} + \ldots \frac{\sin 1^\circ}{\cos 44^\circ \cos 45^\circ}\right]$

$\frac{1}{\sin 1^\circ}\left[\frac{\sin(1^\circ - 0^\circ)}{\cos 0^\circ \cos 1^\circ} + \frac{\sin(2-1)^\circ}{\cos 1^\circ \cos 2^\circ} + \ldots \frac{\sin(45^\circ - 44^\circ)}{\cos 44^\circ \cos 45^\circ}\right]$

$\frac{1}{\sin 1^\circ}[\tan 1^\circ - \tan 0^\circ + \tan 2^\circ - \tan 1^\circ + \ldots \tan 45^\circ - \tan 44^\circ]$

$= \frac{1}{\sin 1^\circ}[\tan 45^\circ]$

$= \frac{1}{0.0174524} = 57.2987$

Integral part = 57

25. Squaring $\sin^2\theta + \cos^2\theta + 2\sin 2\theta = \sin^2 2\theta$

$\Rightarrow \sin^2 2\theta - \sin 2\theta - 1 = 0$

$\sin 2\theta = -\left(\frac{\sqrt{5}-1}{2}\right)$, $\sin 2\theta = \frac{\sqrt{5}+1}{2}$ (reject)

two solution exist between

$\frac{-\pi}{2}$ to $\frac{-\pi}{4}$ & $\frac{\pi}{4}$ to 0 (2 solution)

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Chapter 2

Algebra

1. Let A denote the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, where $i^2 = -1$, and let I denote the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Then $I + A + A^2 + \ldots + A^{2010}$ is- **[2010]**

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

2. Suppose the sides of a triangle form a geometric progression with common ratio r. Then r lies in the interval- **[2010]**

(A) $\left(0, \frac{-1+\sqrt{5}}{2}\right]$

(B) $\left[\frac{1+\sqrt{5}}{2}, \frac{2+\sqrt{5}}{2}\right]$

(C) $\left[\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$

(D) $\left(\frac{2+\sqrt{5}}{2}, \infty\right)$

3. The number of rectangles that can be obtained by joining four of the twelve vertices of a 12-sided regular polygon is- **[2010]**

(A) 66 (B) 30

(C) 24 (D) 15

4. Let 1, ω and ω^2 be the cube roots of unity. The least possible degree of a polynomial, with real coefficients, having $2\omega^2$, $3 + 4\omega$, $3 + 4\omega^2$ and $5 - \omega - \omega^2$ as roots is- **[2010]**

(A) 4 (B) 5

(C) 6 (D) 8

5. Three vertices are chosen randomly from the seven vertices of a regular 7-sided polygon. The probability that they form the vertices of an isosceles triangle is- **[2010]**

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$ (C) $\frac{3}{7}$ (D) $\frac{3}{5}$

6. How many six-digit numbers are there in which no digit is repeated, even digits appear at even places, odd digits appear at odd places and the number is divisible by 4 ? **[2010]**

(A) 3600 (B) 2700 (C) 2160 (D) 1440

7. The number of natural numbers n in the interval [1005, 2010] for which the polynomial $1 + x + x^2 + x^3 + \ldots + x^{n-1}$ divides the polynomial $1 + x^2 + x^3 + x^4 + \ldots + x^{2010}$ is- **[2010]**

(A) 0 (B) 100 (C) 503 (D) 1006

8. Let $a_0 = 0$ and $a_n = 3a_{n-1} + 1$ for $n \geq 1$. Then the remainder obtained dividing a_{2010} by 11 is- **[2010]**

(A) 0 (B) 7 (C) 3 (D) 4

9. Arrange the expansion of $\left(x^{1/2} + \frac{1}{2x^{1/4}}\right)^n$ in decreasing powers of x. Suppose the coefficient of the first three terms form an arithmetic progression. Then the number of terms in the expansion having integer powers of x is- **[2010]**

(A) 1 (B) 2

(C) 3 (D) more than 3

10. Let r be a real number and $n \in N$ be such that the polynomial $2x^2 + 2x + 1$ divides the polynomial $(x + 1)^n - r$. Then (n, r) can be- **[2010]**

(A) $(4000, 4^{1000})$ (B) $\left(4000, \frac{1}{4^{1000}}\right)$

(C) $\left(4^{1000}, \frac{1}{4^{1000}}\right)$ (D) $\left(4000, \frac{1}{4000}\right)$

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11. Suppose m, n are positive integers such that $6^m + 2^{m+n}\ 3^w + 2^n = 332$. The value of the expression $m^2 + mn + n^2$ is- **[2010]**
(A) 7 (B) 13 (C) 19 (D) 21

12. Suppose $\log_a b + \log_b a = c$. The smallest possible integer value of c for all $a, b > 1$ is - **[2011]**
(A) 4 (B) 3 (C) 2 (D) 1

13. Suppose n is a natural number such that $|i + 2i^2 + 3i^3 + ... + ni^n| = 18\sqrt{2}$, where i is the square root of -1. Then n is - **[2011]**
(A) 9 (B) 18 (C) 36 (D) 72

14. Let P be an $m \times m$ matrix such that $P^2 = P$. Then $(1 + P)^n$ equals - **[2011]**
(A) $I + P$ (B) $I + nP$
(C) $I + 2^nP$ (D) $I + (2^n - 1)P$

15. Consider the cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b, c are real numbers. Which of the following statements is correct ? **[2011]**
(A) If $a^2 - 2b < 0$, then the equation has one real and two imaginary roots
(B) If $a^2 - 2b \geq 0$, then the equation has all real roots
(C) If $a^2 - 2b > 0$, then the equation has all real and distinct roots
(D) If $4a^3 - 27b^2 > 0$, then the equation has real and distinct roots

16. Two players play the following game : A writes 3, 5, 6 on three different cards ; B writes 8, 9, 10 on three different cards. Both draw randomly two cards from their collections. Then A computes the product of two numbers he/she has drawn, and B computes the sum of two numbers he/she has drawn. The player getting the larger number wins. What is the probability that A wins? **[2011]**
(A) $\frac{1}{3}$ (B) $\frac{5}{9}$ (C) $\frac{4}{9}$ (D) $\frac{1}{9}$

17. The sum of $(1^2 - 1 + 1)(1!) + (2^2 - 2 + 1)(2!) + ... + (n^2 - n + 1)(n!)$ is - **[2011]**
(A) $(n + 2)!$ (B) $(n - 1)((n + 1)!) + 1$
(C) $(n + 2)! - 1$ (D) $n((n + 1)!) - 1$

18. Let X be a nonempty set and let $P(X)$ denote the collection of all subsets of X. Define

$$f: X \times P(X) \to R \text{ by } f(x, A) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

Then $f(x, A \cup B)$ equals - **[2011]**
(A) $f(x, A) + f(x, B)$
(B) $f(x, A) + f(x, B) - 1$
(C) $f(x, A) + f(x, B) - f(x, A) f(x, B)$
(D) $f(x, A) + |f(x, A) - f(x, B)|$

19. Let A and B be any two n × n matrices such that the following conditions hold : AB = BA and there exist positive integers k and ℓ such that $A^k = I$ (the identity matrix) and $B^\ell = 0$ (the zero matrix). Then- **[2011]**
(A) A + B = I
(B) det (AB) = 0
(C) det (A + B) ≠ 0
(D) $(A + B)^m = 0$ for some integer m

20. The minimum value of n for which $\frac{2^2 + 4^2 + 6^2 + ... + (2n)^2}{1^2 + 3^2 + 5^2 + ... + (2n - 1)^2} < 1.01$ **[2011]**
(A) is 101 (B) is 121
(C) is 151 (D) does not exist

21. The arithmetic mean and the geometric mean of two distinct 2-digit numbers x and y are two integers one of which can be obtained by reversing the digits of the other (in base 10 representation). Then x + y equals- **[2011]**
(A) 82 (B) 116 (C) 130 (D) 148

22. Three children, each accompanied by a guardian, seek admission in a school. The principal want to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this be done **[2012]**
(A) 60 (B) 90 (C) 120 (D) 180

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23. In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ has **[2012]**
(A) No solution
(B) Exactly two distinct solutions
(C) Exactly four distinct solutions
(D) Infinitely may solutions

24. The maximum value M of $3^x+5^x-9^x+15^x-25^x$, as x varies over reals, satisfies **[2012]**
(A) $3 < M < 5$ (B) $0 < M < 2$
(C) $9 < M < 25$ (D) $5 < M < 9$

25. Let a, b, c, d be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a-c)^2 + b - d$ is **[2012]**
(A) 0 (B) 5 (C) 30 (D) 36

26. Let n be a natural number and let 'a' be a real number. The number of zeros of $x^{2n+1} - (2n+1)x + a = 0$ in the interval $[-1, 1]$ is **[2012]**
(A) 2 if $a > 0$
(B) 2 if $a < 0$
(C) At most one for every value of a
(D) At least three for every value of a

27. Let $S_n = \sum_{k=1}^{n} k$ denote the sum of the first n positive integers. The numbers $S_1, S_2, S_3, \ldots S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is **[2012]**
(A) 1/2 (B) 49/100
(C) 49/99 (D) 48/99

28. A purse contains 4 copper coins and 3 silver coins. A second purse contains 6 copper coins and 4 silver coins. A purse is chosen randomly and a coin is taken out of it. What is the probability that it is a copper coin **[2012]**
(A) 41/70 (B) 31/70
(C) 27/70 (D) 1/3

29. The number of ordered pairs (m, n), where $m, n \in \{1, 2, 3, \ldots\ldots, 50\}$, such that $6^m + 9^n$ is a multiple of 5 is **[2012]**
(A) 1250 (B) 2500 (C) 625 (D) 500

30. Suppose $a_1, a_2, a_3, \ldots\ldots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers ? **[2012]**
(A) 0 (B) 1509 (C) 3018 (D) 6036

31. Let $S = \{1, 2, 3, .., n\}$ and $A = \{(a, b) \mid 1 \le a, b \le n\} = S \times S$. A subset B of A is said to be a good subset if $(x, x) \in B$ for every $x \in S$. Then the number of good subsets of A is **[2012]**
(A) 1 (B) 2^n (C) $2^{n(n-1)}$ (D) 2^{n^2}

32. Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has **[2012]**
(A) Two distinct positive real roots
(B) Two equal roots
(C) One positive and one negative root
(D) No real roots

33. The coefficient of x^{2012} in $\frac{1+x}{(1+x^2)(1-x)}$ is **[2012]**
(A) 2010 (B) 2011 (C) 2012 (D) 2013

34. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let P(K) be the probability of scoring at least K points. The largest value of K such that $P(K) > \frac{1}{2}$ is **[2012]**
(A) 14 (B) 15 (C) 16 (D) 17

35. The sum of non-real roots of the polynomial equation $x^3 + 3x^2 + 3x + 3 = 0$. **[2013]**
(A) equals 0
(B) lies between 0 and 1
(C) lies between −1 and 0
(D) has absolute value bigger than 1

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36. Let n be a positive integer such that $\log_2\log_2\log_2\log_2\log_2(n) < 0 < \log_2\log_2\log_2\log_2(n)$. Let ℓ be the number of digits in the binary expansion of n. Then the minimum and the maximum possible values of ℓ are- **[2013]**

(A) 5 and 16 (B) 5 and 17
(C) 4 and 16 (D) 4 and 17

37. Let ω be a cube root of unity not equal to 1. Then the maximum possible value of $|a + b\omega + c\omega^2|$ where $a, b, c \in \{+1, -1\}$ is- **[2013]**

(A) 0 (B) 2
(C) $\sqrt{3}$ (D) $1 + \sqrt{3}$

38. A box contains coupons labeled 1, 2, 3, ..., n. A coupon is picked at random and the number x is noted. The coupon is put back into the box and a new coupon is picked at random. The new number is y. Then the probability that one of the numbers x, y divides the other is (in the options below [r] denotes the largest integer less than or equal to r) **[2013]**

(A) $\frac{1}{2}$ (B) $\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

(C) $-\frac{1}{n}+\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$ (D) $-\frac{1}{n}+\frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

39. Let $n \geq 3$. A list of numbers $0 < x_1 < x_2 < \ldots < x_n$ has mean μ and standard deviation σ. A new list of numbers is made as follows : $y_1 = 0$, $y_2 = x_2$, ..., $y_{n-1} = x_{n-1}$, $y_n = x_1 + x_n$. The mean and the standard deviation of the new list are $\hat{\mu}$ and $\hat{\sigma}$. Which of the following is necessarily true? **[2013]**

(A) $\mu = \hat{\mu}$, $\sigma \leq \hat{\sigma}$
(B) $\mu = \hat{\mu}$, $\sigma \geq \hat{\sigma}$
(C) $\sigma = \hat{\sigma}$
(D) μ may or may not be equal to $\hat{\mu}$

40. The number of integers n with $100 \leq n \leq 999$ and containing at most two distinct digits is- **[2013]**

(A) 252 (B) 280 (C) 324 (D) 360

41. For an integer n let $S_n = \{n + 1, n + 2, ..., n + 18\}$. Which of the following is true for all $n \geq 10$? **[2013]**

(A) S_n has a multiple of 19
(B) S_n has a prime
(C) S_n has at least four multiples of 5
(D) S_n has at most six primes

42. Let a, b, c, d be real numbers such that $\sum_{k=1}^{n}(ak^3 + bk^2 + ck + d) = n^4$, for every natural number n. Then $|a| + |b| + |c| + |d|$ is equal to **[2013]**

(A) 15 (B) 16 (C) 31 (D) 32

43. Let $f(x) = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$. The number of real roots of $f(x) = 0$ is **[2013]**

(A) 0 (B) 1 (C) 2 (D) 4

44. A box contains coupons labelled 1, 2, ..., 100. Five coupons are picked at random one after another without replacement. Let the numbers on the coupons be $x_1, x_2, ..., x_5$. What is the probability that $x_1 > x_2 > x_3$ and $x_3 < x_4 < x_5$? **[2013]**

(A) 1/120 (B) 1/60
(C) 1/20 (D) 1/10

45. In a tournament with five teams, each team plays against every other team exactly once. Each game is won by one of the playing teams and the winning team scores one point, while the losing team scores zero. Which of the following is NOT necessarily true? **[2013]**

(A) There are at least two teams which have at most two points each
(B) There are at least two teams which have at least two points each
(C) There are at most three teams which have at least three points each
(D) There are at most four teams which have at most two points each

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46. Let C_0 be a circle of radius 1. For $n \geq 1$, let C_n be a circle whose area equals the area of a square inscribed in C_{n-1}. Then $\sum_{i=0}^{\infty}$ Area (C_i) equals **[2014]**

(A) π^2 (B) $\frac{\pi-2}{\pi^2}$ (C) $\frac{1}{\pi^2}$ (D) $\frac{\pi^2}{\pi-2}$

47. For a real number r we denote by [r] the largest integer less than or equal to r. If x, y are real numbers with x, $y \geq 1$ then which of the following statements is always true? **[2014]**

(A) $[x+y] \leq [x] + [y]$ (B) $[xy] \leq [x]\,[y]$

(C) $[2^x] \leq 2^{[x]}$ (D) $\left[\frac{x}{y}\right] \leq \frac{[x]}{[y]}$

48. For each positive integer n, let $A_n = \max\left\{\binom{n}{r} \mid 0 \leq r \leq n\right\}$. Then the number of elements n in {1,2,...,20} for which $1.9 \leq \frac{A_n}{A_{n-1}} \leq 2$ is **[2014]**

(A) 9 (B) 10 (C) 11 (D) 12

49. Two persons A and B throw a (fair) die (six-faced cube with faces numbered from 1 to 6) alternately, starting with A. The first person to get an outcome different from the previous one by the opponent wins. The probability that B wins is **[2014]**

(A) $\frac{5}{6}$ (B) $\frac{6}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{9}$

50. Let $n \geq 3$. A list of numbers $x_1, x_2, \ldots, x_n$ has mean μ and standard deviation σ. A new list of numbers $y_1, y_2, \ldots, y_n$ is made as follows: $y_1 = \frac{x_1 + x_2}{2}$, $y_2 = \frac{x_1 + x_2}{2}$ and $y_j = x_j$ for $j = 3, 4, \ldots, n$. The mean and the standard deviation of the new list are $\hat{\mu}$ and $\hat{\sigma}$. Then which of the following is necessarily true? **[2014]**

(A) $\mu = \hat{\mu}$ and $\sigma \leq \hat{\sigma}$ (B) $\mu = \hat{\mu}$ and $\sigma \geq \hat{\sigma}$

(C) $\sigma = \hat{\sigma}$ (D) $\mu \neq \hat{\mu}$

51. Let $S = \{(a, b) : a, b \in Z, 0 \leq a, b \leq 18\}$. The number of elements (x, y) in S such that $3x + 4y + 5$ is divisible by 19 is **[2014]**

(A) 38 (B) 19 (C) 18 (D) 1

52. For a real number r let [r] denote the largest integer less than or equal to r. Let $a > 1$ be a real number which is not an integer and let k be the smallest positive integer such that $[a^k] > [a]^k$. Then which of the following statements is always true? **[2014]**

(A) $k \leq 2([a]+1)^2$ (B) $k \leq ([a]+1)^4$

(C) $k \leq 2^{[a]+1}$ (D) $k \leq \frac{1}{a-[a]} + 1$

53. Let X be a set of 5 elements. The number d of ordered pairs (A, B) of subsets of X such that $A \neq \phi$, $B \neq \phi$, $A \cap B = \phi$ satisfies **[2014]**

(A) $50 \leq d \leq 100$

(B) $101 \leq d \leq 150$

(C) $151 \leq d \leq 200$

(D) $201 \leq d$

54. Let $n \geq 3$ be an integer. For a permutation $\sigma = (a_1, a_2, \ldots, a_n)$ of $(1, 2, \ldots, n)$ we let $f_\sigma(x) = a_n x^{n-1} + a_{n-1}x^{n-2} + \ldots + a_2 x + a_1$. Let S_σ be the sum of the roots of $f_\sigma(x) = 0$ and let S denote the sum over all permutations σ of $(1, 2, \ldots, n)$ of the numbers S_σ. Then - **[2014]**

(A) $S < n!$ (B) $-n! < S < 0$

(C) $0 < S < n!$ (D) $n! < S$

55. If n is a positive integer and $\omega \neq 1$ is a cube root of unity, the number of possible values of $\left|\sum_{k=0}^{n}\binom{n}{k}\omega^k\right|$ **[2014]**

(A) 2 (B) 3 (C) 4 (D) 6

56. Suppose a parabola $y = ax^2 + bx + c$ has two x intercepts, one positive and one negative, and its vertex is (2, –2). Then which of the following is true? **[2014]**

(A) $ab > 0$ (B) $bc > 0$

(C) $ca > 0$ (D) $a + b + c > 0$

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57. The number of integers n for which $3x^3 - 25x + n = 0$ has three real roots is - **[2014]**
(A) 1 (B) 25 (C) 55 (D) infinite

58. Choose a number n uniformly at random from the set {1, 2, ..., 100}. Choose one of the first seven days of the year 2014 at random and consider n consecutive days starting from the chosen day. What is the probability that among the chosen n days, the number of Sundays is different from the number of Mondays? **[2014]**
(A) $\frac{1}{2}$ (B) $\frac{2}{7}$ (C) $\frac{12}{49}$ (D) $\frac{43}{175}$

59. If z is a complex number satisfying $|z^3 + z^{-3}| \le 2$, then the maximum possible value of $|z + z^{-1}|$ is - **[2015]**
(A) 2 (B) $\sqrt[3]{2}$ (C) $2\sqrt{2}$ (D) 1

60. The largest perfect square that divides $2014^3 - 2013^3 + 2012^3 - 2011^3 + ... + 2^3 - 1^3$ is- **[2015]**
(A) 1^2 (B) 2^2
(C) 1007^2 (D) 2014^2

61. Let p(x) be a polynomial such that $p(x) - p'(x) = x^n$, where n is a positive integer. Then p(0) equals - **[2015]**
(A) n! (B) (n – 1)!
(C) $\frac{1}{n!}$ (D) $\frac{1}{(n-1)!}$

62. An urn contains marbles of four colours : red, white, blue and green. When four marbles are drawn without replacement, the following events are equally likely : **[2015]**
(1) the selection of four red marbles
(2) the selection of one white and three red marbles
(3) the selection of one white, one blue and two red marbles
(4) the selection of one marble of each colour
The smallest total number of marbles satisfying the given condition is
(A) 19 (B) 21 (C) 46 (D) 69

63. There are 6 boxes labelled $B_1, B_2, ... , B_6$. In each trial, two fair dice D_1, D_2 are thrown. If D_1 shows j and D_2 shows k, then j balls are put into the box B_k. After n trials, what is the probability that B_1 contains at most one ball ? **[2015]**
(A) $\left(\frac{5^{n-1}}{6^{n-1}}\right) + \left(\frac{5^n}{6^n}\right)\left(\frac{1}{6}\right)$
(B) $\left(\frac{5^n}{6^n}\right) + \left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$
(C) $\left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$
(D) $\left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$

64. If $\log_{(3x-1)}(x-2) = \log_{(9x^2-6x+1)}(2x^2 - 10x - 2)$, then x equals - **[2015]**
(A) $9 - \sqrt{15}$ (B) $3 + \sqrt{15}$
(C) $2 + \sqrt{5}$ (D) $6 - \sqrt{5}$

65. Suppose a, b, c are positive integers such that $2^a + 4^b + 8^c = 328$. Then $\frac{a+2b+3c}{abc}$ is equal to- **[2015]**
(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{17}{24}$ (D) $\frac{5}{6}$

66. Let $x = (\sqrt{50} + 7)^{1/3} - (\sqrt{50} - 7)^{1/3}$. Then - **[2015]**
(A) x = 2
(B) x = 3
(C) x is a rational number, but not an integer
(D) x is an irrational number

67. Let $(1 + x + x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$ $... ...+ a_{4028}x^{4028}$, and let **[2015]**
$A = a_0 - a_3 + a_6 - .... + a_{4026}$,
$B = a_1 - a_4 + a_7 - ..... - a_{4027}$,
$C = a_2 - a_5 + a_8 - .... + a_{4028}$,
Then -
(A) |A| = |B| > |C| (B) |A| = |B| < |C|
(C) |A| = |C| > |B| (D) |A| = |C| < |B|

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68. Let $X_n = \{1, 2, 3, ..., n\}$ and let a subset A of X_n be chosen so that every pair of elements of A differ by at least 3. (For example, if n = 5, A can be $\varnothing$, {2} or {1,5} among others). When n = 10, let the probability that $1 \in A$ be p and let the probability that $2 \in A$ be q. Then - **[2015]**

(A) $p > q$ and $p - q = \frac{1}{6}$

(B) $p < q$ and $q - p = \frac{1}{6}$

(C) $p > q$ and $p - q = \frac{1}{10}$

(D) $p < q$ and $q - p = \frac{1}{10}$

69. The remainder when the determinant $\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$ is divided by 5 is - **[2015]**

(A) 1 (B) 2 (C) 3 (D) 4

70. The number of triples (x, y, z) of real numbers satisfying the equation $x^4 + y^4 + z^4 + 1 = 4xyz$ is **[2016]**

(A) 0 (B) 4

(C) 8 (D) more than 8

71. For any real number r, let $A_r = \{e^{i\pi rn} : n \text{ is a natural number}\}$ be a set of complex numbers. Then - **[2016]**

(A) A_1, $A_{\frac{1}{\pi}}$, $A_{0.3}$ are all infinite sets

(B) A_1 is a finite set and $A_{\frac{1}{\pi}}$, $A_{0.3}$ are infinite sets

(C) A_1, $A_{\frac{1}{\pi}}$, $A_{0.3}$ are all finite sets

(D) A_1, $A_{0.3}$ are finite sets and $A_{\frac{1}{\pi}}$ is an infinite sets

72. A box contains b blue balls and r red balls. A ball is drawn randomly from the box and is returned to the box with another ball of the same colour. The probability that the second ball drawn from the box is blue is - **[2016]**

(A) $\frac{b}{r+b}$ (B) $\frac{b^2}{(r+b)^2}$

(C) $\frac{b+1}{r+b+1}$ (D) $\frac{b(b+1)}{(r+b)(r+b+1)}$

73. The number of noncongruent integer-sided triangles whose sides belong to the set {10, 11, 12, ..., 22} is - **[2016]**

(A) 283 (B) 446

(C) 448 (D) 449

74. Suppose we have to cover the xy-plane with identical tiles such that no two tiles overlap and no gap is left between the tiles. Suppose that we can choose tiles of the following shapes; equilateral triangle, square, regular pentagon, regular hexagon. Then the tiling can be done with tiles of - **[2016]**

(A) all four shapes

(B) exactly three of the four shapes

(C) exactly two of the four shapes

(D) exactly one of the four shapes

75. The remainder when the polynomial $1 + x^2 + x^4 + x^6 + ... + x^{22}$ is divided by $1 + x + x^2 + x^3 + ... + x^{11}$ is - **[2016]**

(A) 0

(B) 2

(C) $1 + x^2 + x^4 + ... + x^{10}$

(D) $2(1 + x^2 + x^4 + ... + x^{10})$

76. For each positive real number λ, let A_λ be the set of all natural numbers n such that $|\sin(\sqrt{n+1} - \sin(\sqrt{n})| < \lambda$. Let A_λ^c be the complement of A_λ in the set of all natural numbers. Then - **[2016]**

(A) $A_{\frac{1}{2}}, A_{\frac{1}{3}}, A_{\frac{2}{5}}$ are all finite sets

(B) $A_{\frac{1}{3}}$ is a finite set but $A_{\frac{1}{2}}$, $A_{\frac{2}{5}}$ are infinite sets

(C) $A^c_{\frac{1}{2}}, A^c_{\frac{1}{3}}, A^c_{\frac{2}{5}}$ are all finite sets

(D) $A_{\frac{1}{3}}, A_{\frac{2}{5}}$ are finite sets and $A_{\frac{1}{2}}$ is an infinite set

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77. Three schools send 2, 4 and 6 students, respectively, to a summer camp. The 12 students must be accommodated in 6 rooms numbered 1, 2, 3, 4, 5, 6 in such a way that each room has exactly 2 students and both from the same school. The number of ways, the students can be accommodated in the rooms is - **[2016]**

(A) 60 (B) 45
(C) 32400 (D) 2700

78. Let a be a fixed non-zero complex number with $|a| < 1$ and $w = \left(\frac{z-a}{1-\bar{a}z}\right)$, where z is a complex number. Then **[2016]**

(A) there exists a complex number z with $|z| < 1$ such that $|w| > 1$
(B) $|w| > 1$ for all z such that $|z| < 1$
(C) $|w| < 1$ for all z such that $|z| < 1$
(D) there exists z such with $|z| < 1$ and $|w| = 1$

79. Ten ants are on the real line. At time t = 0, the k-th ant starts at the point k^2 and travelling at uniform speed, reaches the point $(11 - k)^2$ at time t = 1. The number of distinct times at which at least two ants are at the same location is **[2016]**

(A) 45 (B) 11
(C) 17 (D) 9

80. The number of solution pairs (x, y) of the simultaneous equations
$\log_{1/3}(x + y) + \log_3(x - y) = 2$ and $2y^2 = 512^{x+1}$ is **[5 Nov, 2017]**

(A) 0 (B) 1
(C) 2 (D) 3

81. The fractional part of a real number x is $x - [x]$, where $[x]$ is the greatest integer less than or equal to x. Let F_1 and F_2 be the fractional parts of $(44 - \sqrt{2017})^{2017}$ and $(44 + \sqrt{2017})^{2017}$ respectively. Then $F_1 + F_2$ lies between the numbers **[5 Nov, 2017]**

(A) 0 and 0.45 (B) 0.45 and 0.9
(C) 0.9 and 1.35 (D) 1.35 and 1.8

82. Suppose p, q, r are real numbers such that $q = p(4 - p)$, $r = q(4 - q)$, $p = r(4 - r)$.
The maximum possible value of $p + q + r$ is **[5 Nov, 2017]**

(A) 0 (B) 3
(C) 9 (D) 27

83. A shooter can hit a given target with probability $\frac{1}{4}$. She keeps firing a bullet at the target until she hits it successfully three times and then she stops firing. The probability that she fires exactly six bullets lies in the interval **[5 Nov, 2017]**

(A) (0.5272, 0.5274) (B) (0.2636, 0.2638)
(C) (0.1317, 0.1319) (D) (0.0658, 0.0660)

84. Consider the following events :
E_1 : Six fair dice are rolled and at least one die shows six.
E_2 : Twelve fair dice are rolled and at least two dice show six.
Let p_1 be the probability of E_1 and p_2 be the probability of E_2. Which of the following is true ? **[5 Nov, 2017]**

(A) $p_1 > p_2$ (B) $p_1 = p_2 = 0.6651$
(C) $p_1 < p_2$ (D) $p_1 = p_2 = 0.3349$

85. For how many different values of a does the following system have at least two distinct solutions ?
$ax + y = 0$
$x + (a + 10)y = 0$ **[5 Nov, 2017]**

(A) 0 (B) 1
(C) 2 (D) Infinitely many

86. Let a, b, c, d, e, be real numbers such that $a + b < c + d$, $b + c < d + e$, $c + d < e + a$, $d + e < a + b$. Then **[5 Nov, 2017]**

(A) The largest is a and the smallest is b
(B) The largest is a and the smallest is c
(C) The largest is c and the smallest is e
(D) The largest is c and the smallest is b

87. If a fair coin is tossed 5 times, the probability that heads does not occur two or more times in a row is **[5 Nov, 2017]**

(A) $\frac{12}{2^5}$ (B) $\frac{13}{2^5}$
(C) $\frac{14}{2^5}$ (D) $\frac{15}{2^5}$

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88. Let $a_i = i + \frac{1}{i}$ for $i = 1,2, \ldots\ldots, 20$. Put

$$p = \frac{1}{20}(a_1 + a_2 + \ldots\ldots + a_{20}) \text{ and}$$

$$q = \frac{1}{20}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots\ldots + \frac{1}{a_{20}}\right). \text{ Then}$$

[5 Nov, 2017]

(A) $q \in \left(0, \frac{22-p}{21}\right)$

(B) $q \in \left(\frac{22-p}{21}, \frac{2(22-p)}{21}\right)$

(C) $q \in \left(\frac{2(22-p)}{21}, \frac{22-p}{7}\right)$

(D) $q \in \left(\frac{22-p}{7}, \frac{4(22-p)}{21}\right)$

89. Let x,y,z be positive integers such that HCF (x,y,z) = 1 and $x^2 + y^2 = 2z^2$. Which of the following statements are true?

I. 4 divides x or 4 divides y.

II. 3 divides x + y or 3 divides x − y.

III. 5 divides $z(x^2 - y^2)$ **[5 Nov, 2017]**

(A) I and II only (B) II and III only

(C) II only (D) III only

90. The quotient when $1 + x^2 + x^4 + x^6 + \ldots\ldots + x^{34}$ is divided by $1 + x + x^2 + x^3 + \ldots\ldots + x^{17}$ is **[5 Nov, 2017]**

(A) $x^{17} - x^{15} + x^{13} - x^{11} \ldots\ldots + x$

(B) $x^{17} + x^{15} + x^{13} + x^{11} + \ldots\ldots + x$

(C) $x^{17} + x^{16} + x^{15} + x^{14} + \ldots\ldots + 1$

(D) $x^{17} - x^{16} + x^{15} - x^{14} + \ldots\ldots - 1$

91. Among the inequalities below, which ones are true for all natural numbers n greater than 1000 ? **[19 Nov, 2017]**

I. $n! \le n^n$

II. $(n!)^2 \le n^n$

III. $10^n \le n!$

IV. $n^n \le (2n)!$

(A) I and IV only (B) I, III and IV only

(C) II and IV only (D) I, II, III and IV

92. Let

$$S = \left\{\frac{a^2 + b^2 + c^2}{ab + bc + ca} : a, b, c \in R, ab + bc + ca \neq 0\right\},$$

where R is the set of real numbers. Then S equals **[19 Nov, 2017]**

(A) $(-\infty, -1] \cup [1, \infty)$

(B) $(-\infty, 0) \cup (0, \infty)$

(C) $(-\infty, -1] \cup [2, \infty)$

(D) $(-\infty, -2] \cup [1, \infty)$

93. Let S be the infinite sum given by

$$S = \sum_{n=0}^{\infty} \frac{a_n}{10^{2n}}$$

where $\{a_n\}_{n \ge 0}$ is a sequence defined by $a_0 = a_1 = 1$ and $a_j = 20a_{j-1} - 108\, a_{j-2}$ for $j \ge 2$.

If S is expressed in the form $\frac{a}{b}$, where a, b are coprime positive integers, then a equals **[19 Nov, 2017]**

(A) 2017 (B) 2020

(C) 2023 (D) 2025

94. Define a function $f(x) = \frac{16x^2 - 96x + 153}{x - 3}$ for all real $x \neq 3$. The least positive value of f(x) is **[19 Nov, 2017]**

(A) 16 (B) 18

(C) 22 (D) 24

95. Let $n > 2$ be an integer and define a polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x + a_0$ where $a_0, a_1, \ldots\ldots a_{n-1}$ are integers. Suppose we know that $np(x) = (1 + x)p'(x)$. If $b = p(1)$, then **[19 Nov, 2017]**

(A) b is divisible by 10

(B) b is divisible by 3

(C) b is a power of 2

(D) b is a power of 5

96. The number of 5-tuples (a, b, c, d, e) of positive integers such that

I. a,b,c,d,e are the measures of angles of a convex pentagon in degrees ;

II. $a \le b \le c \le d \le e$;

III a, b, c, d, e are in arithmetic progression is **[19 Nov, 2017]**

(A) 35 (B) 36

(C) 37 (D) 126

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97. Thirty two persons $X_1, X_2, \ldots, X_{32}$ are randomly seated around a circular table at equal intervals. Two persons X_i and X_j are said to be within earshot of each other if there are at most three persons between them on the minor arc joining X_i and X_j. The probability that X_1 and X_2 are within earshot of each other is, $\left(\text{Here} \binom{n}{r} = \frac{n!}{(n-r)!r!}\right)$ **[19 Nov, 2017]**

(A) $\frac{\binom{32}{2}30!}{8(32!)}$ (B) $\frac{\binom{32}{2}30!}{4(32!)}$

(C) $\frac{8}{31}$ (D) $\frac{4}{31}$

98. Let n be the smallest positive integer such that $1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{n} \geq 4$.
Which one of the following statements is true ? **[19 Nov, 2017]**

(A) $20 < n \leq 60$ (B) $60 < n \leq 80$
(C) $80 < n \leq 100$ (D) $100 < n \leq 120$

99. A pair of 12-sided fair dice with faces numbered 1,2,3, ..., 12 is rolled. The probability that the sum of the numbers appearing has remainder 2 when divided by 9 is **[19 Nov, 2017]**

(A) $\frac{7}{72}$ (B) $\frac{5}{48}$

(C) $\frac{11}{144}$ (D) $\frac{1}{9}$

100. Let $x_1, x_2, \ldots\ldots, x_6$ be the roots of the polynomial equation $x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64 = 0$. Then **[19 Nov, 2017]**

(A) $|x_i| = 2$ for exactly one value of i
(B) $|x_i| = 2$ for exactly two values of i
(C) $|x_i| = 2$ for all values of i
(D) $|x_i| = 2$ for no value of i

101. In the complex plane, let $z_1 = \sqrt{3} + i$ and $z_2 = \sqrt{3} - i$ be two adjacent vertices of an n-sided regular polygon centered at origin. Then n equals **[19 Nov, 2017]**

(A) 4 (B) 6
(C) 8 (D) 12

102. Let $A^{-1} = \begin{bmatrix} 1 & 2017 & 2 \\ 1 & 2017 & 4 \\ 1 & 2018 & 8 \end{bmatrix}$. Then $|2A| - |2A^{-1}|$ is equal to **[19 Nov, 2017]**

(A) 3 (B) –3
(C) 12 (D) –12

103. Let $z_1, z_2, \ldots\ldots, z_7$ be the vertices of a regular heptagon that is inscribed in the unit circle with centre at the origin in the complex plane. Let $w = \sum_{1 \leq i < j \leq 7} z_i z_j$, then $|w|$ is equal to **[19 Nov, 2017]**

(A) 0 (B) 1
(C) 2 (D) 3

104. Let m, n be two distinct integers chosen randomly from the set { 0, 1, 2, ...,99}. Then the probability that $4^m + 4^n + 3$ is divisible by 5 lies in the interval **[19 Nov, 2017]**

(A) (0, 0.25] (B) (0.25, 0.5]
(C) (0.5, 0.75] (D) (0.75,1)

105. Let A be the set of all permutations $a_1, a_2, \ldots, a_6$ of 1, 2, ..., 6 such that $a_1, a_2, \ldots. a_k$ is not a permutation of 1, 2, ..., k for any k, $1 \leq k \leq 5$. Then the number of elements in A is **[19 Nov, 2017]**

(A) 192 (B) 408 (C) 312 (D) 528

106. Let f(x) be a polynomial with integer coefficients satisfying f(1) = 5 and f(2) = 7. The smallest possible positive value of f(12) is **[19 Nov, 2017]**

(A) 5 (B) 7
(C) 27 (D) 15

107. Suppose four balls labelled 1, 2, 3, 4 are randomly placed in boxes B_1, B_2, B_3, B_4. The probability that exactly one box is empty is **[19 Nov, 2017]**

(A) $\frac{8}{256}$ (B) $\frac{9}{16}$

(C) $\frac{27}{256}$ (D) $\frac{9}{64}$

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ANSWERS

1. (C)	2. (C)	3. (D)	4. (B)	5. (D)	6. (D)	7. (C)
8. (A)	9. (C)	10. (B)	11. (C)	12. (C)	13. (C)	14. (D)
15. (A)	16. (C)	17. (B)	18. (C)	19. (B)	20. (C)	21. (C)
22. (B)	23. (D)	24. (B)	25. (B)	26. (C)	27. (C)	28. (A)
29. (A)	30. (D)	31. (B)	32. (D)	33. (*)	34. (B)	35. (C)
36. (A)	37. (B)	38. (D)	39. (A)	40. (A)	41. (D)	42. (A)
43. (A)	44. (C)	45. (D)	46. (D)	47. (D)	48. (C)	49. (B)
50. (B)	51. (B)	52. (B)	53. (C)	54. (B)	55. (C)	56. (B)
57. (C)	58. (*)	59. (A)	60. (C)	61. (A)	62. (B)	63. (D)
64. (B)	65. (C)	66. (A)	67. (D)	68. (C)	69. (D)	70. (B)
71. (D)	72. (A)	73. (C)	74. (B)	75. (D)	76. (C)	77. (C)
78. (C)	79. (C)	80. (B)	81. (C)	82. (C)	83. (D)	84. (A)
85. (C)	86. (A)	87. (B)	88. (A)	89. (B)	90. (D)	91. (B)
92. (D)	93. (D)	94. (D)	95. (C)	96. (B)	97. (C)	98. (A)
99. (D)	100. (C)	101. (B)	102. (C)	103. (A)	104. (A)	105. (D)
106. (C)	107. (B)					

SOLUTIONS

1. $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$; $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$;

$A^3 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I + A + A^2 + A^3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

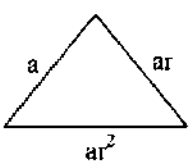
$\therefore I + A + A^2 + A^3 + \ldots\ldots A^{2010}$

$= (I + A + A^2 + A^3) + A^4(I + A + A^2 + A^3) + \ldots$

$\ldots + A^{2008}(I + A + A^2)$

$= \begin{pmatrix} 1 & i \\ i & I \end{pmatrix}$

2.



$a + ar > ar^2$

$r^2 - r - 1 < 0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ ...(1)

$ar^2 + ar > a$

$r^2 + r - 1 > 0$

$r > \frac{-1+\sqrt{5}}{2}$, $r < \frac{-1-\sqrt{5}}{2}$ ...(2)

$ar^2 + a > ar$; $r^2 - r + 1 > 0$ always true

Solving (1) & (2)

$r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

3. Number of diagonals passing through centre = 6

Number of rectangles = ${}^5C_2 = 15$

4. roots $\rightarrow$ $2\omega^2$, $3 + 4\omega$, $3 + 4\omega^2$, $5 - \omega - \omega^2$

α β γ δ

$\delta = 5 - (\omega + \omega^2) = 5 - (-1) = 6$

If $\alpha = 2\omega^2$ is a root then 2ω has to be a root too.

total $\rightarrow$ min. 5 roots, hence min. degree $\rightarrow$ 5

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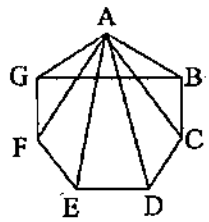
5.



ΔAGB, ΔAFC & ΔAED are isosceles

$$P = \frac{{}^7C_1 \times 3}{{}^7C_3} = \frac{7\times 3}{\frac{7\times 6\times 5}{3\times 2}} = \frac{3}{5}$$

6.

3 ways	3 ways	4 ways	4 ways	5 ways	2 ways
				(1,3,5,7,9)	(2,6)

$= 3 \times 3 \times 4 \times 4 \times 5 \times 2 = 1440$

7. $1 + x^2 + x^4 + \ldots.. x^{2010}$

$$= \frac{1(1-x^{2012})}{1-x^2}$$

$$= \frac{(1-x^{1006})(1+x^{1006})}{(1-x)(1+x)}$$

$$= (1+x^{1006})\left(\frac{(1-x^{503})}{(1-x)}\right)\left(\frac{(1+x^{503})}{(1+x)}\right)$$

$= (1 + x^{1006})(1 + x + x^2 + \ldots. x^{502})$

$(1 - x + x^2 - x^3 + \ldots. x^{502})$

this is divisible by $1 + x + x^2 + \ldots.. x^{n-1}$

if $n - 1 = 502$

$\boxed{n = 503}$

8. $a_n = 3a_{n-1} + 1$

$a_{2010} = 3a_{2009} + 1$

$= 3(3a_{2008} + 1) + 1 = 3^2 a_{2008} + 3 + 1$

$= 3^3 a_{2007} + 3 + 3 + 1$

.

.

.

$$3^{2010} a_0 + \underbrace{(3+3+\ldots3)}_{2009 \text{ times}} + 1$$

$= 0 + 6027 + 1$

$= 6028$

$$\text{Remainder}\left(\frac{6028}{11}\right) = 0$$

9. $$T_{r+1} = {}^nC_n (x^{1/2})^{n-r} \frac{1}{(2x^{1/4})^r} = \frac{{}^nC_r}{2^r} x^{\frac{2n-3r}{4}}$$

$T_1, T_2, T_3 \rightarrow$ AP

$$\frac{2\,{}^nC_1}{2} = {}^nC_0 + \frac{{}^nC_2}{2^2}$$

$$n - 1 = \frac{n(n-1)}{8} \Rightarrow n = 8$$

$$\frac{16-3r}{4} = \text{Integers} \quad \boxed{r = 0, 4, 8}$$

10. $2x^2 + 2x + 1 = 0$

$$x = \frac{-1+i}{2}, \frac{-1-i}{2}$$

x satisfies $(x + 1)^n - r = 0$

$$\left(\frac{-1\pm i}{2} + 1\right)^n - r = 0$$

$$\left(\frac{1\pm i}{2}\right)^n - r = 0$$

$$\left(\frac{1}{\sqrt{2}}\right)^n \left(\frac{1+i}{\sqrt{2}}\right)^n = r$$

$$\left(\frac{1}{\sqrt{2}}\right)^n \left(e^{\pm\frac{i\pi}{4}}\right)^n = r$$

RHS = real

LHS = real only when n = multiply of 4

$n = 4000$

$$r = \left(\frac{1}{\sqrt{2}}\right)^{4000} = \frac{1}{4^{1000}}$$

11. $6^m + 2^{m+n} \quad 3^w + 2^n = 332$

maximum possible value of m is 3

checking for m = 3, 2 and 1

we get $m = 2, n = 3, w = 2$

$m^2 + mn + n^2 = 4 + 6 + 9 = 19$

12. $$c = \log_a b + \frac{1}{\log_a b} \geq 2$$

13. $S = i + 2i^2 + 3i^3 + \ldots.. + ni^n$

$iS = \quad i^2 + 2i^3 + \ldots.. + (n-1)i^n + ni^{n+1}$

$S(1-i) = i + i^2 + i^3 + \ldots.. + i^n - ni^{n+1}$

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$$S(1-i)=\frac{i(1-i^n)}{1-i}-ni^{n+1}$$

$$\Rightarrow S=\frac{1-i^n}{-2i}-\frac{ni^{n+1}}{1-i}$$

$$\downarrow \qquad \downarrow$$

z_1(say) $\quad z_2$(say)

$$|z_1|=\frac{1}{\sqrt{2}} \text{ or } 0; \quad |z_2|=\frac{n}{\sqrt{2}}=\frac{n}{2}\sqrt{2}$$

$$\frac{n}{2}=18 \Rightarrow n=36$$

14. $P^2=P$

$P^{-1}P^2=P^{-1}P$

$P=I$

$(I+P)^n=(2P)^n=2^nP^n$

$=2^nP$

$=P+(2^n-1)P$

$=I+(2^n-1)P$

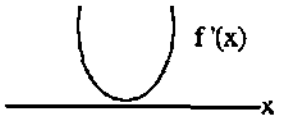
15. $f(x)=x^3+ax^2+bx+c$

$f'(x)=3x^2+2ax+b$

$D=4a^2-4.3.b=4(a^2-3b)$

If $a^2<2b \Rightarrow a^2<3b \Rightarrow D<0$

$\Rightarrow f'(x)=0$ has non real roots



Hence $f(x)=0$ has 1 real and two imaginary roots

16. For A to win, A can draw either 3, 6 or 5, 6. If A draws 3, 6 then B can draw only 8 & 9

$$\text{Prob.}=\frac{1}{3}.\frac{1}{3}=\frac{1}{9}$$

If A draws 5, 6 then B can draw, any two

$$\text{Prob.}=\frac{2}{3}.1=\frac{1}{3}$$

$$\text{Prob.}=\frac{1}{9}+\frac{1}{3}=\frac{4}{9}$$

17. $T_n=(n^2-n+1)n!$

$=(n^2-1)n!-(n-2)n!$

$T_n=(n-1)(n+1)!-(n-2)n!$

Sum $=1+(n-1)(n+1)!$

18. $f(x,A\cup B)=\begin{cases}1 & \text{if } x\in A\cup B\\ 0 & \text{if } x\notin A\cup B\end{cases}$

if $x\in A, x\in B$
if $x\in A, x\notin B$
if $x\notin A, x\in B$

$\Rightarrow f(x, A\cup B)=1$

None of the option (A,B,D) satisfy

if $x\notin A, x\notin B$

$\Rightarrow f(x, A\cup B)=0$

$\Rightarrow$ C(only C satisfy)

19. $A^k=I, B^t=0$ (det (B) = 0)

$\Rightarrow$ det (AB) = 0

20. $$\frac{x}{\frac{2n(2n+1)(4n+1)}{6}-x}<1.01$$

$$2.01x<(1.01)\frac{2n(2n+1)(4n+1)}{6}$$

$$2.01.\frac{4n(n+1)(2n+1)}{6}<(1.01)\frac{2n(2n+1)(4n+1)}{6}$$

$$\frac{2.01}{1.01}<\frac{4n+1}{2n+2} \Rightarrow n>150.5$$

21. $\frac{x+y}{2}=10a+b, \ (a,b\in N)$

$\sqrt{xy}=10b+a$

$xy=(10b+a)^2$

$(x-y)^2=4(11a+11b)(9a-9b)$

$=4.11.(a+b).9(a-b)$

$\Rightarrow a+b=11, a-b=1$

$\Rightarrow a=6, b=5$

($(x-y)^2$ is perfect square of an integer)

$x\div y=130$

22. $\frac{6!}{(2!)^3}=90$

23. $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1; x\geq 1$

$\sqrt{(x-1)-2\times 2\sqrt{x-1}+4}+\sqrt{(x-1)-6\sqrt{x-1}+9}=1$

$|\sqrt{x-1}-2|+|\sqrt{x-1}-3|=1$

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Case - I

$\sqrt{x-1}-2+\sqrt{x-1}-3=1$ when $x \geq 10$

$2\sqrt{x-1}=6$

$x=10$

Case -II

$\sqrt{x-1}-2-\sqrt{x-1}+3=1$ when $5 \leq x \leq 10$

Case -III

$-\sqrt{x-1}+2-\sqrt{x-1}+3=1$ when $1 \leq x \leq 5$

$2\sqrt{x-1}=4$

$x=5$

24. $M=a+b-a^2+ab-b^2$

but $\frac{a^2+b^2}{2} \geq ab$

$\Rightarrow a^2+b^2 \geq 2ab$ or $-(a^2+b^2) \leq -2ab$

$M \leq a+b-ab$

So $M<1-(a-1)(b-1)$

So maximum value of M can be 1.

at $x=0$, else where the value of the function is less than 1. So answer must be option (B) but KVPY has mentioned option (C) as answer in its answer key.

25. $y=2x^3+ax+b$ $y=2x^3+cx+d$

No Solution

$2x^3+ax+b \neq 2x^3+cx+d$

$ax+b \neq cx+d$ for no real x

$(a-c)x \neq d-b$

$x \neq \frac{d-b}{a-c}$ $a=c$

$(a-c)^2+(b-d)=0+6-1=5$

26. $f(x)=x^{2n+1}-(2n+1)x+a$

$f'(x)=(2n+1)x^{2n}-(2n+1)$

$=(2n+1)(x^{2n}-1) \leq 0$ when $x \in [-1, 1]$

f(x) is strictly decreasing in $[-1, 1]$

f(x) cut x axis at most one point in given interval

27. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105 till 98 terms

48 terms are even and 48 terms odd

99^{th} term $=\frac{99\times100}{2}=$ even

Total even terms $=48+1=49$

Probability $=\frac{49}{99}$

28. P_1 : 4 copper coins 3 silver coins

P_2 : 6 copper coins 4 silver coins

E = Event of copper coin

$P(E)=P(P_1).P(E/P_1)+P(P_2).P(E/P_2)$

$=\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{10}=\frac{41}{70}$

29. 6^m+9^n

$6^1=6$ $9^1=9$

$6^2=6$ $9^2=1$

$6^3=6$ $9^3=9$

$6^4=6$ $9^4=1$

m can be any value and n will be odd number then sum is multiple of 5

$50\times25=1250$

30. $a_1, a_2, a_3, \ldots\ldots, a_{2012}=3018$ ...(1)

$\frac{a_1+a_3}{2}=a_2$

$2a_2+2a_4+2a_6+\ldots.+2a_{2012}=6036$

$(a_1+a_3)+(a_3+a_5)+(a_5+a_7)+\ldots+(a_{101}+a_1)=6036$

$2(a_1+a_3+a_5+\ldots+a_{2011})=6036$

$a_1+a_3+a_5+\ldots.+a_{2011}=3018$ ...(2)

Add (1) and (2)

Sum of all number $=3018+3018=6036$

31. As total number of elements in set A will be n^2.

Now set B will be

Set B $=\{(1, 1), (2, 2) \ldots\ldots (n, n)\}$

Number of elements in B is 'n'.

So number of good subsets $=2^n$

So answer is (B) although KVPY has Mentioned option (C) in its answer key.

32. $x^2+2ax+b^2=0$ $x^2+2bx+c^2=0$

$D_1>0$ $D_2>0$

$4a^2+b^2>0$ $4b^2-4c^2>0$

$a^2>b^2$.....(1) $b^2>c^2$.....(2)

From (1) and (2)

$a^2>b^2>c^2 \Rightarrow a^2>c^2 \Rightarrow c^2-a^2<0$

$x^2+2cx+a^2=0$

$D=4c^2-4a^2<0$ No real roots

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33. Coeff. of x^{2012} is $\frac{(1+x)^2}{(1+x^2)(1-x^2)}$

$=(1+x)^2(1-x^4)^{-1}$

$=(1+2x+x^2)(1-x^4)^{-1}$

Coeff. of x^{2012} + 2 Coeff of x^{2011} + Coeff of x^{2010} in the expansion of $(1-x^4)^{-1}$

x^{2011} and x^{2010} not possible in $(1-x^4)^{-1}$

= only coeff. of x^{2012} in the expansion of $(1-x^4)^{-1}$

$= {}^{1+503-1}C_{503} = 1$

34. Ways to make the sum K is coefficient of x^K in $(x+x^2)^{10}$

Coefficient of x^K in $x^{10}(1+x)^{10}$

Coefficient of x^{K-10} in $(1+x)^{10}$

which is ${}^{10}C_{K-10}$

So ways to make sum minimum K is

${}^{10}C_{K-10} + {}^{10}C_{K-9} + {}^{10}C_{K-8} + \ldots\ldots + {}^{10}C_{10}$

Probability

$$P(K) = \frac{{}^{10}C_{K-10} + {}^{10}C_{K-9} + \ldots\ldots\ldots\ldots + {}^{10}C_{10}}{2^{10}}$$

$$P(K) = \frac{2^{10} - ({}^{10}C_0 + \ldots\ldots + {}^{10}C_{K-11})}{2^{10}}$$

$$= 1 - \frac{{}^{10}C_0 + \ldots\ldots + {}^{10}C_{K-11}}{2^{10}} > \frac{1}{2}$$

So ${}^{10}C_0 + {}^{10}C_1 + \ldots\ldots {}^{10}C_{K-11} < 2^9$

So K – 11 = 4

$\Rightarrow$ K = 15

35. $f(x) = x^3 + 3x^2 + 3x + 3$

$f'(x) = 3x^2 + 6x + 3$

$f'(x) = 3(x^2 + 2x + 1) \geq 0$

regular increasing function

and f(–2) f(–3) < 0

so real root lies between –2 and –3

i.e. $-2 < \alpha < -3$

$\alpha + \beta + \gamma = -3$

$\beta + \gamma = -3 - \alpha$

$-1 < \beta + \gamma < 0$

36. After solving logarithmic inequation

$2^4 < n < 2^{16}$

number of binary digits in $2^n = n + 1$

$\therefore 5 < \ell < 16$

37. $|a + b(\omega + \omega^2)| \leq |a| + |b|\,|\omega + \omega^2|$

$\leq 1 + 1.1$

≤ 2

38. Note : No. of numbers divisible by 'k' among $\{1, 2, 3, \ldots\ldots n\}$ are $\left[\frac{n}{k}\right]$

Some case for y	Let x = 1, y = 1
y = 1	.
y = 2	.
.	.
.	x = n, y = n
.	comes two time so that
y = n	$-\frac{1}{n} + \frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

adding then

39. $\frac{x_1 + x_2 + \ldots + x_n}{n} = \mu$

$$\sigma = \sqrt{\frac{x_1^2 + x_2^2 + \ldots\ldots + x_n^2}{n}}$$

$y_1 = 0, y_2 = x_2, y_3 = x_3, \ldots\ldots, y_{n-1} = x_{n-1}, y_n = x_1 + x_n$

Clearly, $\hat{\mu} = \mu$ and

$$\hat{\sigma} = \sqrt{\frac{0^2 + x_2^2 + x_3^2 + \ldots\ldots + x_{n-1}^2 + (x_1 + x_n)^2}{n}}$$

$$= \sqrt{\frac{x_1^2 + x_2^2 + \ldots\ldots + x_n^2 + 2x_1x_n}{n}}$$

So $\hat{\sigma} > \sigma$

40. Required integers = 900 – 9 × 9 × 8

= 900 – 648

= 252

41. (A) Put n = 19 $S_n = \{20, 21, \ldots\ldots 37\}$

So (A) is wrong

(B) By observation we can say there exist some S_n don't have any prime number

(C) Let n = 10 {11, 12, ... 28}

so option C is wrong

(D) Let n = 10, S_n = {11, 12, ... 28}

prime number = 11, 13, 17, 19, 23

(five prime)

(at most six)

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42. $\sum_{k=1}^{n}(ak^3+bk^2+ck+d)=n^4$

$$\Rightarrow \frac{a(n^2)(n^2+2n+1)}{4}+\frac{bn(2n^2+3n+1)}{6}+\frac{c(n^2+n)}{2}+dn=n^4$$

(1) $\frac{a}{4}=1 \Rightarrow a=4$

(2) $\frac{2a}{4}+\frac{2b}{6}=0 \Rightarrow b=-6$

(3) $\frac{a}{4}+\frac{3b}{6}+\frac{c}{2}=0$

$\Rightarrow 1-3+\frac{c}{2}=0$

$\Rightarrow c=4$

(4) $\frac{b}{6}+\frac{c}{2}+d=0 \Rightarrow d=-1$

$\therefore |a|+|b|+|c|+|d|=15$

43. Minimum value of f(x) is +ve.
So f(x) doesn't have any solution.

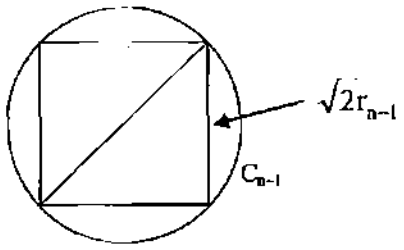
44. $\frac{{}^{100}C_5\times\frac{4!}{2!2!}}{{}^{100}C_5\times 5!}=\frac{1}{20}$

45. Total point $={}^5C_2=10$
Maximum points of one point $=4$
possible distribution of points

4	4	2	0	0
4	4	1	1	0
4	3	2	1	0
4	3	3	0	0
4	3	1	1	1
4	2	2	2	0
4	2	2	1	1
3	3	3	1	0
3	3	2	2	0
3	3	2	1	1
3	2	2	2	1
2	2	2	2	2

Clearly answer is "D"

46. $\sum_{i=0}^{\infty}\text{Area}(C_i)=\pi r_0^2+\pi r_1^2+\pi r_2^2+\pi r_3^2+\ldots\infty$



Area of $C_n=\pi r_n^2=(\sqrt{2}\,r_{n-1})^2$

$r_n^2=\frac{2}{\pi}r_{n-1}^2$

so $r_1^2=\frac{2}{\pi}r_0^2$,

$r_2^2=\frac{2}{\pi}r_1^2=\frac{2}{\pi}\left(\frac{2}{\pi}r_0^2\right)$

$r_3^2=\frac{2}{\pi}(r_2^2)=\frac{2}{\pi}\left(\frac{2}{\pi}\frac{2}{\pi}r_0^2\right)$

So $\sum_{i=0}^{\infty}\text{Area}(C_i)$

$$=\pi\left[r_0^2+\frac{2}{\pi}r_0^2+\frac{2}{\pi}.\frac{2}{\pi}r_0^2+\ldots\infty\right]$$

$$=\frac{\pi r_0^2}{1-\frac{2}{\pi}}$$

$$=\frac{\pi^2 r_0^2}{\pi-2}\quad \forall\, r_0=1$$

$$=\frac{\pi^2}{\pi-2}$$

47. (A) $[x+y]\le[x]+[y]$
Let $x=0.1, y=0.9$
$[0.1+0.9]\le[0.1]+[0.9]$
$1\le 0+0$ wrong

(B) $[xy]\le[x][y]$
$x=2; y=\frac{1}{2}$

$\left[2.\frac{1}{2}\right]<[2]\left[\frac{1}{2}\right]$

$\Rightarrow 1\le 0$ wrong

(C) $[2^x]\le 2^{[x]}$
$x=0.99\quad [2^{0.99}]\le 2^{[0.99]}$
$[2^{0.99}]<2^0=1$ wrong

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(D) $\left[\frac{x}{y}\right] \le \frac{[x]}{[y]}$

given $x, y \ge 1$

if $x < y$ $\left[\frac{x}{y}\right] = 0 \quad 0 \le \frac{[x]}{[y]}$ true

if $x \ge y$ $\left[\frac{x}{y}\right] \le \frac{[x]}{[y]}$ always true

48. Case (1) n = even

$$\frac{A_n}{A_{n-1}} = \frac{{}^{n}C_{n/2}}{{}^{n-1}C_{\frac{n-1-1}{2}}} = 2$$

so for all n even given relation is true.

Case (2) n = odd

$$\frac{A_n}{A_{n-1}} = \frac{{}^{n}C_{\frac{n-1}{2}}}{{}^{n-1}C_{\frac{n-1}{2}}} = \frac{2n}{n+1}$$

which satisfies only for n = 19

49. $P = \frac{6}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \ldots\ldots\infty$

$$= \frac{5}{6} + \frac{5}{6^3} + \frac{5}{6^5} + \ldots\ldots$$

$$= \frac{5/6}{1-\frac{1}{36}} = \frac{30}{35} = \frac{6}{7}$$

50. $\mu = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$

$\hat{\mu} = \frac{y_1 + y_2 + \ldots\ldots + y_n}{n}$

$$= \frac{\frac{x_1 + x_2}{2} + \frac{x_1 + x_2}{2} + x_3 + \ldots + x_n}{n}$$

$\hat{\mu} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n} = \mu \Rightarrow \boxed{\hat{\mu} = \mu}$

$\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$

$\sigma^2 = \frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n} - \mu^2 \quad \ldots(1)$

$\hat{\sigma}^2 = \frac{y_1^2 + y_2^2 + \ldots + y_n^2}{n} - \mu^2$

$$= \frac{\left(\frac{x_1 + x_2}{2}\right)^2 + \left(\frac{x_1 + x_2}{2}\right)^2 + x_3^2 + \ldots x_n^2}{n} - \mu^2$$

$\hat{\sigma}^2 = \frac{\frac{x_1^2 + x_2^2}{2} + x_1x_2 + x_3^2 + \ldots + x_n^2}{n} - \mu^2 \ldots(2)$

$\sigma^2 - \hat{\sigma}^2 = \frac{x_1^2 + x_2^2}{n} - \left(\frac{x_1^2 + x_2^2 + 2x_1x_2}{2n}\right)$

$= \frac{x_1^2 + x_2^2 - 2x_1x_2}{2n}$

$= \frac{(x_1 - x_2)^2}{2n} \ge 0$

$\Rightarrow \sigma \ge \hat{\sigma}$ & $\mu = \hat{\mu}$

51. $3x + 4y + 5 = 19\,I$

$5 \le (3x + 4y + 5) \le 131$

$5 \le 19\,I \le 131$

Case (i) $3x + 4y + 5 = 19$ $3x + 4y = 14$ $y = \frac{14 - 3x}{4}$ $x = 2$	Case (ii) $3x + 4y + 5 = 38$ $3x + 4y = 33$ $4y = 33 - 3x$ $y = \frac{33 - 3x}{4}$ $x = 3, 7, 11$
Case (iii) $3x + 4y = 52$ $x = \frac{52 - 3x}{4}$ $x = 0, 4, 8, 12, 16$	Case (iv) $3x + 4y = 71$ $4y = 71 - 3x$ $y = \frac{71 - 3x}{4}$ $x = 1, 5, 9, 13, 17$
Case (v) $3x + 4y = 90$ $y = \frac{90 - 3x}{4}$ $x = 6, 10, 14, 18$	Case (vi) $3x + 4y = 109$ $y = \frac{109 - 3x}{4}$ $x = 15$ is only possibility.

Total Solution = 19

52. By taking different values of a & k.
option (B) is possible.

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53. $^5C_2 \cdot 2! + {}^5C_3\left(\frac{3!}{1!\,2!}\times 2!\right)$

$+ {}^5C_4\left[\frac{4!}{1!\,3!}\times 2! + \frac{4!}{2!\,2!}\times\frac{2!}{2!}\right]$

$+ {}^5C_5\left[\frac{5!}{1!\,4!}\times 2! + \frac{5!}{2!\,3!}\times 2!\right]$

$= 10(2) + 10(6) + 5(8+6) + (10+20)$

$= 20 + 60 + 70 + 30 = 180$

54. $S = -\left[\frac{\lambda - a_n}{a_n} + \frac{\lambda - a_{n-1}}{a_{n-1}} + \ldots\ldots + \frac{\lambda - a_1}{a_1}\right]$

$\forall\ \lambda = a_1 + a_2 + \ldots\ldots + a_n$

$S = -\left[(a_1 + a_2 + \ldots + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right) - n\right]$

$S = n - (a_1 + a_2 + \ldots + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)$

from A.M. $\geq$ H.M.

$(a_1 + a_2 + \ldots + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right) \geq n^2$

$\boxed{S \leq -n(n-1)}$

55. $\sum_{k=0}^{n} {}^nC_k\omega^k = {}^nC_0 + {}^nC_1\omega + \ldots\ldots + {}^nC_n\omega^n$

$= (1+\omega)^n = (-\omega^2)^n$

$= (-1)^n\,\omega^{2n}$

$\therefore \quad \left|e^{(-1)^n\omega^{2n}}\right| = \left|e^{(-\omega^2)^n}\right|$

$= \left|e^{\left(-\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right)^n}\right|$

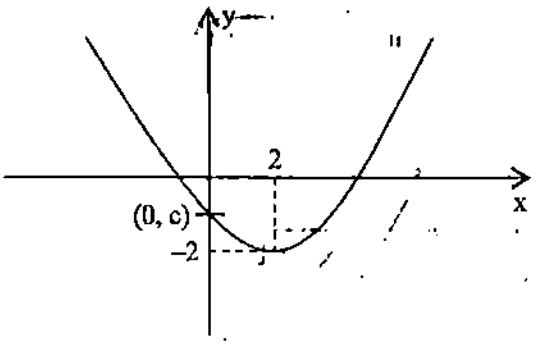
$= \left|e^{\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}}\right|$

$= \left|e^{\cos\frac{n\pi}{3}}\right|$ can have values

$= \{e^1, e^{1/2}, e^{-1/2}, e^{-1}\}$

Four values.

56. The graph according to the question is



Clearly it can be observed

$c < 0$

$a > 0$

$\frac{-b}{a} > 0$

$\Rightarrow -b > 0 \Rightarrow b < 0$

$f(1) < 0$

$\Rightarrow a + b + c < 0$

$ab < 0$

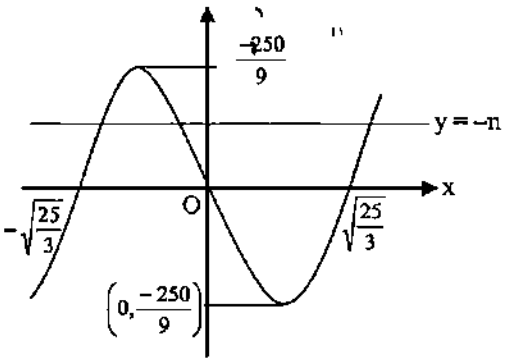
$ac < 0$

$bc > 0$

57. $x(3x^2 - 25) = -n$

$3x\left(x^2 - \frac{25}{3}\right) = -n$

$3x\left(x - \sqrt{\frac{25}{3}}\right)\left(x + \sqrt{\frac{25}{3}}\right) = -n$



$\therefore\ n \in \left(-\frac{250}{9}, \frac{250}{9}\right)$

$\therefore$ But $n \in I$

Total integer = 55

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59. $|z^3 + z^{-3}| \le 2$

$$\left|z^3 + \frac{1}{z^3}\right| \le 2$$

$$\left|\left(z+\frac{1}{z}\right)\left(z^2+\frac{1}{z^2}-1\right)\right| \le 2$$

$$\left|\left(z+\frac{1}{z}\right)\left(\left(z+\frac{1}{z}\right)^2-3\right)\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left|\left(z+\frac{1}{z}\right)^2-3\right| \le 2$$

$$\left|z+\frac{1}{z}\right|\left\{\left|\left|z+\frac{1}{z}\right|^2-3\right|\right\} \le 2$$

$\{\because |z_1 - z_2| \ge ||z_1| - |z_2||\}$

$t|t^2-3| \le 2 \quad (t \ge 0)$ where $t = \left|z+\frac{1}{z}\right|$

$t \ge \sqrt{3}$	$0 \le t < \sqrt{3}$
$t(t^2-3) \le 2$	$t(3-t^2) \le 2$
$t^3 - 3t - 2 \le 0$	$3t - t^3 \le 2$
$(t-2)(t+1)^2 \le 0$	$t^3 - 3t + 2 \ge 0$
	$(t-1)^2(t+2) \ge 0$
$t - 2 \le 0$	$t \ge -2$
$t \le 2$	$t \in [0, \sqrt{3})$

$$\left|z+\frac{1}{z}\right|_{max} = 2$$

60. $2\{(2014)^3 + (2012)^2 + \ldots + 2^3\}$
$\quad - \{(2014)^3 + (2013)^3 + \ldots + 1^3\}$
$= 2 \times 8\{(1007)^2 + (1006)^2 + \ldots + 1^3\}$
$\quad - \{(2014)^3 + (2013)^2 + \ldots + 1^3\}$

$$= 2 \times 8 \times \left(\frac{(1007)(1008)}{2}\right)^2 - \left(\frac{(2014)(2015)}{2}\right)^2$$

$$= 2 \times 8 \times \frac{(1007)^2(1008)^2}{4} - \frac{(2014)^2(2015)^2}{4}$$

$= (1007)^2(2016)^2 - (1007)^2(2015)^2$
$= (1007)^2\{2016 - 2015\}\{2016 + 2015\}$
$= (1007)^2(4031)$
= divisible by $(1007)^2$

61. Let $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$
$P'(x) = na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1}$
$P(x)P'(x) = a_0x^n + (a_1 - na_0)x^{n-1}$
$\quad + (a_2 - (n-1)a_1)x^{n-2} + \ldots$
$\quad \ldots + (a_n - a_{n-1})$
given $P(x) - P'(x) = a_0x^n$.
so that

$$a_1 - na_0 = 0 \Rightarrow \frac{a_1}{a_0} = n$$

$$a_2 - (n-1)a_1 = 0 \Rightarrow \frac{a_2}{a_1} = (n-1)$$

$$a_n - a_{n-1} = 0 \Rightarrow \frac{a_n}{a_{n-1}} = 1$$

$$P(0) = a_n = \left(\frac{a_n}{a_{n-1}}\right)\left(\frac{a_{n-1}}{a_{n-2}}\right)\ldots\left(\frac{a_1}{a_0}\right)$$
$$= 1 \times 2 \times 3 \times \ldots \times n$$
$$= n!$$

62. Let Red Balls = x
White Balls = y
Blue Balls = z
Green Balls = w

$$\frac{{}^xC_4}{{}^{x+y+z+w}C_4} = \frac{{}^xC_3 \cdot {}^yC_1}{{}^{x+y+z+w}C_4}$$

$$= \frac{{}^xC_2\,{}^yC_1\,{}^zC_1}{{}^{x+y+z+w}C_4} = \frac{{}^xC_1 \times {}^yC_1 \times {}^zC_1 \times {}^wC_1}{{}^{x+y+z+w}C_4}$$

${}^xC_4 = {}^xC_3\,{}^yC_1$
$x - 3 = 4y$
$x = 4y + 3$

${}^xC_3\,{}^yC_1 = {}^xC_2\,{}^yC_1\,{}^zC_1$
$x - 2 = 3z$
$x = 3z + 2$

${}^xC_2\,{}^yC_1\,{}^zC_1 = {}^xC_1\,{}^yC_1\,{}^zC_1\,{}^wC_1$
$x - 1 = 2w$
$x = 2w + 1$

Cleary for y = 1 not possible
at $y = 2 \quad x = 11$
$z = 3 \quad x = 11$
$w = 5 \quad x = 11$
so, minimum number of Ball
$= 11 + 2 + 3 + 5 = 21$

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91. (A) $\frac{n^n}{n!}=\left(\frac{n}{n}\right)\left(\frac{n}{n-1}\right)\left(\frac{n}{n-2}\right)\ldots\ldots\left(\frac{n}{1}\right)\geq 1$

$n^n \geq n!$ (correct)

(C) $\frac{n!}{10^n}=\frac{(n)(n-1)(n-2)\ldots\ldots}{(10)(10)\ldots\ldots n \text{ times} \quad (10)}$

given that $n > 1000$

clearly $\frac{n!}{10^n} \geq 1$

$n! \geq 10^n$

(D) $\frac{(1.2.3.4\ldots\ldots n)(n+1)(n+2)(n+3)\ldots\ldots(n+n)}{n^n}$

$(n!)\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\ldots\left(1+\frac{n}{n}\right)$

clearly ≥ 1.

$2n! \geq n^n$

92. **Case-I**

$(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

$a^2+b^2+c^2-ab-bc-ca \geq 0$

$a^2+b^2+c^2 \geq ab+bc+ca$

If $ab+bc+ca > 0$

Then $\frac{a^2+b^2+c^2}{ab+bc+ca} \geq 1$

Case-II

Let $ab+bc+ca < 0$

$\frac{(a+b+c)^2}{ab+bc+ca} \leq 0$

$\frac{a^2+b^2+c^2+2(ab+bc+ca)}{ab+bc+ca} \leq 0$

$\frac{a^2+b^2+c^2}{ab+bc+ca}+2 \leq 0$

$\frac{a^2+b^2+c^2}{ab+bc+ca} \leq -2$

So, Range is $(-\infty,-2]\cup[1,\infty)$

93. $a_n = 20a_{n-1}-108a_{n-2}$

$\frac{a_n}{10^{2n}}=\frac{20a_{n-1}}{10^{2n}}-\frac{108a_{n-2}}{10^{2n}}$

$\frac{a_n}{10^{2n}}=\frac{20}{100}\frac{a_{n-1}}{10^{2(n-1)}}-\frac{108}{10000}\frac{a_{n-2}}{10^{2(n-2)}}$

apply summation

$\sum_{n=2}^{\infty}\frac{a_n}{10^{2n}}=\frac{1}{5}\sum_{n=2}^{\infty}\frac{a_{n-1}}{10^{2(n-1)}}-\frac{27}{2500}\sum_{n=2}^{\infty}\frac{a_{n-2}}{10^{2(n-2)}}$

$S-1-\frac{1}{100}=\frac{1}{5}(S-1)-\frac{27}{2500}S.$

$S-1-\frac{1}{100}=\frac{1}{5}S-\frac{1}{5}-\frac{27}{2500}S.$

$S\left(1-\frac{1}{5}+\frac{27}{2500}\right)=-\frac{1}{5}+1+\frac{1}{100}$

$S\left(\frac{2500-500+27}{2500}\right)=\frac{81}{100}$

$S=\frac{81\times 25}{2027}$

$S=\frac{2025}{2027}$

94. $y=\frac{16x^2-96x+153}{x-3}$

make it quadratic in x

$16x^2-x(96+y)+(153+3y)=0$

$D \geq 0$

Solve $y^2-576 \geq 0$

$y \in (-\infty,-24]\cup[24,\infty)$

So, least positive is 24

95. $n[x^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}-a_1x+a_0]$

$=(1+x)(nx^{n-1}+a_{n-1}(n-1)x^{n-2}$

$+a_{n-2}x^{n-3}(n-2)$

$+a_{n-3}(n-3)x^{n-4}+\ldots\ldots)$

compare coefficient of x^{n-1}

$na_{n-1}=(n-1)a_{n-1}+n$

Solve $a_{n-1}=n$ or ${}^nC_1$

compare coefficient of x^{n-2}

$na_{n-2}=(n-2)a_{n-2}+(n-1)a_{n-1}$

$a_{n-2}=\frac{n(n-1)}{2}={}^nC_2$

similarly $a_{n-3}={}^nC_3$ & So ..... on

$b=P(1)=1+a_{n-1}+a_{n-2}+\ldots..a_1+a_0$

$={}^nC_0+{}^nC_1+{}^nC_2+\ldots\ldots{}^nC_n=2^n$

96. $a+b+c+d+e=540$

Let say first term = a

Common difference d

$\frac{n}{2}[2a+(n-1)d]=540$

$\Rightarrow a+2d=108$

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Case-1 $d = 0$

$a = 108$

(108, 108, 108, 108, 108)

Case-2 $d = 1$

(106, 107, 108, 109, 110)

Similarly it goes up to $d = 35$

for $d > 35$, interior angle $> 180°$

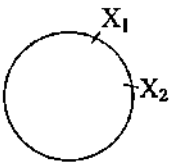
which is not possible

So form $d = 0$ to $d = 35$

total 36 tuples are possible

97. **Case -1**

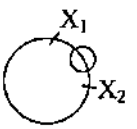
No person between X_1 & X_2



$$\frac{30!\times 2!}{31!} = \frac{2}{31}$$

Case-2

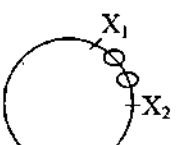
One person between X_1 & X_2



$$\frac{^{30}C_1(29)!\times 2!}{31!} = \frac{2}{31}$$

Case-3

When 2 person between X_1 & X_2



$$\frac{^{30}C_2\times 28!\times 2!\times 2!}{31!} = \frac{2}{31}$$

Case-4

When 3 person between X_1 & X_2

$$\frac{^{30}C_3\times 27!\times 2!\times 3!}{31!} = \frac{2}{31}$$

$$\text{Total} = \frac{8}{31}$$

98. $1 \div x < 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

$1 + x < e^x$

$\ln(1+x) < x$

$x = \frac{1}{y}$

$\ln\left(1 + \frac{1}{y}\right) < \frac{1}{y}$

$\ln(y+1) - \ln y < \frac{1}{y}$

put $y = 1$

$\ln(2) - \ln(1) < \frac{1}{1}$

put $= y = 2$

$\ln 3 - \ln 2 < \frac{1}{2}$

put $= y = 3$

$\ln 4 - \ln 3 < \frac{1}{3}$

put $y = n$

$\ln(n+1) - \ln n < \frac{1}{4}$

$\ln(n+1) - \ln(1) < 1 + \frac{1}{2} + \frac{1}{3} + \ldots. \frac{1}{n}$

$\ln(n+1) \le 4$

$n \le e^4 - 1$

$n \le 60$

99. $x_1 + x_2 = 11$ or $x_1 + x_2 = 20$

possible cases

(1,10)	(8,12)
(2,9)	(9,11)
(3,8)	(10,10)
(4,7)	
(5,6)	

$$\frac{10}{144} + \frac{6}{144} = \frac{16}{144} = \frac{1}{9}$$

100. It form an G.P.

$$\frac{x^6\left(1-\left(\frac{2}{x}\right)^7\right)}{\left(1-\frac{2}{x}\right)} = 0$$

solve that

$x^7 = 2^7$

$x = 2$

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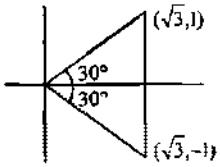
101.



$$\frac{2\pi}{n} = \frac{\pi}{3}$$

$n = 6$

102. $2^3|A| - 2^3\frac{1}{|A|}$

$$|A^{-1}| = \begin{vmatrix} 1 & 2017 & 2 \\ 1 & 2017 & 4 \\ 1 & 2018 & 8 \end{vmatrix}$$

$$\frac{1}{|A|} = -2 \Rightarrow |A| = \frac{-1}{2}$$

Put the value answer is = 12

103. given expression can be written as

$$\frac{(z_1+z_2+....+z_7)^2 - \sum_{i=1}^{7} z_i^2}{2} = \sum_{1\le i<j\le 7} z_i z_j$$

Sum of all seven root of unity = 0 (By property)

$z_1^2 + z_2^2 + z_3^2 + ... + z_7^2 = 0$ (By property)

104. possible case

Case-1

If unit digit is sum of $4^m + 4^n$ is 7

possible case are

(m, n) = (0, 2) (0, 4) ..... (0, 98) = 49

(n, m) = (0,2) (0, 4) .....(0,98) = 49

total = 98

Case-2

Unit digit is sum of $4^m + 4^n$ is 2

Possible case

(m, n) = (2,4) (2,6).....(2, 98) = 48

(m, n) = (4, 6) (4, 8) .....(4.98) = 47

⋮

(m, n) = (96, 98) ⇒ 1Case

same case repetition (n, m)

$$2 \times \left(\frac{48\times 49}{2}\right) = 2352$$

Now, Case 1 + Case 2

⇒ 2352 + 98 = 2450

Total number of ways m, n can be selected

= 100 × 99

= 9900

$$\text{Probability} = \frac{2450}{9900} = 0.2474$$

106. $f(x) = ax + b$

$5 = a + b$ ...(i)

$7 = 2a + b$ ....(ii)

solve $a = 2$

$b = 3$

$f(x) = 2x + 3$

$f(12) = 24 + 3$

$f(12) = 27$

107. required probability =

$$\frac{{}^4C_1 \times \frac{4!}{(1!)^2\times 2!} \times \frac{1}{2!} \times 3!}{4^4} = \frac{9}{16}$$

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Chapter 3

2D-Coordinates

1. A circle touches the parabola $y^2 = 4x$ at (1, 2) and also touches its directrix. The y-coordinates of the point of contact of the circle and the directrix is- **[2010]**

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

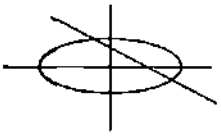
2. Let P be an arbitrary point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b > 0$. Suppose F_1 and F_2 are the foci of the ellipse. The locus of the centroid of the triangle PF_1F_2 as P moves on the ellipse is- **[2010]**

(A) a circle (B) a parabola

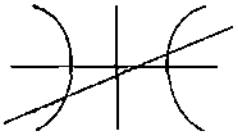
(C) an ellipse (D) a hyperbola

3. Suppose a, b are real numbers such that $ab \neq 0$. Which of the following four figures represents the curve $(y - ax - b)(bx^2 + ay^2 - ab) = 0$? **[2010]**

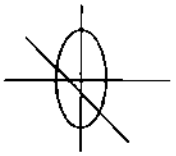
(A)



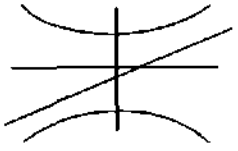
(B)



(C)



(D)



4. Among all cyclic quadrilaterals inscribed in a circle of radius R with one of its angles equal to 120°. Consider the one with maximum possible area. Its area is- **[2010]**

(A) $\sqrt{2}\ R^2$ (B) $\sqrt{3}\ R^2$

(C) $2\ R^2$ (D) $2\sqrt{3}\ R^2$

5. All the points (x, y) in the plane satisfying the equation $x^2 + 2x \sin(xy) + 1 = 0$ lie on - **[2011]**

(A) a pair of straight lines

(B) a family of hyperbolas

(C) a parabola

(D) an ellipse

6. Let $A = (4, 0)$, $B = (0, 12)$ be two points in the plane. The locus of a point C such that the area of triangle ABC is 18 sq. units is - **[2011]**

(A) $(y + 3x + 12)^2 = 81$

(B) $(y + 3x + 81)^2 = 12$

(C) $(y + 3x - 12)^2 = 81$

(D) $(y + 3x - 81)^2 = 12$

7. In a rectangle $ABCD$, the coordinates of A and B are (1, 2) and (3, 6) respectively and some diameter of the circumscribing circle of $ABCD$ has equation $2x - y + 4 = 0$. Then the area of the rectangle is - **[2011]**

(A) 16 (B) $2\sqrt{10}$

(C) $2\sqrt{5}$ (D) 20

8. In the xy-plane, three distinct lines l_1, l_2, l_3 concur at a point $(\lambda, 0)$. Further the lines l_1, l_2, l_3 are normals to the parabola $y^2 = 6x$ at the points $A = (x_1, y_1)$, $B = (x_2, y_2)$, $C = (x_3, y_3)$ respectively. Then we have - **[2011]**

(A) $\lambda < -5$ (B) $\lambda > 3$

(C) $-5 < \lambda < -3$ (D) $0 < \lambda < 3$

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9. The locus of the point P = (a, b) where a, b are real numbers such that the roots of $x^3 + ax^2 + bx + a = 0$ are in arithmetic progression is- **[2011]**
(A) an ellipse
(B) a circle
(C) a parabola whose vertex in on the y-axis
(D) a parabola whose vertex is on the x-axis

10. The smallest possible positive slope of a line whose y-intercept is 5 and which has a common point with the ellipse $9x^2 + 16y^2 = 144$ is **[2011]**
(A) $\frac{3}{4}$ (B) 1
(C) $\frac{4}{3}$ (D) $\frac{9}{16}$

11. Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. then $|p| =$ **[2012]**
(A) 0 (B) 3
(C) 5 (D) 17

12. Consider the conic $ex^2 + \pi y^2 - 2c^2x - 2\pi^2 y + c^3 + \pi^3 = \pi c$. Suppose P is any point on the conic and S_1, S_2 are the foci of the conic, then the maximum value of $(PS_1 + PS_2)$ is **[2012]**
(A) πc (B) $\sqrt{\pi e}$
(C) $2\sqrt{\pi}$ (D) $2\sqrt{e}$

13. Let (x, y) be a variable point on the curve $4x^2 + 9y^2 - 8x - 36y + 15 = 0$. Then
$\min(x^2 - 2x + y^2 - 4y + 5)$
$+ \max(x^2 - 2x + y^2 - 4y + 5)$ is **[2012]**
(A) $\frac{325}{36}$ (B) $\frac{36}{325}$
(C) $\frac{13}{25}$ (D) $\frac{25}{13}$

14. If a, b are positive real numbers such that the lines $ax + 9y = 5$ and $4x + by = 3$ are parallel, then the least possible value of a + b is- **[2013]**
(A) 13 (B) 12
(C) 8 (D) 6

15. Two line segments AB and CD are constrained to move along the x and y axes, respectively, in such a way that the points A, B, C, D are concyclic. If AB = a and CD = b, then the locus of the centre of the circle passing through A, B, C, D in polar coordinates is- **[2013]**
(A) $r^2 = \frac{a^2 + b^2}{4}$ (B) $r^2 \cos 2\theta = \frac{a^2 - b^2}{4}$
(C) $r^2 = 4(a^2 + b^2)$ (D) $r^2 \cos 2\theta = 4(a^2 - b^2)$

16. Consider a triangle ABC in the xy-plane with vertices A = (0, 0), B = (1, 1) and C = (9, 1). If the line x = a divides the triangle into two parts of equal area, then a equals- **[2013]**
(A) 3 (B) 3.5
(C) 4 (D) 4.5

17. The vertices of the base of an isosceles triangle lie on a parabola $y^2 = 4x$ and the base is a part of the line $y = 2x - 4$. If the third vertex of the triangle lies on the x-axis, its coordinates are **[2013]**
(A) $\left(\frac{5}{2}, 0\right)$ (B) $\left(\frac{7}{2}, 0\right)$
(C) $\left(\frac{9}{2}, 0\right)$ (D) $\left(\frac{11}{2}, 0\right)$

18. In a triangle ABC, let G denote its centroid and let M, N be points in the interiors of the segments AB, AC, respectively, such that M, G, N are collinear. If r denotes the ratio of the area of triangle AMN to the area of ABC then **[2013]**
(A) $r = \frac{1}{2}$ (B) $r > \frac{1}{2}$
(C) $\frac{4}{9} \le r < \frac{1}{2}$ (D) $\frac{4}{9} < r$

19. Let b, d > 0. The locus of all points $P(r, \theta)$ for which the line OP (where O is the origin) cuts the line $r \sin\theta = b$ in Q such that PQ = d ism **[2014]**
(A) $(r - d)\sin\theta = b$ (B) $(r \pm d)\sin\theta = b$
(C) $(r - d)\cos\theta = b$ (D) $(r \pm d)\cos\theta = b$

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20. Let C be the circle $x^2 + y^2 = 1$ in the xy-plane. For each $t \geq 0$, let L_t be the line passing through (0, 1) and (t, 0). Note that L_t intersects C in two points, one of which is (0, 1). Let Q_t be the other point. As t varies between 1 and $1 + \sqrt{2}$, the collection of points Q_t sweeps out an arc on C. The angle subtended by this arc at (0, 0) is **[2014]**

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{3\pi}{8}$

21. In an ellipse, its foci and the ends of its major axis are equally spaced. If the length of its semi-minor axis is $2\sqrt{2}$, then the length of its semi-major axis is **[2014]**

(A) 4 (B) $2\sqrt{3}$

(C) $\sqrt{10}$ (D) 3

22. Let $n \geq 3$ and let $C_1, C_2, \ldots, C_n$, be circles with radii $r_1, r_2, \ldots, r_n$, respectively. Assume that C_i and C_{i+1} touch externally for $1 \leq i \leq n-1$. It is also given that the x-axis and the line $y = 2\sqrt{2}x + 10$ are tangential to each of the circles. Then $r_1, r_2, \ldots, r_n$ are in - **[2014]**

(A) an arithmetic progression with common difference $3 + \sqrt{2}$

(B) a geometric progression with common ratio $3 + \sqrt{2}$

(C) an arithmetic progression with common difference $2 + \sqrt{3}$

(D) a geometric progression with common ratio $2 + \sqrt{3}$

23. An ellipse inscribed in a semi-circle touches the circular arc at two distinct points and also touches the bounding diameter. Its major axis is parallel to the bounding diameter. When the ellipse has the maximum possible area, its eccentricity is - **[2014]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

24. Let $S = \{(a, b) | a, b \in Z, 0 \leq a, b \leq 18\}$. The number of lines in R^2 passing through (0, 0) and exactly one other point in S is - **[2014]**

(A) 16 (B) 22

(C) 28 (D) 32

25. The number of ordered pairs (x, y) of real numbers that satisfy the simultaneous equations $x + y^2 = x^2 + y = 12$ is **[2015]**

(A) 0 (B) 1

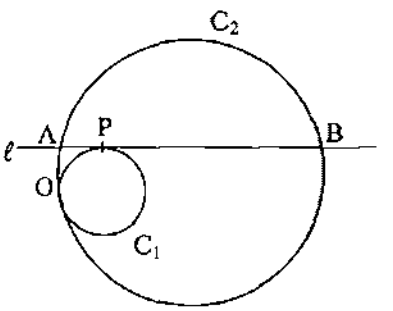
(C) 2 (D) 4

26. Suppose OABC is a rectangle in the xy plane where O is the origin and A, B lie on the parabola $y = x^2$. Then C must lie on the curve - **[2015]**

(A) $y = x^2 + 2$ (B) $y = 2x^2 + 1$

(C) $y = x^2 + 2$ (D) $y = -2x^2 + 1$

27. Circles C_1 and C_2, of radii r and R respectively, touch each other as shown in the figure. The line ℓ, which is parallel to the line joining the centres of C_1 and C_2, is tangent to C_1 at P and intersects C_2 at A, B. If $R^2 = 2r^2$, then $\angle AOB$ equals - **[2015]**



(A) $22\frac{1}{2}°$ (B) 45°

(C) 60° (D) $67\frac{1}{2}°$

28. A mirror in the first quadrant is in the shape of a hyperbola whose equation is $xy = 1$. A light source in the second quadrant emits a beam of light that hits the mirror at the point (2, 1/2). If the reflected ray is parallel to the y-axis, the slope of the incident beam is - **[2015]**

(A) $\frac{13}{8}$ (B) $\frac{7}{4}$

(C) $\frac{15}{8}$ (D) 2

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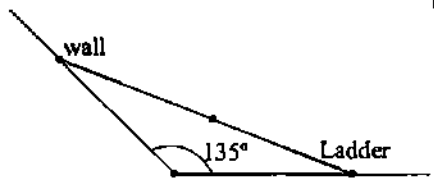
29. Suppose the tangent to the parabola $y = x^2 + px + q$ at (0, 3) has slope –1. Then $p + q$ equals **[2016]**
(A) 0 (B) 1 (C) 2 (D) 3

30. Let O = (0, 0); let A and B be points respectively on x-axis and y-axis such that $\angle OBA = 60°$. Let D be a point in the first quadrant such that OAD is an equilateral triangle. Then the slope of DB is - **[2016]**
(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

31. Suppose the parabola $(y - k)^2 = 4(x - h)$, with vertex A, passes through O = (0, 0) and L = (0, 2). Let D be an end point of the latus rectum. Let the y-axis intersect the axis of the parabola at P. Then $\angle PDA$ is equal to **[2016]**
(A) $\tan^{-1}\frac{1}{19}$ (B) $\tan^{-1}\frac{2}{19}$
(C) $\tan^{-1}\frac{4}{19}$ (D) $\tan^{-1}\frac{8}{19}$

32. In a circle with centre O, suppose A, P, B are three points on its circumference such that P is the mid-point of minor arc AB. Suppose when $\angle AOB = \theta$, $\frac{\text{area}(\Delta AOB)}{\text{area}(\Delta APB)} = \sqrt{5} + 2$.
If $\angle AOB$ is doubled to 2θ, then the ratio $\frac{\text{area}(\Delta AOB)}{\text{area}(\Delta APB)}$ is **[2016]**
(A) $\frac{1}{\sqrt{5}}$ (B) $\sqrt{5} - 2$
(C) $2\sqrt{3} + 3$ (D) $\frac{\sqrt{5} - 1}{2}$

33. A wall is inclined to the floor at an angle of 135°. A ladder of length ℓ is resting on the wall. As the ladder slides down, its mid-point traces an arc of an ellipse. Then the area of the ellipse is **[2016]**



(A) $\frac{\pi\ell^2}{4}$ (B) $\pi\ell^2$
(C) $4\pi\ell^2$ (D) $2\pi\ell^2$

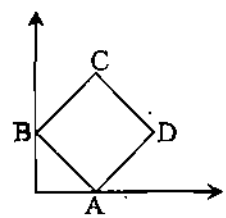
34. Let BC be a fixed line segment in the plane. The locus of a point A such that the triangle ABC is isosceles, is (with finitely many possible exceptional points) **[5 Nov, 2017]**
(A) a line
(B) a circle
(C) the union of a circle and a line
(D) the union of two circles and a line

35. Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$ be two points in the plane with integer coordinates. Which one of the following is not a possible value of the distance between A and B ? **[5 Nov, 2017]**
(A) $\sqrt{65}$ (B) $\sqrt{74}$
(C) $\sqrt{83}$ (D) $\sqrt{97}$

36. How many different (mutually noncongruent) trapeziums can be constructed using four distinct side lengths from the set {1,3,4,5,6}? **[5 Nov, 2017]**
(A) 5 (B) 11
(C) 15 (D) 30

37. Let R be the region of the disc $x^2+y^2 \le 1$ in the first quadrant. Then the area of the largest possible circle contained in R is **[5 Nov, 2017]**
(A) $\pi(3 - 2\sqrt{2})$ (B) $\pi(4 - 3\sqrt{2})$
(C) $\frac{\pi}{6}$ (D) $\pi(2\sqrt{2} - 2)$

38. Consider a rigid square ABCD as in the figure with A and B on the x and y axis respectively. **[19 Nov, 2017]**



When A and B slide along their respective axes, the locus of C forms a part of
(A) a circle
(B) a parabola
(C) a hyperbola
(D) an ellipse which is not a circle

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39. An ellipse with its minor and major axis parallel to the coordinate axes passes through (0,0), (1,0) and (0,2). One of its foci lies on the y-axis. The eccentricity of the ellipse is **[19 Nov, 2017]**

(A) $\sqrt{3}-1$ (B) $\sqrt{5}-2$

(C) $\sqrt{2}-1$ (D) $\frac{\sqrt{3}-1}{2}$

40. The sound of a cannon firing is heard one second later at a position B than at position A. If the speed of sound is uniform, then **[19 Nov, 2017]**

(A) The positions A and B are foci of a hyperbola, with cannon's position on one branch of the hyperbola

(B) the position A and B are foci of an ellipse with cannon's position on the ellipse

(C) One of the positions A,B is focus of a parabola with cannon's position on the parabola

(D) It is not possible to describe the positions of A, B and the cannon with the given information

41. Let AB be the latus rectum of the parabola $y^2 = 4ax$ in the xy-plane. Let T be the region bounded by the finite arc AB of the parabola and the line segment AB. A rectangle PQRS of maximum possible area is inscribed in T with P, Q on line AB, and R, S on arc AB. Then area(PQRS)/area(T) equals **[19 Nov, 2017]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{3}}$

42. An ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$ and the parabola $x^2 = 4(y + b)$ are such that the two foci of the ellipse and the end points of the latus rectum of parabola are the vertices of a square. The eccentricity of the ellipse is **[19 Nov, 2017]**

(A) $\frac{1}{\sqrt{13}}$ (B) $\frac{2}{\sqrt{13}}$

(C) $\frac{1}{\sqrt{11}}$ (D) $\frac{2}{\sqrt{11}}$

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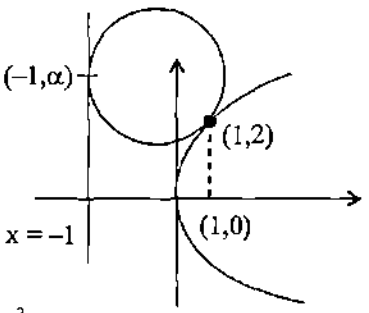
ANSWERS

1. (C)	2. (C)	3. (B)	4. (B)	5. (A)	6. (C)	7. (A)
8. (B)	9. (C)	10. (B)	11. (C)	12. (C)	13. (A)	14. (B)
15. (B)	16. (A)	17. (C)	18. (C)	19. (B)	20. (B)	21. (C)
22. (D)	23. (D)	24. (*)	25. (D)	26. (A)	27. (B)	28. (C)
29. (C)	30. (D)	31. (B)	32. (A)	33. (A)	34. (D)	35. (C)
36. (B)	37. (A)	38. (D)	39. (C)	40. (A)	41. (D)	42. (B)

SOLUTIONS

1.



$y^2 = 4x$

$2y\frac{dy}{dx} = 4$

$m_T = \frac{2}{y} = \frac{2}{2} = 1$

Circle $\rightarrow S + \lambda L = 0$

$(x+1)^2 + (y-\alpha)^2 + \lambda(x+1) = 0$ ....(1)

differentiate

$2(x+1) + 2(y-\alpha)\frac{dy}{dx} + \lambda = 0$

$x = 1, y = 2$

$4 + 2(2-\alpha)m_T + \lambda = 0$

$\lambda = 2\alpha - 8$ ....(2)

(1, 2) satisfies eq.(1)

$2^2 + (2-\alpha)^2 + 2\lambda = 0$

$\alpha^2 - 4\alpha + 8 + 2(2\alpha - 8) = 0$

$\alpha^2 = 8$

$\alpha = 2\sqrt{2}$

2. $P \rightarrow (a\cos\theta, b\sin\theta)$

$G \rightarrow \left(\frac{\Sigma x_i}{3}, \frac{\Sigma y_i}{3}\right)$

$F_1 \rightarrow (ae, 0)$ $F_2 \rightarrow (-ae, 0)$

$h = \frac{a\cos\theta + ae - ae}{3}$; $\cos\theta = \frac{3h}{a}$

$k = \frac{b\sin\theta}{3}$; $\sin\theta = \frac{3k}{b}$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{9h^2}{a^2} + \frac{9k^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{(a^2/9)} + \frac{y^2}{(b^2/9)} = 1}$

(Ellipse)

3. $y = ax + b$ and $\frac{x^2}{a} + \frac{y^2}{b} = 1$

slope = a

for the line, y intercept = b

Fig.1 for line $\rightarrow a < 0, b > 0$

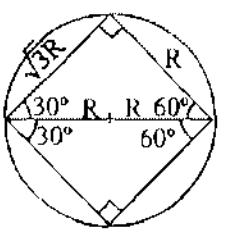
Hence the other fig. cannot be an ellipse

Fig.2 $a > 0, b < 0$

Hence the fig. is a hyperbola

Similarly you can check rest 2 options

4.



$A = 2 \times \frac{1}{2} \times \sqrt{3}\,R \times R$

$= \sqrt{3}\,R^2$

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5. $x^2 + 2x \sin(xy) + 1 = 0$

$2\sin(xy) = -\left(x + \frac{1}{x}\right)$

R.H.S. ≥ 2 or ≤ -2

$\Downarrow$ $\Downarrow$

L.H.S. = R.H.S. = 2 $\sin(xy) = -1$

$\Downarrow$ $(x = 1)$

$x = -1$

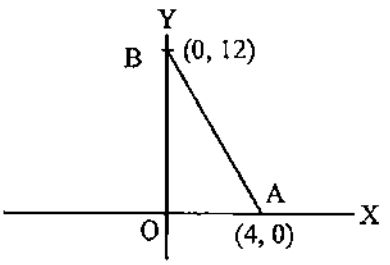
$\sin(-y) = 1$ $\sin y = -1$

$\sin y = -1$

$y = 2n\pi - \frac{\pi}{2}, n \in I$

Hence pair of straight lines.

6.



$\frac{1}{2}\begin{vmatrix} 1 & x & y \\ 1 & 0 & 12 \\ 1 & 4 & 0 \end{vmatrix} = \pm 18$

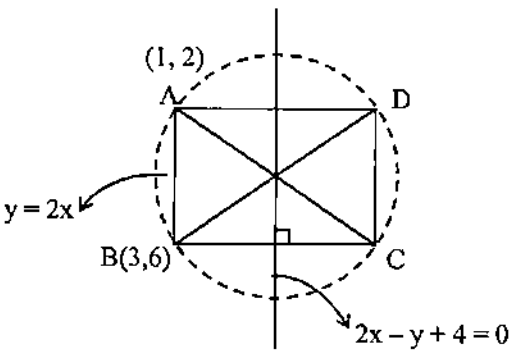
$\Rightarrow 1(-48) - x(-12) + y(4) = \pm 36$

$\Rightarrow 12x + 4y - 48 = \pm 36$

$\Rightarrow 3x + y - 12 = \pm 9$

$\Rightarrow (3x + y - 12)^2 = 81$

7.



Slope of AB $= \frac{4}{2} = 2$

Slope of BC $= -\frac{1}{2}$

$\ell(AB) = \sqrt{4+16} = 2\sqrt{5}$

distance between $2x - y + 4 = 0$ & $2x - y = 0$

$\Rightarrow \frac{4}{\sqrt{5}}$

Area $= 2\sqrt{5}.\frac{8}{\sqrt{5}} = 16$

8. Any normal

$y = mx - 2am - am^3$ Here $a = 3/2$

through $(\lambda, 0)$

$0 = m\lambda - 2am - am^3$

$m = 0, \quad \lambda = 2a + am^2$

$m^2 = \frac{\lambda}{a} - 2 > 0$

$\lambda > 2a \Rightarrow \lambda > 3$

9. Let roots $\alpha - d, \alpha, \alpha + d$

Sum $3\alpha = -a \Rightarrow \alpha = -\frac{a}{3}$

Product

$\alpha(\alpha^2 - d^2) = -a$

$\alpha^2 - d^2 = 3$

pair product $b = \alpha^2 - \alpha d + \alpha^2 + \alpha d + \alpha^2 - d^2$

$b = 2\alpha^2 + 3$

$b - 3 = \frac{2}{9}\ a^2$

$\Rightarrow$ Locus $x^2 = \frac{9}{2}(y - 3)$ Parabola

10. Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Any tangent $\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$

y intercept $= 5 \Rightarrow \sin\theta = \frac{3}{5}; \ \theta \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow \cos\theta = -\frac{4}{5}$

tangent $\Rightarrow -\frac{x}{5} + \frac{y}{5} = 1$

$\Rightarrow$ slope $= 1$

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11. $(x-3)^2+(y-p)^2=9-17+p^2$

Director circle is

$(x-3)^2+(y-p)^2=2(p^2-8)$

Passes through (0, 0)

$9+p^2=2p^2-16$

$\Rightarrow p^2=25 \Rightarrow p=\pm 5 \Rightarrow |p|=5$

12. $ex^2+\pi y^2-2e^2x-2\pi^2y+e^3+\pi^3=\pi e$

$e(x^2-2ex+e^2)+\pi(y^2-2\pi y+\pi^2)=\pi e$

$$\frac{(x-e)^2}{\pi}+\frac{(y-\pi)^2}{e}=1$$

$a^2=\pi \Rightarrow a=\sqrt{\pi}$ $\quad\quad \pi > e$

$PS_1+PS_2=2a$ $\quad\quad$ Major axis is $\|$ to axis

$PS_1+PS_2=2\sqrt{\pi}$

13. $4x^2+9y^2-8x-36y+15=0$

$4(x^2-2x)+9(y^2-4y)=-15$

$4(x^2-2x+1)+9(y^2-4y+4)=-15+4+36$

$4(x-1)^2+9(y-2)^2=25$

$$\frac{(x-1)^2}{\left(\frac{5}{2}\right)^2}+\frac{(y-2)^2}{\left(\frac{5}{3}\right)^2}=1 \quad\quad ...(1)$$

x^2-2x+y^2-4y+5

$=(x-1)^2+(y-2)^2$

$$\text{min of } ((x-1)^2+(y-2)^2)=\frac{25}{9}$$

$$\text{max of } ((x-1)^2+(y-2)^2)=\frac{25}{4}$$

$\therefore$ min of $((x-1)^2+(y-2)^2)+$ max of $((x-1)^2+(y-2)^2)$

$$=\frac{25}{9}+\frac{25}{4}=\frac{325}{36}$$

14. $$\therefore \frac{a}{4}=\frac{9}{b}$$

$\Rightarrow ab=36$

We know

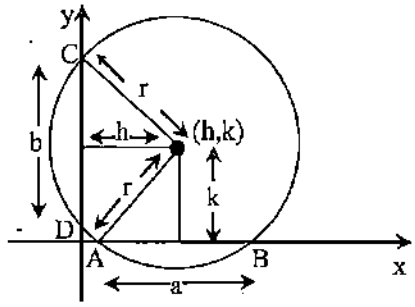
A.M. $\geq$ G.M.

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$a+b \geq 12$

$(a+b)_{min}=12$

15.



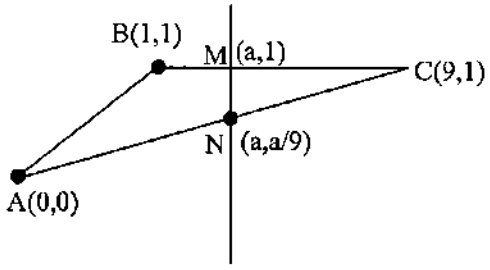
Equating radius

$$\frac{a^2}{4}+k^2=\frac{b^2}{4}+h^2$$

$$x^2-y^2=\left(\frac{a^2-b^2}{4}\right)$$

$$r^2(\cos 2\theta)=\frac{a^2-b^2}{4}$$

16.



area $\Delta ABC=\frac{1}{2}\,8\times 1=4$

area $\Delta CMN=\frac{1}{2}(9-a)\left(1-\frac{a}{9}\right)=2$

$$\Rightarrow 9-a-a+\frac{a^2}{9}=4$$

$$\Rightarrow \frac{a^2}{9}-2a+5=0$$

$\Rightarrow a^2-18a+45=0$

$\Rightarrow (a-15)(a-3)=0$

$\Rightarrow a=15, 3$

$\Rightarrow a=3$

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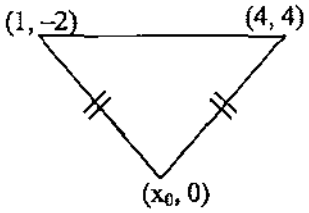
17. Let point satisfying parabola and line is $(t^2, 2t)$ which is vertex of triangle.

$2t^2 - 2t - 4 = 0$

$t^2 - t - 2 = 0$

$t = -1, t = 2$



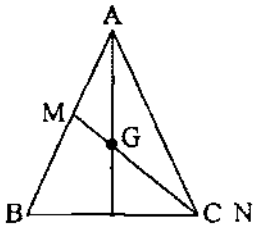
$(x_0 - 1)^2 + 4 = (x_0 - 4)^2 + 16$

$-2x_0 + 1 + 4 = -8x_0 + 16 + 16$

$6x_0 = 27$

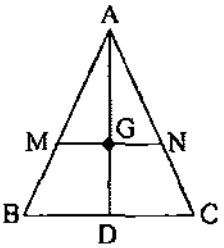
$x_0 = \frac{27}{6} = \frac{9}{2}$

18.



$ar(\Delta AMN) = \frac{\Delta}{2}$

$\Rightarrow r_{max} < \frac{1}{2}$



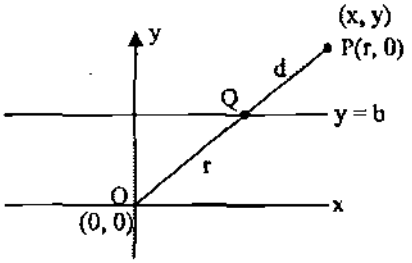
$\frac{AG}{AD} = \frac{2}{3}$

$\frac{ar(\Delta AMN)}{ar(\Delta ABC)} = \frac{4}{9}$

$= \frac{2^2}{3^2} = r_{min}$

$\frac{4}{9} \le r < \frac{1}{2}$

19.



Equation of OP is $y = x \tan\theta$

Point Q is $(b \cot\theta, b)$

$\therefore$ Point P is $y = b \pm d \sin\theta$

$r \sin\theta = b \pm d \sin\theta$

$(r \mp d) \sin\theta = b$

20. $x^2 + y^2 = 1$

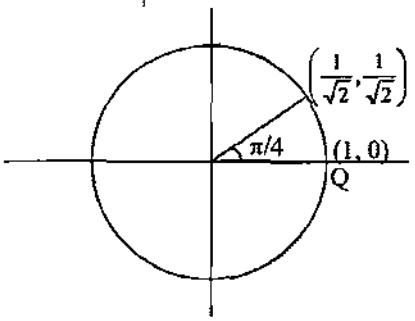
$L_t \quad \frac{x}{t} + \frac{y}{1} = 1$

$y = 1 - \frac{x}{t}$

$x^2 + 1 + \frac{x^2}{t^2} - \frac{2x}{t} = 1$

$x^2\left(1 + \frac{1}{t^2}\right) - \frac{2x}{t} = 0$

$x = 0,$	$x\left(1 + \frac{1}{t^2}\right) = \frac{2}{t}$
$y = 1$	$x = \frac{2t}{t^2+1}; \; y = 1 - \frac{2}{t^2+1}$
$(0, 1)$	$y = \frac{t^2-1}{t^2+1}$
	$Q_t\left(\frac{2t}{1+t^2}, \frac{t^2-1}{t^2+1}\right)$
$1 \le t \le 1 + \sqrt{2}$	$t = \tan\theta, \; Q_t(\sin 2\theta, -\cos 2\theta)$
$\theta \in \left(45°, 67\frac{1°}{2}\right)$	lies on circle C



so angle at centre $= \frac{\pi}{4}$

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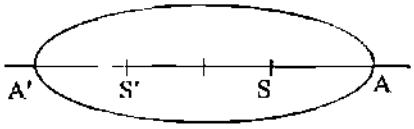
21.



$A'S' = SS' = SA$

$2ae = a - ae$

$3ae = a$

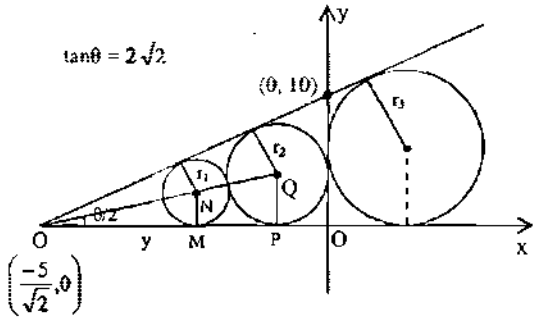
$e = 1/3$

$1 - \frac{b^2}{a^2} = \frac{1}{9} \Rightarrow \frac{b^2}{a^2} = \frac{8}{9}$

$\Rightarrow \frac{8}{a^2} = \frac{8}{9}$

$\Rightarrow a = 3$

22.



$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

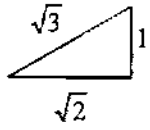
$2\sqrt{2} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$\sqrt{2}\tan^2\frac{\theta}{2} + \tan\theta - \sqrt{2} = 0$

$\tan\frac{\theta}{2} = \frac{-1\pm\sqrt{1+8}}{2\sqrt{2}}$

$= \frac{-1\pm 3}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$ or $-\sqrt{2}$

$\therefore \tan\frac{\theta}{2} = \frac{1}{\sqrt{2}}$



In ΔOMN,

$\sin\frac{\theta}{2} = \frac{r_1}{ON}$

$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{\sqrt{3}}$

$\Rightarrow ON = \sqrt{3}r_1$

In ΔOPQ,

$\sin\frac{\theta}{2} = \frac{r_2}{ON + r_1 + r_2}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{\sqrt{3}r_1 + r_1 + r_2}$

$\Rightarrow \sqrt{3}r_1 + r_1 + r_2 = \sqrt{3}r_2$

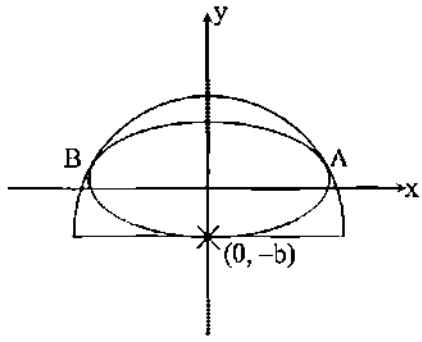
$\Rightarrow r_1(\sqrt{3}+1) = r_2(\sqrt{3}-1)$

$\Rightarrow \frac{r_2}{r_1} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}+1)^2}{2}$

$= 2 + \sqrt{3}$

23.



Let ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and circle $x^2 + (y+b)^2 = r^2$ {let radius = r}

put $x^2 = a^2 - \frac{a^2y^2}{b^2}$

in circle $a^2 - \frac{a^2y^2}{b^2} + (y+b)^2 = r^2$

$\Rightarrow \left(1 - \frac{a^2}{b^2}\right)y^2 + 2by + (a^2 + b^2 - r^2) = 0$

$D = 0 \Rightarrow r^2 = \frac{a^4}{a^2 - b^2}$

$\Rightarrow b = a\sqrt{1 - \frac{a^2}{r^2}}$

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$$\text{Area} = \Delta = \pi ab = \pi a^2\sqrt{1-\frac{a^2}{r^2}}$$

$$\frac{d\Delta}{da} = 0 \Rightarrow a^2 = \frac{2r^2}{3}$$

$$\Rightarrow a = \sqrt{\frac{2}{3}}\,r$$

$$\therefore b = a\sqrt{1-\frac{2}{3}} = \frac{a}{\sqrt{3}}$$

$$\Rightarrow e = \sqrt{\frac{2}{3}}$$

25. $x + y^2 = x^2 + y = 12$

Curve (1)

$x + y^2 = 12$

$y^2 = -(x - 12)$

Intersection on x-axis (12, 0)

Intersection on x-axis = $(\pm\sqrt{12}, 0)$

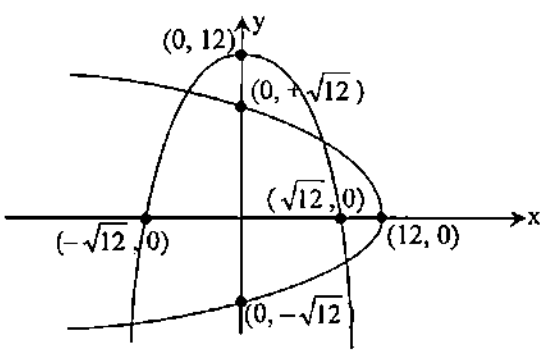
Curve (2)

$x^2 + y = 12$

$x^2 = -(y - 12)$

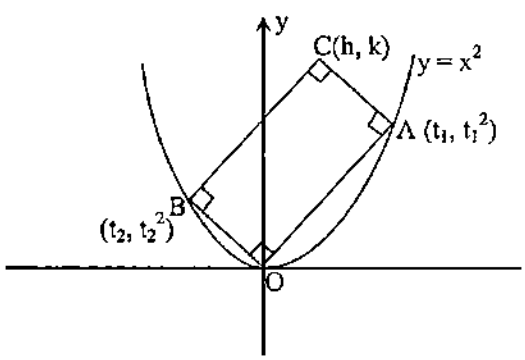
Intersection on y-axis $(0, \pm\sqrt{12})$

Intersection on y-axis = (0, 12)



Four intersections

26.



$\because$ OB ⊥ OA

So, $t_1t_2 = -1$

Now $\frac{h}{2} = \frac{t_1 + t_2}{2}$

$t_1 + t_2 = h$ ...(1)

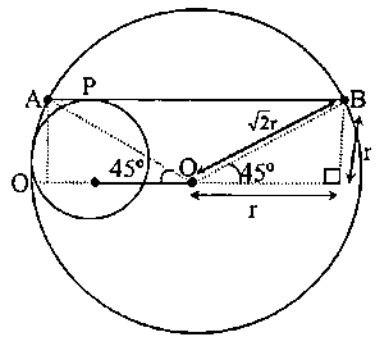
also $t_1^2 + t_2^2 = k$

$(t_1 + t_2)^2 - 2t_1t_2 = k$

$h^2 + 2 = k$

Locus is $x^2 + 2 = y$

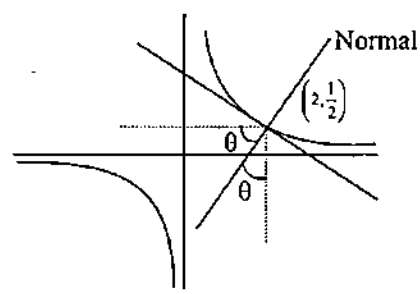
27.



Chose AB subtend 90° at centre.

so that AB subtend 45° at O (circumference of circle)

28.



Curve $y = \frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{x^2}$

Slope of tangent $= -\frac{1}{4}$ at $\left(2, \frac{1}{2}\right)$

Slope of normal = 4

where foot of normal is $\left(2, \frac{1}{2}\right)$

Let slope at incident beam is 'm'

$$\left|\frac{4-m}{1+4m}\right| = \left|\frac{\infty - 4}{1 - 4.\infty}\right|$$

$$\frac{4-m}{1+4m} = \pm\frac{1}{4}$$

$$m = \frac{15}{8}$$

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29. (0, 3) lies on the curve

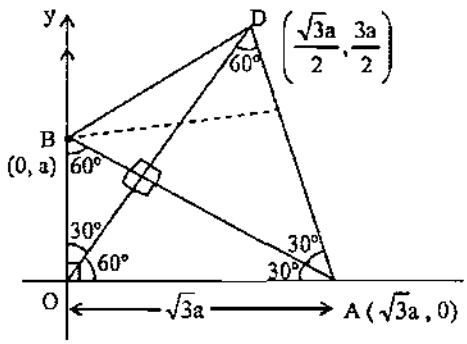
So $q = 3$

Now, $\frac{dy}{dx} = 2x + p$;

$\left(\frac{dy}{dx}\right)_{(0,3)} = p = -1$

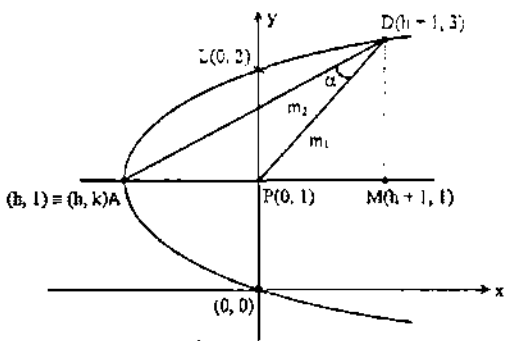
$\therefore p + q = -1 + 3 = 2$

30.



$M_{BD} = \frac{1}{\sqrt{3}}$

31.



Curve, S : $(y-k)^2 = 4(x-h)$

LLR = 4; Clearly k = 1;

⇒ A(h, 1) & 'M' is focus (h + 1, 1)

So D (h +1, 3)

$S_{(0,0)} = 0 \Rightarrow k^2 = -4h$

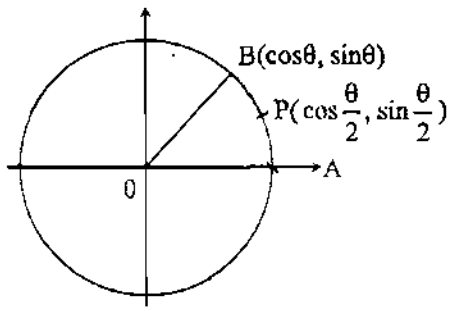
$\Rightarrow h = \frac{-1}{4} \Rightarrow D\left(\frac{3}{4}, 3\right)$

Now ; $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{8}{3} - 2}{1 + \frac{8}{3} \times 2}\right| = \frac{2}{19}$

where, $m_1 = \frac{3-1}{\frac{3}{4} - 0} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$

$\Rightarrow \quad m_2 = \frac{3-1}{1} = 2$

32.



$\frac{\Delta(AOB)}{\Delta APB} = 2 + \sqrt{5}$

$\frac{\frac{1}{2} \cdot 1 \cdot \sin\theta}{\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 1 \\ \cos\theta & \sin\theta & 1 \end{vmatrix}} = 2 + \sqrt{5}$ on solving

$\frac{\cos\frac{\theta}{2}}{1 - \cos\frac{\theta}{2}} = 2 + \sqrt{5}$

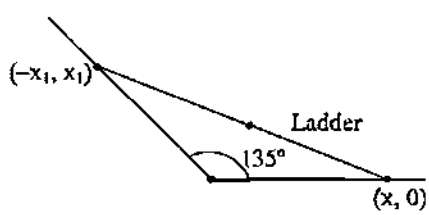
$\Rightarrow \cos\frac{\theta}{2} = \frac{1 + \sqrt{5}}{4}$

$\Rightarrow$ So $\cos\theta = \frac{\sqrt{5} - 1}{4}$

If $\theta \to 2\theta$

$\Rightarrow \frac{\Delta AOB}{\Delta APB} = \frac{\cos\theta}{1 - \cos\theta} = \frac{1}{\sqrt{5}}$

33.



Mid point (h, k) = $\left(\frac{x - x_1}{2}, \frac{x_1}{2}\right)$

Now $(x + x_1)^2 - x_1^2 = \ell^2$

As $2h - 4k = x + x_1$, $2k = x$,

So required locus is

$4(h + 2k)^2 + 4k^2 = \ell^2$

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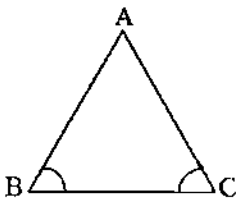
$\Rightarrow h^2 + 5k^2 + 4hk = \frac{\ell^2}{4}$

$\Rightarrow x^2 + 5y^2 + 4xy = \frac{\ell^2}{4}$

Whose area is $\frac{\pi\ell^2}{4}$

34.

Case (i) :

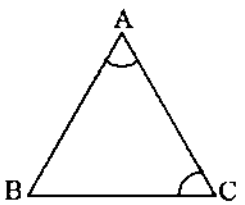


If $\angle B = \angle C$

locus of A is $\perp$ bisector of BC

So it is straight line

Case (ii) :



If $\angle A = \angle C$

BC fixed B(a, 0), C(0, a)

BC = AB

So, $(x-a)^2 + y^2 = 2a^2$

Circle

Case (iii) :

$\angle A = \angle B$

AC = BC

$\sqrt{h^2 + (k-a)^2} = \sqrt{2a^2}$

$x^2 + (y-a)^2 = 2a^2$

also a circle

So union of two circle and a line.

35. $AB = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

Square + Square = $\sqrt{65}$ possible when

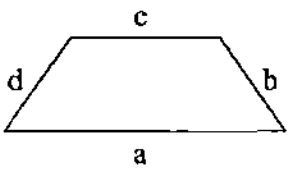
$= 64 + 1$

$\sqrt{74} = 49 + 25$

$\sqrt{97} = 81 + 16$

But $\sqrt{83}$ not possible

36.



$|a - c| < b + d < a + c$

(a, c) (b, d) (1, 3) (5, 6)

(1, 3) (4, 5)

and (1, 3) (4, 6)

Now make different combination. Total of 11 combination are possible.

37. Required equation of circle

$(x-h)^2 + (y-h)^2 = h^2$

Both circle touch internally

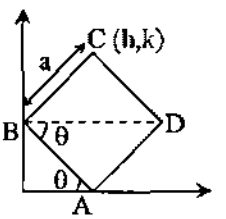
$C_1C_2 = |r_1 - r_2|$

$\sqrt{h^2 + h^2} = |h - 1|$

Solve this $h = \sqrt{2} - 1$

Area $\pi(\sqrt{2} - 1)^2 = \pi(3 - 2\sqrt{2})$

38.



C(h, k)

$h = a\sin\theta \Rightarrow \sin\theta = \frac{h}{a}$ ...(i)

$k = a\sin\theta + a\cos\theta$

$k = h + a\cos\theta$

$\frac{k-h}{a} = \cos\theta$ ....(2)

$\sin^2\theta + \cos^2\theta = 1$

$\frac{h^2}{a^2} + \frac{(k-h)^2}{a^2} = 1$

$h^2 + (k-h)^2 = a^2$

locus

$x^2 + (y-x)^2 = a^2$

solving we get

$x^2 + y^2 + x^2 - 2xy = a^2$

$2x^2 + y^2 - 2xy = a^2$

check $h^2 - ab < 0$

ellipse

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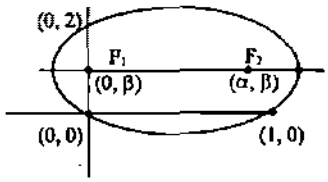
39.



Centre is

$$\frac{\left(x-\frac{\alpha}{2}\right)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$$

pass through (0, 0) (0, 2) & (1, 0)

Distance between F_1 & $F_2 \Rightarrow \alpha = 2ae$

$$\alpha^2 = 4a^2e^2$$

Pass through (0, 0)

$$\Rightarrow \frac{\alpha^2}{4a^2}+\frac{\beta^2}{b^2}=1$$

$$\frac{\alpha^2}{4a^2}=e^2$$

Pass through (0, 2)

$$\frac{\alpha^2}{4a^2}+\frac{(2-\beta)^2}{b^2}=1$$

from these two $\Rightarrow \beta = 1$

Use properties $PF_1 + PF_2 = 2a$

$F_1(0,1)$ $F_2(\alpha, 1)$ $P(0, 2)$

$$1+\sqrt{\alpha^2+1}=2a$$

$F_1(0,1)$ $F_2(\alpha, 1)$ $P(1, 0)$

$$\sqrt{1+1}+\sqrt{(\alpha-1)^2+1}=2a$$

From these two

$$\alpha = -2\alpha + 2\sqrt{2}a$$

put α in any of above two equations

$$a=\frac{\sqrt{2}+1}{2} \Rightarrow 2a=\sqrt{2}+1$$

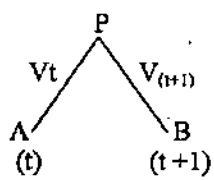
$\alpha = 1$

$\alpha = 2ae$

$\alpha = 1, 2a = \sqrt{2}+1$

find $e = \sqrt{2}$

40.



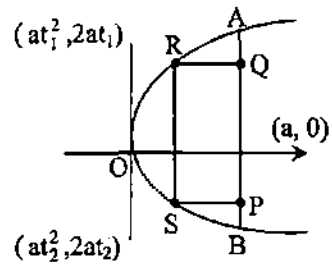
PB – PA = Vt + v – vt

PB – PA = V (const)

locus of P is Hyperbola

A & B are foci of Hyperbola

41.



$$T = \int_0^a 2\sqrt{a}\ \sqrt{x}\ .\ dx = \frac{8a^2}{3}$$

$t_1 = t_2$

Area of PQRS

$$= 2a\,|t_1 - t_2| \times |a - at_1^2|$$

But $t_1 \cdots - t_2$

$$= 4a^2 t_1 (1 - t_1^2)$$

Differentiation with respect t_1

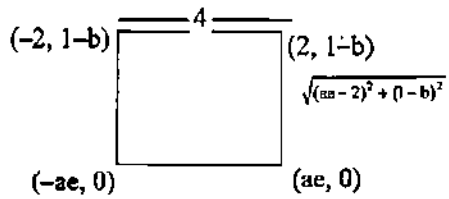
We will get $t_1^2 = \frac{1}{3}$

Now put $t_1 = \frac{1}{\sqrt{3}}$ get Area of

$$PQRS = \frac{8a^2}{3\sqrt{3}}$$

ratio Becomes $\frac{1}{\sqrt{3}}$

42.



As it is square

$ae = 2$

$$4 = \sqrt{(ae-2)^2+(1-b)^2}$$

$b = -3$	$b = 5$
$a^2 = 13$	$a^2 = 29$

(from realation $b^2 = a^2(1 - e^2)$

$ae = 2$

$$e = \frac{2}{\sqrt{13}}$$

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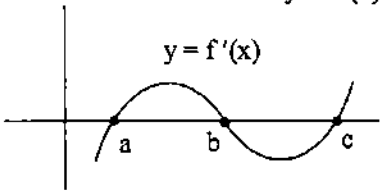
Chapter

4 Differential Calculus

1. Let $f : R \to R$ be a differentiable function such that $f(a) = 0 = f(b)$ and $f'(a)\, f'(b) > 0$ for some $a < b$. Then the minimum number of roots of $f'(x) = 0$ in the interval (a, b) is- **[2010]**
(A) 3 (B) 2 (C) 1 (D) 0

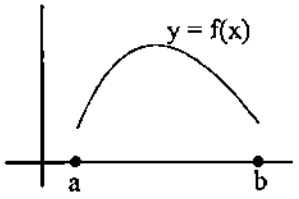
2. The roots of
$(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$ are - **[2010]**
(A) all necessarily real
(B) non-real except one positive real root
(C) non-real except three positive real roots
(D) non-real except for three real roots of which exactly one is positive

3. The figure shown below is the graph of the derivative of some function $y = f'(x)$. **[2010]**



Then-
(A) f has local minima at x = a, b and a local maximum at x = c
(B) f has local minima at x = b, c and a local maximum at x = a
(C) f has local minima at x = c, a and a local maximum at x = b
(D) the given figure is insufficient to conclude any thing about the local minima and local maxima of f

4. The following figure shows the graph of a differentiable function y = f(x) on the interval [a, b] (not containing 0).



Let g(x) = f(x)/x which of the following is a possible graph of y = g(x) ? **[2010]**

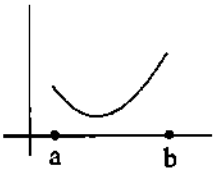


Fig.1

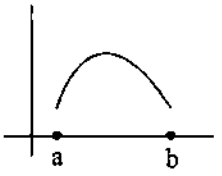


Fig.2

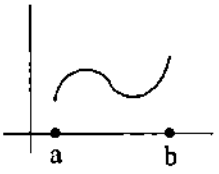


Fig.3

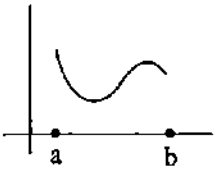


Fig.4

(A) Fig. 1 (B) Fig. 2 (C) Fig. 3 (D) Fig. 4

5. Let V_1 be the volume of a given right circular cone with O as the centre of the base and A as its apex. Let V_2 be the maximum volume of the right circular cone inscribed in the given cone whose apex is O and whose base is parallel to the base of the given cone. Then the ratio V_2/V_1 is- **[2010]**
(A) $\frac{3}{25}$ (B) $\frac{4}{9}$ (C) $\frac{4}{27}$ (D) $\frac{8}{27}$

6. Let $f(x) = \cos 5x + A\cos 4x + B\cos 3x + C\cos 2x + D\cos x + E$, and

$$T = f(0) - f\left(\frac{\pi}{5}\right) + f\left(\frac{2\pi}{5}\right) - f\left(\frac{3\pi}{5}\right) + \ldots\ldots - f\left(\frac{8\pi}{5}\right) + f\left(\frac{9\pi}{5}\right)$$

Then T **[2011]**
(A) depends on A, B, C, D, E
(B) depends on A, C, E but independent of B and D
(C) depends on B, D but independent of A, C, E
(D) is independent of A, B, C, D, E

7. Which of the following intervals is a possible domain of the function $f(x) = \log_{\{x\}}[x] + \log_{[x]}\{x\}$, where $[x]$ is the greatest integer not exceeding x and $\{x\} = x - [x]$? **[2011]**
(A) (0, 1) (B) (1, 2)
(C) (2, 3) (D) (3, 5)

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8. If $f(x) = (2011 + x)^n$, where x is a real variable and n is a positive integer, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}$ is **[2011]**

(A) $(2011)^n$ (B) $(2012)^n$
(C) $(2012)^n - 1$ (D) $n(2011)^n$

9. The minimum distance between a point on the curve $y = e^x$ and a point on the curve $y = \log_e x$ is - **[2011]**

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$
(C) $\sqrt{3}$ (D) $2\sqrt{2}$

10. Let $f(x) = x^{12} - x^9 + x^4 - x + 1$. Which of the following is true ? **[2011]**

(A) f is one-one
(B) f has a real root
(C) f' never vanishes
(D) f takes only positive values

11. The maximum possible value of $x^2 + y^2 - 4x - 6y$, x, y real, subject to the condition $|x + y| + |x - y| = 4$ **[2011]**

(A) is 12 (B) is 28
(C) is 72 (D) does not exist

12. Let $f(x) = \frac{\sin(x-a) + \sin(x+a)}{\cos(x-a) - \cos(x+a)}$, then **[2012]**

(A) $f(x + 2\pi) = f(x)$ but $f(x + \alpha) \neq f(x)$ for any $0 < \alpha < 2\pi$
(B) f is a strictly increasing function
(C) f is strictly decreasing function
(D) f is a constant function

13. The mid- point of the domain of the function $f(x) = \sqrt{4 - \sqrt{2x+5}}$ for real x is **[2012]**

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$
(C) $\frac{2}{3}$ (D) $-\frac{2}{5}$

14. Let $f : R \to R$ be the function
$f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_3)(x - a_1)$
with $a_1, a_2, a_3 \in R$. Then $f(x) \geq 0$ if and only if **[2012]**

(A) At least two of a_1, a_2, a_3 are equal
(B) $a_1 = a_2 = a_3$
(C) a_1, a_2, a_3 are all distinct
(D) a_1, a_2, a_3, are all positive and distinct

15. A polynomial P(x) with real coefficients has the property that $P''(x) \neq 0$ for all x. Suppose $P(0) = 1$ and $P'(0) = -1$. What can you say about P(1) ? **[2012]**

(A) $P(1) \geq 0$ (B) $P(1) \neq 0$
(C) $P(1) \leq 0$ (D) $-\frac{1}{2} < P(1) < \frac{1}{2}$

16. Define a sequence (a_n) by
$a_1 = 5$, $a_n = a_1 a_2 \ldots a_{n-1} + 4$ for $n > 1$. Then $\lim_{n\to\infty} \frac{\sqrt{a_n}}{a_{n-1}}$ **[2012]**

(A) equals $\frac{1}{2}$ (B) equals 1
(C) equals $\frac{2}{5}$ (D) does not exist

17. Let $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$. Let
$f^1(x) = f(x), f^2(x) = f(f(x))$ and generally
$f^n(x) = f(f^{n-1}(x))$ for $n > 1$
Let $P = f^1(2) f^2(3) f^3(4) f^4(5)$
Which of the following is a multiple of P ? **[2012]**

(A) 125 (B) 375
(C) 250 (D) 147

18. Let $f : R \to R$ be a function such that $\lim_{x\to\infty} f(x) = M > 0$. Then which of the following is false ? **[2013]**

(A) $\lim_{x\to\infty} x \sin(1/x) f(x) = M$
(B) $\lim_{x\to\infty} (f(x)) = \sin M$
(C) $\lim_{x\to\infty} x \sin(e^{-x}) f(x) = M$
(D) $\lim_{x\to\infty} \frac{\sin x}{x} . f(x) = 0$

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19. Let $f(x = \sqrt{2-x-x^2}$ and $g(x) = \cos x$. Which of the following statements are true ? **[2013]**

(I) Domain of $f((g(x))^2)$ = Domain of $f(g(x))$

(II) Domain of $f(g(x)) + g(f(x))$ = Domain of $g(f(x))$

(III) Domain of $f(g(x))$ = Domain of $g(f(x))$

(IV) Domain of $g((f(x))^3)$ = Domain of $f(g(x))$

(A) Only (I)

(B) Only (I) and (II)

(C) Only (III) and (IV)

(D) Only (I) and (IV)

20. Let $f(x)$ be a non-constant polynomial with real coefficients such that $f\left(\frac{1}{2}\right) = 100$ and $f(x) \le 100$ for all real x. Which of the following statements is NOT necessarily true ? **[2013]**

(A) The coefficient of the highest degree term in $f(x)$ is negative

(B) $f(x)$ has at least real roots

(C) If $x \neq \frac{1}{2}$ then $f(x) < 100$

(D) At least one of the coefficients of $f(x)$ is bigger than 50

21. Consider the function

$$f(x) = \begin{cases} \frac{x+5}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

Then $f(f(x))$ is discontinuous **[2014]**

(A) at all real numbers

(B) at exactly two values of x

(C) at exactly one value of x

(D) at exactly three values of x

22. Let $f: [0, \pi] \to R$ be defined as

$$f(x) = \begin{cases} \sin x, & \text{if } x \text{ is irrational and } x \in [0, \pi] \\ \tan^2 x, & \text{if } x \text{ is rational and } x \in [0, \pi] \end{cases}$$

The number of points in $[0, \pi]$ at which the function f is continuous is **[2014]**

(A) 6 (B) 4

(C) 2 (D) 0

23. The value of the limit $\lim\limits_{x \to 0} \left(\frac{x}{\sin x}\right)^{6/x^2}$ is - **[2015]**

(A) e (B) e^{-1}

(C) $e^{-1/6}$ (D) e^6

24. Among all sectors of a fixed perimeter, choose the one with maximum area. Then the angle at the center of this sector (i.e., the angle between the bounding radii) is - **[2015]**

(A) $\frac{\pi}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2

25. Let $a > 0$ be a real number. Then the limit $\lim\limits_{x \to 2} \frac{a^x + a^{3-x} - (a^2 + a)}{a^{3-x} - a^{x/2}}$ is - **[2015]**

(A) 2 log a (B) $-\frac{4}{3}$ a

(C) $\frac{a^2+a}{2}$ (D) $\frac{2}{3}(1-a)$

26. Let $f(x) = \alpha x^2 - 2 + \frac{1}{x}$ where α is a real constant. The smallest α for which $f(x) \ge 0$ for all $x > 0$ is - **[2015]**

(A) $\frac{2^2}{3^3}$ (B) $\frac{2^3}{3^3}$

(C) $\frac{2^4}{3^3}$ (D) $\frac{2^5}{3^3}$

27. If $P(x)$ be a polynomial with real coefficients such that $P(\sin^2 x) = P(\cos^2 x)$, for all $x \in [0, \pi/2]$. Consider the following statements : **[2016]**

I. $P(x)$ is an even function.

II. $P(x)$ can be expressed as a polynomial in $(2x-1)^2$

I. $P(x)$ is a polynomial of even degree

Then.

(A) all are false

(B) only I and II are true

(C) only II and III are true

(D) all are true

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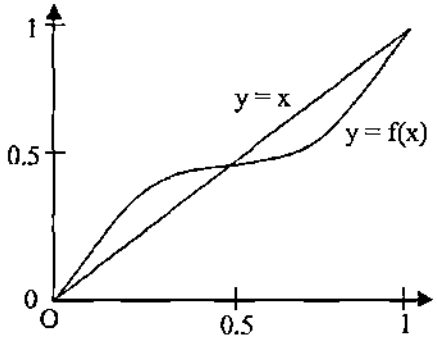
28. Number of integers k for which the equation $x^3 - 27x + k = 0$ has at least two distinct integer roots is - **[2016]**

(A) 1 (B) 2
(C) 3 (D) 4

29. The graph of the function $f(x) = x + \frac{1}{8}\sin(2\pi x)$, $0 \le x \le 1$ is shown below. Define
$f_1(x) = f(x)$, $f_{n+1}(x) = f(f_n(x))$, for $n \ge 1$



Which of the following statements are true ?

I. There are infinitely many $x \in [0, 1]$ for which $\lim_{n\to\infty} f_n(x) = 0$

II. There are infinitely many $x \in [0, 1]$ for which $\lim_{n\to\infty} f_n(x) = \frac{1}{2}$

III. There are infinitely many $x \in [0, 1]$ for which $\lim_{n\to\infty} f_n(x) = 1$

IV. There are infinitely many $x \in [0, 1]$ for which $\lim_{n\to\infty} f_n(x)$ does not exist **[2016]**

(A) I and III only (B) II only
(C) I, II, III only (D) I, II, III and IV

30. Limit $\lim_{x\to\infty} x^2 \int_0^x e^{t^3 - x^3} dt$ equals **[2016]**

(A) $\frac{1}{3}$ (B) 2 (C) ∞ (D) $\frac{2}{3}$

31. The polynomial equation
$x^3 - 3ax^2 + (27a^2 + 9)x + 2016 = 0$ has **[2016]**

(A) exactly one real root for any real a
(B) three real roots for any real a
(C) three real roots for any $a \ge 0$, and exactly one real root for any $a < 0$
(D) three real roots for any $a \le 0$, and exactly one real root for any $a > 0$

32. The range of the polynomial $p(x) = 4x^3 - 3x$ as x varies over the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is **[2016]**

(A) [–1, 1] (B) (–1, 1]
(C) (–1, 1) (D) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

33. Let f(x) be a non-negative differentiable function on $[0, \infty)$ such that $f(0) = 0$ and $f'(x) \le 2f(x)$ for all $x > 0$. Then, on $[0, \infty)$ **[2016]**

(A) f(x) is always a constant function
(B) f(x) is strictly increasing
(C) f(x) is strictly decreasing
(D) f'(x) changes sign

34. The value of the limit $\lim_{x\to-\infty}\left(\sqrt{4x^2 - x} + 2x\right)$ is **[5 Nov, 2017]**

(A) $-\infty$ (B) $-\frac{1}{4}$ (C) 0 (D) $\frac{1}{4}$

35. Let R be a relation on the set of all natural numbers given by $a\ R\ b \Leftrightarrow a$ divides b^2.
Which of the following properties does R satisfy ? **[5 Nov, 2017]**

I. Reflexivity
II. Symmetry
III. Transitivity

(A) I only (B) III only
(C) I and III only (D) I and II only

36. Let R be the set of real numbers and $f : R \to R$ be defined by $f(x) = \frac{\{x\}}{1 + [x]^2}$, where [x] is the greatest integer less than or equal to x, and $\{x\} = x - [x]$. Which of the following statements are true ?

I. The range of f is a closed interval
II. f is continuous on R.
III. f is one-one on R. **[5 Nov, 2017]**

(A) I only
(B) II only
(C) III only
(D) None of I, II and III

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37. Let $x_n = (2^n + 3^n)^{1/2n}$ for all natural numbers n. Then **[5 Nov, 2017]**

(A) $\lim_{n\to\infty} x_n = \infty$

(B) $\lim_{n\to\infty} x_n = \sqrt{3}$

(C) $\lim_{n\to\infty} x_n = \sqrt{3} + \sqrt{2}$

(D) $\lim_{n\to\infty} x_n = \sqrt{5}$

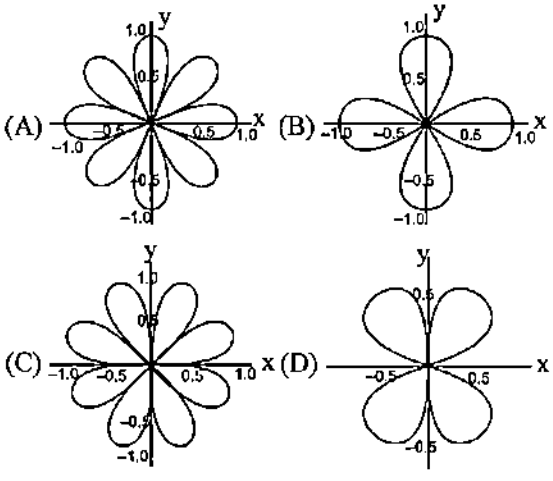
38. Consider the following parametric equation of a curve :

$x(\theta) = |\cos 4\theta| \cos\theta$

$y(\theta) = |\cos 4\theta| \sin\theta$

for $0 \le \theta \le 2\pi$

Which one of the following graphs represents the curve ? **[5 Nov, 2017]**



39. A solid hemisphere is mounted on a solid cylinder, both having equal radii. If the whole solid is to have a fixed surface area and the maximum possible volume, then the ratio of the height of the cylinder to the common radius is **[5 Nov, 2017]**

(A) 1:1 (B) 1:2

(C) 2:1 (D) $\sqrt{2}$:1

40. Let R be the set of real numbers and $f : R \to R$ be given by $f(x) = \sqrt{|x|} - \log(1 + |x|)$. We now make the following assertions:

I. There exists a real number A such that $f(x) \le A$ for all x.

II. There exists a real number B such that $f(x) \ge B$ for all x. **[5 Nov, 2017]**

(A) I is true and II is false

(B) I is false and II is true

(C) I and II both are true

(D) I and II both are false

41. The number of continuous functions $f:[0,1] \to R$ that satisfy **[5 Nov, 2017]**

(A) 0 (B) 1

(C) 2 (D) infinity

42. A sector is removed from a metallic disc and the remaining region is bent into the shape of a circular conical funnel with volume $2\sqrt{3}\,\pi$. The least possible diameter of the disc is **[19 Nov, 2017]**

(A) 4 (B) 6

(C) 8 (D) 12

43. Let $a_n = \int_{-\pi}^{\pi} |x-1| \cos nx \, dx$ for all natural numbers n. Then the sequence $(a_n)_{n\ge 0}$ satisfies **[19 Nov, 2017]**

(A) $\lim_{n\to\infty} a_n = \infty$

(B) $\lim_{n\to\infty} a_n = -\infty$

(C) $\lim_{n\to\infty} a_n$ exists and is positive

(D) $\lim_{n\to\infty} a_n = 0$

44. Let $f(x) = \log(1 + x^2)$ and A be a constant such that $\frac{|f(x) - f(y)|}{|x - y|} \le A$ for all x, y real and $x \ne y$. Then the least possible value of A is **[19 Nov, 2017]**

(A) equal to 1

(B) bigger than 1 but less than 2

(C) bigger than 0 but less than 1

(D) bigger than 2

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ANSWERS

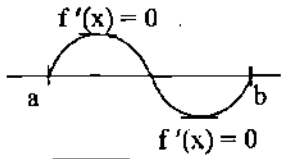
1. (B)	2. (B)	3. (C)	4. (B)	5. (C)	6. (C)	7. (C)
8. (C)	9. (B)	10. (D)	11. (B)	12. (D)	13. (B)	14. (B)
15. (B)	16. (C)	17. (B)	18. (C)	19. (B)	20. (C)	21. (B)
22. (B)	23. (A)	24. (D)	25. (D)	26. (D)	27. (C)	28. (B)
29. (B)	30. (A)	31. (A)	32. (C)	33. (A)	34. (D)	35. (A)
36. (D)	37. (B)	38. (A)	39. (A)	40. (B)	41. (B)	42. (B)
43. (D)	44. (A)					

SOLUTIONS

1. $f'(a) \cdot f'(b) > 0$

so either both are positive or both are negative

$f(a) = f(b) = 0$



2 roots

2. $(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$

$f(x) = (x-41)^{49} + (x-49)^{41} + (x-2009)^{2009}$

$f'(x) = 49(x-41)^{48} + 41(x-49)^{40} + 2009(x-2009)^{48} > 0$

hence f(x) will cut x-axis only once.

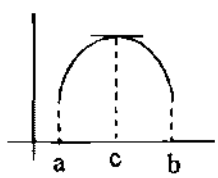
1 real root

3. $f'(a) = f'(b) = f'(c) = 0$

$\left.\begin{matrix} f'(a^-) < 0 & f'(a^+) > 0 \\ f'(c^-) < 0 & f'(c^+) > 0 \end{matrix}\right]$ minima at a & c

$\left.\begin{matrix} f'(b^-) > 0 & f'(b^+) < 0 \end{matrix}\right]$ max. at b.

4.



$f'(c) = 0 \quad f'(c^-) > 0 \quad f'(c^+) < 0$

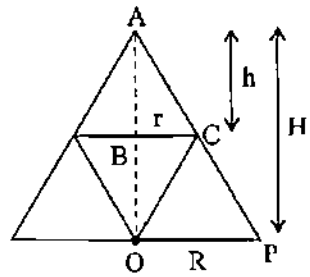
$g(x) \quad \frac{f(x)}{x} \Rightarrow g'(x) = \frac{xf'(x) - 1}{x^2}$

$g'(c^+) = \lim_{h \to 0} \frac{(c+h)f'(c+h) - 1}{(c+h)^2} < 0$

$(f'(c+h) < 0)$

hence fig.(2)

5.



ΔABC and ΔAOP are similar

$$\frac{h}{r} = \frac{H}{R} \quad \Rightarrow h = \frac{rH}{R}$$

$$V_2 = \frac{1}{3}\pi r^2(H-h) = \frac{\pi}{3}r^2 H\left(1 - \frac{r}{R}\right)$$

$$= \frac{\pi H}{3}\left(r^3 - \frac{r^3}{R}\right)$$

$$\frac{dV_2}{dr} = 2r - \frac{3r^2}{R} = 0 \Rightarrow r = \frac{2R}{3}$$

$$V_{2max} = \frac{4\pi R^2 H}{81}$$

$$V_1 = \frac{\pi R^2 H}{3}$$

$$\therefore \frac{V_2}{V_1} = \frac{4}{27}$$

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6. Clearly $f(\pi + x) + f(\pi - x)$
(every term contain cosine)

$$f\left(\frac{\pi}{5}\right) = f\left(\frac{9\pi}{5}\right),\ f\left(\frac{2\pi}{5}\right) = f\left(\frac{8\pi}{5}\right),$$

$$f\left(\frac{3\pi}{5}\right) = f\left(\frac{7\pi}{5}\right),\ f\left(\frac{4\pi}{5}\right) = f\left(\frac{6\pi}{5}\right)$$

$$T = f(0) - 2\left[f\left(\frac{\pi}{5}\right) + f\left(\frac{3\pi}{5}\right)\right] + 2\left[f\left(\frac{2\pi}{5}\right) + f\left(\frac{4\pi}{5}\right)\right] - f(\pi)$$

$f(0) - f(\pi) = 2(1 + B + D)$

$$f\left(\frac{\pi}{5}\right) + f\left(\frac{3\pi}{5}\right) = f\left(\frac{\pi}{5}\right) - f\left(\frac{4\pi}{5}\right)$$

$$= 2\left(1 + B\cos\frac{3\pi}{5} + D\cos\frac{\pi}{5}\right)$$

$$f\left(\frac{2\pi}{5}\right) + f\left(\frac{4\pi}{5}\right) = f\left(\frac{2\pi}{5}\right) - f\left(\frac{3\pi}{5}\right)$$

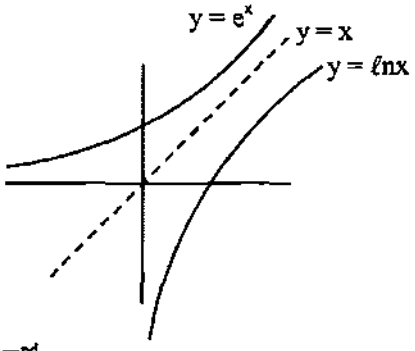
$$= 2\left(1 + B\cos\frac{6\pi}{5} + D\cos\frac{2\pi}{5}\right)$$

$T \Rightarrow$ contains only B, D terms

7. $x \notin I$ & $[x] > 1$
$\Rightarrow x \in (2, 3)$ only option satisfy.

8. $(2011)^n + {}^nC_1(2011)^{n-1} + {}^nC_2(2011)^{n-2} + \ldots$
$\ldots + {}^nC_{n-1}2011 + {}^nC_{n-1}$
$= (2011 + 1)^n - 1$

9.



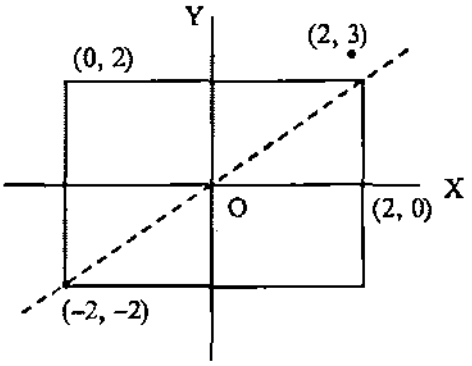
IInd curve

$y' = \frac{1}{x}$

$x = 1 \Rightarrow$ point (1, 0)
similarly I$^{st} \Rightarrow$ point (0, 1)
distance $= \sqrt{2}$

10. $f(x) = x^9(x^3 - 1) + x(x^3 - 1) + 1$
positive for $x \geq 1$ or $x \leq 0$
$= 1 - x + x^4 - x^9 + x^{12}$
positive for $x \in (0, 1)$
$f(x)$ is always positive

11.



$|x + y| + |x - y| = 4$ represent a square
$x^2 + y^2 - 4x - 6y = (x - 2)^2 + (y - 3)^2 - 13$
= (distance point on square from $(2, 3))^2 - 13$
Maximum $= (-2 - 2)^2 + (-2 - 3)^2 - 13$
$= 28$

12. $$f(x) = \frac{\sin(x - a) + \sin(x + a)}{\cos(x - a) - \cos(x + a)}$$

$$= \frac{2\sin(x).\cos a}{2\sin x.\sin a} = \cot a$$

13. $f(x) = \sqrt{4 - \sqrt{2x + 5}}$

$4 - \sqrt{2x + 5} \geq 0$, $\quad 2x + 5 \geq 0$

$\sqrt{2x + 5} \leq 4$, $\quad x \geq -\frac{5}{2}$

$x \leq \frac{11}{2}$

$x \in \left[-\frac{5}{2}, \frac{11}{2}\right]$

$$\text{Mid point} = \frac{-\frac{5}{2} + \frac{11}{2}}{2} = \frac{3}{2}$$

14. Only when $a_1 = a_2 = a_3$
In other cases $f(x)$ will take both positive and negative values

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15. As $P''(x) \neq 0$ for all x. So P(x) can be a polynomial of degree greater than or equal to 2. but situational benefit must be given to P(x) as quadratic expression.

So let $P(x) = ax^2 + bx + c$

$P(0) = 1 \Rightarrow c = 1$

$P'(0) = -1 \Rightarrow b = -1$

$P''(0) \neq 2a \neq 0$

So $P(x) = ax^2 - x + 1$

$\therefore \quad P(1) = a \neq 0$

So $P(1) \neq 0$

but if we consider P(x) as polynomial of degree than 2. then all options (A), (B), (C), (D) can be correct answers, as given information will not be sufficient to find the result. So option (B) is the most appropriate answer. Although KVPY has mentioned only option (C) as answer in its answer key.

16. $a_1 = 5$

$a_n = a_1 a_2 \ldots\ldots a_{n-1} + 4$

$a_2 = a_1 + 4 = 9$

$a_3 = a_1.a_2 + 4 = 5 \times 9 + 4 = 49$

$a_4 = a_1 a_2 a_3 + 4 = 2209$

$a_5 = a_1 a_2 a_3 a_4 + 4 = 4870849 = (2207)^2$

$a_5 = (a_4 - 2)^2$

$a_4 = (49 - 2)^2 = (a_3 - 2)^2$

$a_3 = (9-2)^2 = (a_2 - 2)^2$

$an = (a_{n-1} - 2)^2$

$\sqrt{a_n} = a_{n-1} - 2$

$$\frac{\sqrt{a_n}}{a_{n-1}} = \frac{a_{n-1} - 2}{a_{n-1}}$$

$$= 1 - \lim_{n\to\infty} \frac{2}{a_{n-1}} \quad \because \lim_{n\to\infty} a_{n-1} = \infty$$

$= 1$

17. $f(x) = \dfrac{x+1}{x-1}$

$$f^2(x) = f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = x$$

$$f^3(x) = f(x) = \frac{x+1}{x-1}$$

$f^4(x) = x$

$P = f(2).f^3(3)f^3(4)f^4(5)$

$P = 3 \times 3 \times \frac{5}{3} \times 5 = 75$

Multiple of P is 375

18. (A) $\lim\limits_{x\to\infty} x.\sin\left(\frac{1}{x}\right).f(x)$

$$\lim_{x\to\infty}\left(\frac{\sin(1/x)}{1/x}\right)\lim_{x\to\infty} f(x) = M$$

(B) $\lim\limits_{x\to\infty} \sin(f(x)) = \sin\left[\lim\limits_{x\to\infty} f(x)\right] = \sin M$

(C) $\lim\limits_{x\to\infty} x \sin e^{-x} f(x) = \lim\limits_{x\to\infty}\left(\frac{x}{e^x}\right)\left(\frac{\sin e^{-x}}{e^{-x}}\right)(f(x))$

(D) $\lim\limits_{x\to\infty} \frac{\sin x}{x}.f(x) = 0$

19. $f(x) = \sqrt{2 - x - x^2}$; $g(x) = \cos x$

Domain, $D_f \Rightarrow x^2 + x - 2 \leq 0$

$\Rightarrow (x+2)(x-1) \leq 0 \Rightarrow x \in [-2, 1]$

Range of $f \in \left[0, \frac{3}{2}\right]$

$D_g \in R$

$R_g \in [-1, 1]$

(I) Now; $D((f(g(x))^2) \Rightarrow -1 \leq \cos^2 x \leq 1$

$\Rightarrow \cos^2 x \leq 1$

also $D\,(f(g(x))) \Rightarrow \cos x \in [-1, 1] \rightarrow$ true

(II) D_{fog} R & D_{gof} is $[-2, 1]$

$D_{fog} \cap D_{gof} = D_{gof}$

(III) Clearly false from above

(IV) $D_{gof} \in [-2, 1]$

$D_{fog} \in R$ Hence false.

20. $f\left(\frac{1}{2}\right) = 100$

$$f(x) = a\left(x - \frac{1}{2}\right)(a_0 x^{n-1} + a_1 x^{n-2} + a_2 x^{n-3} + \ldots$$

$$\ldots + a_{n-1}) + 100$$

If $f(x) \leq 100\ \forall\ x \in R$

$\therefore$ $a < 0$ and f(x) must be even degree polynomial.

Since there may be more value of x at which f(x) attains its maximum so C may be false.

21. Discontinuous at $x = 2$

$$f(f(x)) = f\left(\frac{x+5}{x-2}\right) = \frac{\left(\frac{x+5}{x-2} + 5\right)}{\left(\frac{x+5}{x-2} - 2\right)}$$

$$= \frac{6x-5}{-x+9} = \frac{6x-5}{9-x}$$

At $x = 9$ it is discontinuous

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22. $f(x) = \begin{cases} \sin x & x \notin Q \\ \tan^2 x & x \in Q \end{cases}$

if is continuous at $x = 0, \pi$

so 2 points

$\sin x = \tan^2 x \Rightarrow \sin x(\cos^2 x - \sin x) = 0$

$\sin x = 0$

$x = 0, \pi$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-1 \pm \sqrt{5}}{2}$

$\sin x = \dfrac{\sqrt{5}-1}{2}$ 2 values

Total 4 points

23. $\lim\limits_{x\to 0}\left(\dfrac{x}{\sin x}\right)^{6/x^2}$ (1^∞ form)

$= e^{\lim\limits_{x\to 0}\frac{6}{x^2}\left(\frac{x}{\sin x}-1\right)}$

$= e^{\lim\limits_{x\to 0}\frac{6}{x^2}\left(\frac{x-\sin x}{\sin x}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{6}{x^2}\left\{\frac{x-\left(x-\left(\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right)\right)}{\left(x-\left(\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right)\right)}\right\}}$

$= e^{\lim\limits_{x\to 0}\frac{6}{x^2}\frac{\left\{\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right\}}{\left(x-\left(\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right)\right)}}$

$= e^{\lim\limits_{x\to 0}\frac{6x^3}{x^3}\frac{\left\{\frac{1}{3!}+\frac{x^2}{5!}\cdots\right\}}{\left\{1-\left(\frac{x^2}{3!}\cdots\right)\right\}}}$

$= e^1$

24. Given that $2r + r\theta = P \Rightarrow r = \dfrac{P}{2+\theta}$

$\text{Area} = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}\theta\left(\dfrac{P}{2+\theta}\right)^2 = \dfrac{1}{2}\dfrac{P^2\theta}{(2+\theta)^2}$

$\dfrac{dA}{d\theta} = 0 \Rightarrow \theta = 2^C$

25. $\lim\limits_{x\to 2}\dfrac{a^x + a^{3-x} - (a^2+a)}{a^{3-x} - a^{x/2}}\left(\dfrac{0}{0}\right)$

Apply L hospital rule

$= \lim\limits_{x\to 2}\dfrac{a^x\ell na - a^{3-x}\ell na}{-a^{3-x}\ell na - \frac{1}{2}a^{x/2}\ell na}$

$= \dfrac{a^2\ell na - a\ell na}{-a\ell na - \frac{a}{2}\ell na} = \dfrac{a^2-a}{-\frac{3a}{2}} = \dfrac{2}{3}(1-a)$

26. $f(x) = \alpha x^2 - 2 + \dfrac{1}{x}$

$f(x) = \dfrac{\alpha x^3 - 2x + 1}{x}\ \forall\ x\ (0, \infty)$

so $\phi(x) = \alpha x^3 - 2x + 1$ should be positive

$\phi(x) = \alpha x^3 - 2x + 1$

$\phi'(x) = 3\alpha x^2 - 2 = 0$

$\Rightarrow x = \pm\sqrt{\dfrac{2}{3\alpha}}$

Clearly $x = \sqrt{\dfrac{2}{3\alpha}}$ point of minima

$\phi\left(\sqrt{\dfrac{2}{3\alpha}}\right) \geq 0$

$\sqrt{\dfrac{2}{3\alpha}}\left\{\alpha\dfrac{2}{3\alpha} - 2\right\} + 1 \geq 0$

$\sqrt{\dfrac{2}{3\alpha}}\left(-\dfrac{4}{3}\right) + 1 \geq 0$

$\sqrt{\dfrac{2}{3\alpha}}\left(\dfrac{4}{3}\right) \leq 1$

$\sqrt{\dfrac{2}{3\alpha}} \leq \dfrac{3}{4}$

$\dfrac{2}{3\alpha} \leq \dfrac{3^2}{4^2}$

$\alpha \geq \dfrac{32}{27}$

27. $P(\sin^2 x) = P(\cos^2 x)$

$P(\sin^2 x) = P(1 - \sin^2 x)$

$P(x) = P(1 - x)\ \forall\ x \in [0, 1]$

Differentiable both sides w.r.t. x

$P'(x) = -P'(1 - x)$

So $P'(x)$ is symmetric about point $x = \dfrac{1}{2}$

So $P'(x)$ has highest degree odd

$\Rightarrow$ $P(x)$ has highest degree even

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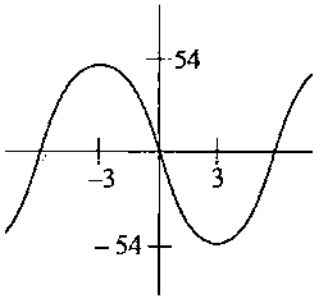
28. Let $f(x) = x^3 - 27x$

$f'(x) = 3x^2 - 27 = 3(x^2 - 9)$



As sum of the roots is zero, so if two roots are integer then 3rd root has to be integer

Now put $x = 6t$

$216\,t^3 - 27 \times 6t + k = 0$

$54\,(4t^3 - 3t) + k = 0$

Put $t = \cos\theta$

$54 \cos 3\theta = -k$

Now for $3\theta = 0, 2\pi$ we get integral solution

So two values of 'k'

29. $\lim\limits_{n\to\infty} f_n(x) = f(f(f(.....\infty \text{ times } (x))$

Now for $x_1 \in \left(0, \frac{1}{2}\right)$

$f(x_1) > x_1$ as $f(x)$ is concave - downward

Thus $f_n \to \frac{1}{2}$ as $n \to \infty$

Similarly for $x_1 \in \left(\frac{1}{2}, 1\right)$

$f(x_1) < x_1$ as $f(x)$ is concave upward

Thus $f_n \to \frac{1}{2}$ as $n \to \infty$

30. $$\lim_{x\to\infty} \frac{x^2 \int_0^x e^{t^3} dt}{e^{x^3}}$$

Apply L Hospital

$$= \lim_{x\to\infty} \frac{2x \int_0^x e^{t^3} + x^2 e^{x^3}}{3x^2 e^{x^3}}$$

$$= \lim_{x\to\infty} \frac{2 \int_0^x e^{t^3} + x e^{x^3}}{3x e^{x^3}}$$

$$= \lim_{x\to\infty} \frac{2e^{x^3} + e^{x^3} + 3x^3 e^{x^3}}{3e^{x^3} + 9x^3 e^{x^3}}$$

$$= \lim_{x\to\infty} \frac{3 + 3x^3}{3 + 9x^3} = \frac{1}{3}$$

31. $f'(x) = 3x^2 - 6ax + 27a^2 + 9$

$= 3[x^2 - 2ax + 9a^2 + 3]$

$= 3((x-a)^2 + 8a^2 + 3)$

$\therefore$ $f'(x)$ is + ve for $x \in R$ so $f(x)$ is monotonic↑ for $x \in R$.

32. $P'(x) = 12x^2 - 3 = 3(4x^2 - 1)$

In $\left(-\frac{1}{2}, \frac{1}{2}\right)$ $\quad P'(x) < 0$

$\Rightarrow$ P(x) is decreasing

$\Rightarrow$ Range $\in$ (P(–1), P(1))

Range $\in$ (–I, 1)

33. $f'(x) \le 2\,f(x)$

$f'(x)e^{-2x} \le 2\,f(x)e^{-2x}$

$\frac{d}{dx}(f(x)e^{-2x}) \le 0$

$g(x) = f(x)e^{-2x}$ is non-Increasing function

$x \ge 0$

$g(x) \le g(0)$

$f(x)e^{-2x} \le f(0)e^{-0}$

$f(x)e^{-2x} \le 0$

$f(x) \le 0$ but given $f(x)$ is not negative

$\therefore$ $f(x) = 0$ Constant function

34. Rationalise

$$\lim_{x\to-\infty}\left((\sqrt{4x^2 - x} + 2x) \times \frac{\sqrt{4x^2 - x} - 2x}{\sqrt{4x^2 - x} - 2x}\right)$$

$$\lim_{x\to-\infty}\left(\frac{-x}{|x|\sqrt{4 - \frac{1}{x}} - 2x}\right) \text{ at } x \to -\infty \quad |x| = -x$$

$$\lim_{x\to-\infty}\left(\frac{-x}{-x\sqrt{4 - \frac{1}{x}} - 2x}\right) = \frac{1}{2+2} = \frac{1}{4}$$

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35. (I) This relation is reflexive relation because every natural no. divides square of itself

a R a $\Leftrightarrow$ a divides a^2

(II) not symmetric eg. 5 R 10 $\Rightarrow$ 5 Divide 100

But 10 R 5 $\not\Rightarrow$ 10 Divide 25

(III) Not transitivity for example

if 8 R 4 & 4 R 2 $\not\Rightarrow$ 8 R 2

only (I) Option

36. $f(x) = \dfrac{\{x\}}{1+[x]^2}$

$$f(x) = \begin{cases} \dfrac{x+1}{2}; & -1 \le x < 0 \\ x; & 0 \le x < 1 \\ \dfrac{x-1}{2}; & 1 \le x < 2 \\ \dfrac{x-2}{5}; & 2 \le x < 3 \\ \vdots & \\ \text{So - on} & \end{cases}$$

Now check accordingly

37. $\lim\limits_{n\to\infty}(3^n)^{1/2n}\left(\left(\dfrac{2}{3}\right)^n + 1\right)^{1/2n}$

Put $\lim\limits_{n\to\infty}\sqrt{3}$

38. Make graph and observe yourself

39. $S = 2\pi R^2 + 2\pi Rh + \pi R^2$

(R = radius of hemisphere & cylinder)

$$V = \frac{2}{3}\pi R^3 + \pi R^2 h$$

$$V = \frac{2}{3}\pi R^3 + \pi R^2 \times \left(\frac{5 - 3\pi R^2}{2\pi R}\right)$$

$$\frac{dV}{dR} = 2\pi R^2 + \frac{5}{2} - \frac{9\pi}{2}R^2$$

for maximum & minimum $\dfrac{dV}{dR} = 0$

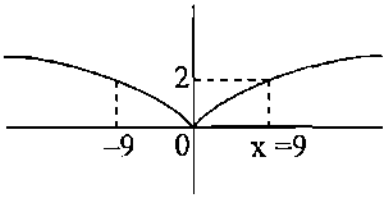
$5\pi R^2 = S$

$5\pi R^2 = 3\pi R^2 + 2\pi Rh$

$2R = 2h$

$h : R = 1 : 1$

40. graph of given function actually look like this



Clear from graph option (B) is right.

41. Given equation can be written as

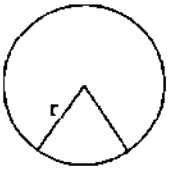
$$-\frac{1}{3} - \int_0^1 \left(\frac{f(x)}{2} - x\right)^2 dx - \int_0^1 x^2\, dx$$

$$\int_0^1 x^2\, dx \;\frac{1}{3} = \int_0^1 \left(\frac{f(x)}{2} - x\right)^2 dx$$

$$0 = \int_0^1 \left(\frac{f(x)}{2} - x\right)^2 dx$$

So, no of continuous function is 1.

42.



for a cone

r will be cone slant height

$V = 2\sqrt{3}$

Let x radius of cone

h be height then

$$\frac{1}{3}\pi r^2 h = 2\sqrt{3}\,\pi$$

$$x^2 h = 6\sqrt{3} \Rightarrow x^2 = \frac{6\sqrt{3}}{h}$$

least Diameter $\Rightarrow$ least slant height of conc

$\ell^2 = x^2 + h^2$

$r^2 = x^2 + h^2$

$$r^2 = \frac{6\sqrt{3}}{h} + h^2$$

Diff. w. to x

$$2r\frac{dr}{dh} = \frac{-6\sqrt{3}}{h^2} + 2h$$

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for maximum & minimum $\frac{dr}{dh} = 0$

$h = \sqrt{3} \Rightarrow h^2 = 3$

$x^2 = \frac{6\sqrt{3}}{\sqrt{3}}$

$x^2 = 6$

$r^2 = 6 + 3$

$r^2 = 9$

$r = 3$

$d = 2r$

$d = 6$

43. $a_n = \underset{-\pi}{\overset{1}{\int}} -(x-1)\cos nx \, dx + \underset{1}{\overset{\pi}{\int}} (x-1)\cos nx \, dx$

I II

Solve I & II part by I LATE

$a_n = 2\pi \sin\frac{n\pi}{n} + \frac{2}{n^2}\cos n\pi - \frac{2}{n^2}\cos n$

$\lim_{x\to\infty} a_n = 0$

44. $f'(x) = \frac{2x}{1+x^2}$

Range of $\frac{2x}{1+x^2} \in [-1, 1]$

$\frac{|f(x)-f(y)|}{|x-y|} \le A$ means

maximum value of $\frac{|f(x)-f(y)|}{|x-y|}$ is always less than or equal to A.

So, least value of A is 1

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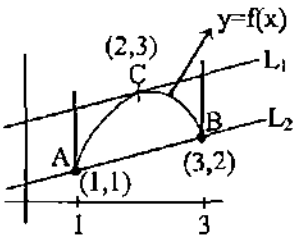
Chapter 5

Integral Calculus

1. The following figure shows the graph of a continuous function $y = f(x)$ on the interval $[1, 3]$. The points A, B, C have coordinates (1, 1), (3, 2), (2, 3) respectively, and the lines L_1 and L_2 are parallel, with L_1 being tangent to the curve at C. If the area under the graph of $y = f(x)$ from $x = 1$ to $x = 3$ is 4 square units, then the area of the shaded region is- **[2010]**



(A) 2 (B) 3 (C) 4 (D) 5

2. Let $I_n = \int_0^1 (\log x)^n \, dx$, where n is a non-negative integer. Then $I_{2001} - 2011\, I_{2010}$ is equal to- **[2010]**

(A) $I_{1000} + 999\, I_{998}$ (B) $I_{890} + 890\, I_{889}$
(C) $I_{100} + 100\, I_{99}$ (D) $I_{53} + 54\, I_{52}$

3. Consider the regions $A = \{(x, y) \mid x^2 + y^2 \le 100\}$ and $= \{(x, y) \mid \sin(x + y) > 0\}$ in the plane. Then the area of the region $A \cap B$ is- **[2010]**

(A) 10π (B) 100
(C) 100π (D) 50π

4. Let $f : R \to R$ be a continuous function satisfying $f(x) = x + \int_0^x f(t)\, dt$, for all $x \in R$. Then the number of elements in the set $S = \{x \in R ; f(x) = 0\}$ is- **[2010]**

(A) 1 (B) 2 (C) 3 (D) 4

5. The value of $\int_0^{2\pi} \min\{|x - \pi|, \cos^{-1}(\cos x)\}\, dx$ is- **[2010]**

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi^2}{8}$ (D) π^2

6. Let $f : (2, \infty) \to N$ be defined by $f(x)$ = the largest prime factor of $[x]$. Then $\int_2^8 f(x)dx$ is equal to - **[2011]**

(A) 17 (B) 22 (C) 23 (D) 25

7. Let $[x]$ denote the largest integer not exceeding x and $\{x\} = x - [x]$. Then $\int_0^{2012} \frac{e^{\cos(\pi\{x\})}}{e^{\cos(\pi\{x\})} + e^{-\cos(\pi\{x\})}} dx$ is equal to **[2011]**

(A) 0 (B) 1006
(C) 2012 (D) 2012π

8. The value of

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-4}} + \ldots + \frac{1}{\sqrt{4n^2-n^2}}\right)$$

is - **[2011]**

(A) $\frac{1}{4}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

9. Let $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are real numbers. If $f(x)$ has a local minimum at $x = 1$ and a local maximum at $x = -\frac{1}{3}$ and $f(2) = 0$, then $\int_{-1}^{1} f(x)\, dx$ equals- **[2011]**

(A) $\frac{14}{3}$ (B) $\frac{-14}{3}$ (C) $\frac{7}{3}$ (D) $\frac{-7}{3}$

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10. For each positive integer n, define $f_n(x) = \text{minimum}\left(\frac{x^n}{n!}, \frac{(1-x)^n}{n!}\right)$, for $0 \le x \le 1$.

Let $I_n = \int_0^1 f_n(x)\,dx$, $n \ge 1$. Then $I_n = \sum_{n=1}^{\infty} I_n$ is equal to- **[2011]**

(A) $2\sqrt{e}-3$ (B) $2\sqrt{e}-2$

(C) $2\sqrt{e}-1$ (D) $2\sqrt{e}$

11. The value $\frac{\int_0^{\pi/2}(\sin x)^{\sqrt{2}+1}\,dx}{\int_0^{\pi/2}(\sin x)^{\sqrt{2}-1}\,dx}$ is **[2012]**

(A) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ (B) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

(C) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (D) $2-\sqrt{2}$

12. The value of $\int_{-2012}^{2012}(\sin(x^3)+x^5+1)dx$ is **[2012]**

(A) 2012 (B) 2013

(C) 0 (D) 4024

13. Let [x] and {x} be the integer part and fractional part of a real number x respectively.

The value of the integral $\int_0^5 [x]\{x\}dx$ is **[2012]**

(A) 5/2 (B) 5

(C) 34.5 (D) 35.5

14. The value of the integral $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, where $a > 0$, is **[2012]**

(A) π (B) $a\pi$ (C) $\pi/2$ (D) 2π

15. Consider

$L = \sqrt[3]{2012}+\sqrt[3]{2013}+....+\sqrt[3]{3011}$

$R = \sqrt[3]{2013}+\sqrt[3]{2014}+....+\sqrt[3]{3012}$

and $I = \int_{2012}^{3012}\sqrt[3]{x}dx$ Then **[2012]**

(A) $L + R < 2I$ (B) $L + R > 2I$

(C) $L + R = 2I$ (D) $\sqrt{LR} = I$

16. For $x, t \in R$, let

$p_t(x) = (\sin t)\,x^2 - (2\cos t)\,x + \sin t$ be a family of quadratic polynomials in x with variable coefficients. Let $A(t) = \int_0^1 p_t(x)dx$. Which of the following statements are true ? **[2013]**

(I) $A(t) < 0$ for all t

(II) A(t) has infinitely many critical points

(III) $A(t) = 0$ for infinitely many t

(IV) $A'(t) < 0$ for all t

(A) (I) and (II) only (B) (II) and (III) only

(C) (III) and (IV) only (D) (IV) and (I) only

17. For real x with $-10 \le x \le 10$ define $f(x) = \int_{-10}^{x} 2^{[t]}\,dt$, where for a real number r we denote by [r] the largest integer less than or equal to r. The number of points of discontinuity of f in the interval (–10, 10) is- **[2013]**

(A) 0 (B) 10 (C) 18 (D) 19

18. For a real number x let [x] denote the largest integer less than or equal to x and $\{x\} = x - [x]$. The smallest possible integer value of n for which $\int_1^n [x]\{x\}\,dx$ exceeds 2013 is- **[2013]**

(A) 63 (B) 64 (C) 90 (D) 91

19. The area bounded by the curve $y = \cos x$, the line joining $\left(-\frac{\pi}{4}, \cos\left(-\frac{\pi}{4}\right)\right)$ and (0, 2) and the line joining $\left(\frac{\pi}{4}, \cos\left(\frac{\pi}{4}\right)\right)$ and (0, 2) is- **[2013]**

(A) $\left(\frac{4+\sqrt{2}}{8}\right)\pi-\sqrt{2}$ (B) $\left(\frac{4+\sqrt{2}}{8}\right)\pi+\sqrt{2}$

(C) $\left(\frac{4+\sqrt{2}}{4}\right)\pi-\sqrt{2}$ (D) $\left(\frac{4+\sqrt{2}}{4}\right)\pi+\sqrt{2}$

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20. Let n be a positive integer. For a real number x, let [x] denote the largest integer not exceeding x and $\{x\} = x - [x]$. Then $\int_1^{n+1} \frac{(\{x\})^{[x]}}{[x]} dx$ is equal to [2013]

(A) $\log_e(n)$ (B) $\frac{1}{n+1}$

(C) $\frac{n}{n+1}$ (D) $1 + \frac{1}{2} + \ldots + \frac{1}{n}$

21. For a real number x, let [x] denote the largest number less than or equal to x. For $x \in R$, let $f(x) = [x] \sin \pi x$. Then [2014]

(A) f is differentiable on R.

(B) f is symmetric about the line $x = 0$.

(C) $\int_{-3}^{3} f(x)dx = 0$.

(D) For each real α, the equation $f(x) - \alpha = 0$ has infinitely many roots.

22. Let $f: [0, 1] \to [0, \infty)$ be a continuous function such that $\int_0^1 f(x)dx = 10$. Which of the following statements is **NOT** necessarily true? [2014]

(A) $\int_0^1 e^x f(x)dx \le 10$

(B) $\int_0^1 \frac{f(x)}{(1+x)^2} dx \le 10$

(C) $-10 \le \int_0^1 \sin(100x) f'(x)dx \le 10$

(D) $\int_0^1 f(x)^2 dx \le 100$

23. A continuous function $f: R \to R$ satisfies the equation $f(x) = x + \int_0^x f(t)dt$. Which of the following options is true? [2014]

(A) $f(x+y) = f(x) + f(y)$

(B) $f(x+y) = f(x)f(y)$

(C) $f(x+y) = f(x) + f(y) + f(x)f(y)$

(D) $f(x+y) = f(xy)$

24. For a real number x let [x] denote the largest integer less than or equal to x and $\{x\} = x - [x]$. Let n be a positive integer. Then $\int_0^n \cos(2\pi[x]\{x\})\, dx$ is equal to [2014]

(A) 0 (B) 1 (C) n (D) $2n - 1$

25. Let $I_n = \int_0^{\pi/2} x^n \cos x\, dx$, where n is a non-negative integer.

Then $\sum_{n=2}^{\infty} \left(\frac{I_n}{n!} + \frac{I_{n-2}}{(n-2)!} \right)$ equals - [2014]

(A) $e^{\pi/2} - 1 - \frac{\pi}{2}$ (B) $e^{\pi/2} - 1$

(C) $e^{\pi/2} - \frac{\pi}{2}$ (D) $e^{\pi/2}$

26. For a real number x let [x] denote the largest integer less than or equal to x. The smallest positive integer n for which the integral $\int_1^n [x][\sqrt{x}]dx$ exceeds 60 is - [2014]

(A) 8 (B) 9 (C) 10 (D) $[60^{2/3}]$

27. Define a function $f: R \to R$ by $f(x) = \max\{|x|, |x-1|, \ldots, |x-2n|\}$ where n is a fixed natural number. Then $\int_0^{2n} f(x)dx$ is - [2015]

(A) n (B) n^2 (C) 3n (D) $3n^2$

28. If p(x) is a cubic polynomial with p(1) = 3, p(0) = 2 and p(−1) = 4, then $\int_{-1}^{1} p(x)dx$ is - [2015]

(A) 2 (B) 3 (C) 4 (D) 5

29. Let $x > 0$ be a fixed real number. Then the integral $\int_0^{\infty} e^{-t} |x - t| dt$ is equal to - [2015]

(A) $x + 2e^{-x} - 1$ (B) $x - 2e^{-x} + 1$

(C) $x + 2e^{-x} + 1$ (D) $-x - 2e^{-x} + 1$

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30. Let $f : R \to R$ be a continuous function satisfying $f(x) + \int_0^x tf(t)dt + x^2 = 0$, for all $x \in R$.

Then - **[2015]**

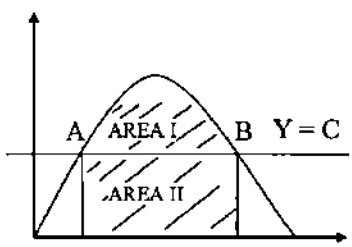
(A) $\lim_{x\to\infty} f(x) = 2$

(B) $\lim_{x\to-\infty} f(x) = -2$

(C) f(x) has more than one point in common with the x-axis

(D) f(x) is an odd functions

31. The figure shows a portion of the graph $y = 2x - 4x^3$. The line $y = c$ is such that the areas of the regions marked I and II are equal. If a, b are the x-coordinates of A, B respectively, then a + b equals - **[2015]**



(A) $\frac{2}{\sqrt{7}}$ (B) $\frac{3}{\sqrt{7}}$ (C) $\frac{4}{\sqrt{7}}$ (D) $\frac{5}{\sqrt{7}}$

32. The area of the region bounded by the curve $y = |x^3 - 4x^2 + 3x|$ and the x-axis, $0 \le x \le 3$, is **[2016]**

(A) $\frac{37}{6}$ (B) $\frac{9}{4}$ (C) $\frac{37}{12}$ (D) 0

33. The number of continuous function $f : [0, 1] \to [0, 1]$ such that $f(x) < x^2$ for all x and $\int_0^1 f(x)dx = \frac{1}{3}$ is : **[2016]**

(A) 0 (B) 1
(C) 2 (D) infinite

34. On the real line R, we define two functions f and g as follows :

$f(x) = \min\{x - [x], 1 - x + [x]\}$

$g(x) = \max\{x - [x], 1 - x + [x]\}$

Where [x] denotes the largest integer not exceeding x. The positive integer n for which $\int_0^1 (g(x) - f(x))dx = 100$ is **[2016]**

(A) 100 (B) 198
(C) 200 (D) 202

35. Let f be a continuous function defined on [0, 1] such that $\int_0^1 f^2(x)dx = \left(\int_0^1 f(x)dx\right)^2$. Then the range of f **[2016]**

(A) has exactly two points
(B) has more than two points
(C) is the interval [0, 1]
(D) is a singleton

36. The parabola $y^2 = 4x + 1$ divides the disc $x^2 + y^2 \le 1$ into two regions with areas A_1 and A_2. Then $|A_1 - A_2|$ equals **[5 Nov, 2017]**

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

37. Let $f(x) = \max\left\{3, x^2, \frac{1}{x^2}\right\}$ for $\frac{1}{2} \le x \le 2$. Then the value of the integral $\int_{1/2}^{2} f(x)dx$ is **[5 Nov, 2017]**

(A) $\frac{11}{3}$ (B) $\frac{13}{3}$ (C) $\frac{14}{3}$ (D) $\frac{16}{3}$

38. Define $g(x) = \int_{-3}^{3} f(x-y)f(y)dy$, for all real x,

where $f(t) = \begin{cases} 1, & 0 \le t \le 1, \\ 0, & \text{elsewhere.} \end{cases}$

Then **[5 Nov, 2017]**

(A) g(x) is not continuous everywhere

(B) g(x) is continuous everywhere but differentiable nowhere

(C) g(x) is continuous everywhere and differentiable everywhere except at x = 0,1

(D) g(x) is continuous everywhere and differentiable everywhere except at x = 0,1,2

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39. Let $I_n = \int_0^1 e^{-y} y^n dy$, where n is a non-negative integer. Then $\sum_{n=1}^{\infty} \frac{I_n}{n!}$ is **[19 Nov, 2017]**

(A) 1 (B) $1 - \frac{1}{e}$

(C) $\frac{1}{e}$ (D) $1 + \frac{1}{e}$

40. The area bounded by the curve $y = \frac{1}{4}|4 - x^2|$ and $y = 7 \quad |x|$ is **[19 Nov, 2017]**

(A) 18 (B) 32

(C) 36 (D) 64

41. Let $g(x) = \int_0^{|x|^{2/4}} t^{2/3} \sin\frac{1}{t} dt$, for all real x. Then $\lim_{x \to 0} \frac{g(x)}{x}$ is equal to **[19 Nov, 2017]**

(A) ∞ (B) ∞

(C) 0 (D) $\frac{3}{4}$

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ANSWERS

1. (A)	2. (C)	3. (D)	4. (A)	5. (B)	6. (B)	7. (B)
8. (D)	9. (B)	10. (A)	11. (D)	12. (D)	13. (B)	14. (C)
15. (A)	16. (B)	17. (A)	18. (D)	19. (A)	20. (C)	21. (D)
22. (D)	23. (C)	24. (B)	25. (A)	26. (B)	27. (D)	28. (D)
29. (A)	30. (B)	31. (A)	32. (C)	33. (A)	34. (C)	35. (D)
36. (B)	37. (C)	38. (D)	39. (C)	40. (B)	41. (C)	

SOLUTIONS

1. Equation of $\ell_2 \rightarrow y - 1 = \frac{2-1}{3-1}(x-1)$

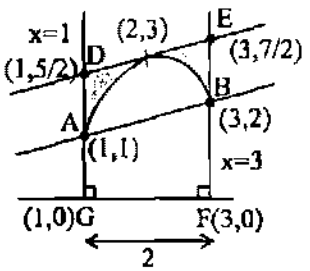
$$2y - 2 = x - 1$$
$$2y - x = 1$$

$\Rightarrow$ slope of $\ell_1 = \frac{1}{2}$

Equation of $\ell_1 = y - 3 = \frac{1}{2}(x-2)$

$$2y - 6 = x - 2$$
$$2y - x = 4$$

$D \rightarrow \left(1, \frac{5}{2}\right)$, $E \rightarrow \left(3, \frac{7}{2}\right)$



Area under $f(x) = 4$

Shaded area = area of trapezium DEFG − area under $f(x)$

$$= \frac{1}{2}\left(\frac{5}{2} + \frac{7}{2}\right) \times 2 - 4$$
$$= 6 - 4$$
$$= 2$$

2. $I_n = \int_1^e 1.(\log x)^n \, dx$

$$\Rightarrow I_n = (\log x)^n \; x\Big|_1^e - \int_1^e \frac{n(\log x)^{n-1}}{x}.x\,dx$$

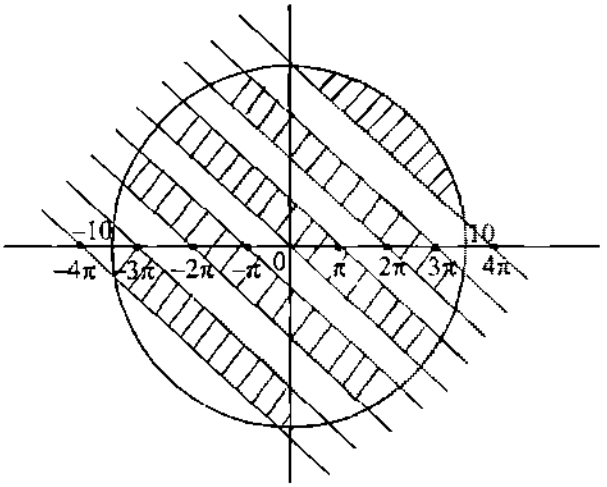
$$\Rightarrow I_n = e - 0 - n\, I_{n-1}$$
$$\Rightarrow I_n + n\, I_{n-1} = e$$
$$\Rightarrow I_{2001+2011}\; I_{2010} = e$$
$$\boxed{I_{100} + 100\, I_{99} = e}$$

3. $x^2 + y^2 \le 100 \rightarrow$ inside of a circle

$\sin(x + y) > 0$

$x + y \in (0, \pi) \cup (2\pi, 3\pi)$ .....

$x + y = c \rightarrow$ equation of a line



Required area = shaded region

$$= \frac{1}{2}\pi(10)^2 = 50\,\pi$$

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4. $f'(x) = 1 + f(x) \Rightarrow f(x) = e^x - 1$

$f(x) = 0 \Rightarrow e^x = 1$ $\boxed{x = 0}$ One solution

5. $I = \int_0^{\pi/2} x\,dx + \int_{\pi/2}^{\pi} (\pi - x)\,dx + \int_{\pi}^{3\pi/2} (x - \pi)\,dx + \int_{3\pi/2}^{2\pi} 2\pi - x\,dx$

$= \frac{\pi^2}{8} + \frac{\pi^2}{8} + \frac{\pi^2}{8} + \frac{\pi^2}{8} = \frac{\pi^2}{2}$

6. $\int_2^8 f(x)\,dx = 2 + 3 + 2 + 5 + 3 + 7 = 22$

7. $I = 2012 \int_0^1 \frac{e^{\cos \pi x}}{e^{\cos \pi x} + e^{-\cos \pi x}} dx$

Using king property

$I = 2012 \int_0^1 \frac{e^{-\cos \pi x}}{e^{-\cos \pi x} + e^{\cos \pi x}} dx$

$\Rightarrow 2I = 2012 \Rightarrow I = 1006$

8. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{4n^2 - r^2}} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{\sqrt{4 - (r/n)^2}}$

$= \int_0^1 \frac{dx}{\sqrt{4 - x^2}} = \left(\sin^{-1}\left(\frac{x}{2}\right) \right)_0^1 = \frac{\pi}{6}$

9. $f'(x) = 3\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right) = 3x^2 - 2x - 1$

$f(x) = x^3 - x^2 - x + \lambda$

$f(2) = 8 - 4 - 2 + \lambda = 0 \Rightarrow \lambda = -2$

$f(x) = x^3 - x^2 - x - 2$

$\int_{-1}^{1} f(x)\,dx = -2\int_0^1 (x^2 + 2)\,dx = -2\left(\frac{1}{3} + 2\right) = \frac{-14}{3}$

10. $I_n = \int_0^{1/2} \frac{x^n}{n!} dx + \int_{1/2}^{1} \frac{(1-x)^n}{n!} dx$

$= \frac{1}{(n+1)!} \left(\left(\frac{1}{2}\right)^{n+1} + \left(\frac{1}{2}\right)^{n+1} \right) = \frac{\left(\frac{1}{2}\right)^n}{(n+1)!}$

$\sum_{n=1}^{\infty} I_n = \left(\frac{1/2}{2!} + \frac{(1/2)^2}{3!} + \ldots\ldots \right) = 2\sqrt{e} - 3$

11. $I_1 = \int_0^{\pi/2} (\sin x)^{\sqrt{2}} . \sin x\,dx$

$I_2 = \int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} dx$

$I_1 = \left((\sin x)^{\sqrt{2}} \int \sin x\,dx \right)_0^{\pi/2} - \int_0^{\pi/2} \left(\sqrt{2} (\sin x)^{\sqrt{2}-1} \cos x \int \sin x\,dx \right)$

$= -\left(\cos x (\sin x)^{\sqrt{2}} \right)_0^{\pi/2} + \sqrt{2} \int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} (1 - \sin^2 x)\,dx$

$\Rightarrow \frac{I_1}{I_2} = \frac{\sqrt{2}}{1+\sqrt{2}} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = 2 - \sqrt{2}$

12. $\int_{-2012}^{2012} (\sin(x^3) + x^5 + 1)dx$

$= \int_{-2012}^{2012} (\sin(x^3)dx + \int_{-2012}^{2012} x^5 dx + \int_{-2012}^{2012} dx = 4024$

13. $\int_0^5 [x]\{x\}dx = \int_0^5 [x](x - [x])dx$

$= \int_0^1 0.dx + \int_1^2 1.(x-1)dx + \int_2^3 2(x-2)dx + \int_3^4 3(x-3)dx + \int_4^5 4(x-4)dx$

$= \left(\frac{(x-1)^2}{2} \right)_1^2 + 2\left(\frac{(x-2)^2}{2} \right)_2^3 + 3\left(\frac{(x-3)^2}{2} \right)_3^4 + 4\left(\frac{(x-4)^2}{2} \right)_4^5$

$= \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2}$

$= 5$

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14. $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$ ...(1)

Since $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$I=\int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}dx$ ...(2)

$I=\int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx$

add equation (1) and (2)

$2I=\int_{-\pi}^{\pi}\cos^2 x\left(\frac{1}{1+a^x}+\frac{a^x}{1+a^x}\right)dx$

$\Rightarrow I=\int_0^{\pi}\cos^2 x\,dx=2\int_0^{\pi/2}\cos^2 x\,dx=\frac{\pi}{2}$

15. $L=\sqrt[3]{2012}+\sqrt[3]{2013}+....+\sqrt[3]{3011}$ ...(1)

$R=\sqrt[3]{2013}+\sqrt[3]{2014}+....+\sqrt[3]{3012}$ ...(2)

$I=\int_{2012}^{3012}x^{1/3}dx$

Let $f(x)=x^{1/3}$

$n=\frac{b-a}{h}=\frac{3012-2012}{1}=1000$

$I=\frac{(b-a)}{h}[f(a)+f(a+h)+f(a+2h)+...$

$...+f(a+(n-1)h)]$

$=[f(2012)+f(2013)+.....+f(3011)]$

$I=(2012)^{1/3}+(2013)^{1/3}+....+(3011)^{1/3}$

$2I=2(2012)^{1/3}+2(2013)^{1/3}+....+2(3011)^{1/3}$

$=(2012)^{1/3}+(2012)^{1/3}+2(2013)^{1/3}...$

$...+(2)(3011)^{1/3}+(3012)^{1/3}+(3012)^{1/3}$

$=(2012)^{1/3}+L+R+(3012)^{1/3}$

$\Rightarrow L+R<2I$

16. $A(t)=(\sin t)\frac{1}{3}-2\cos t\times\frac{1}{2}+\sin t$

$A(t)=\frac{4}{3}\sin t-\cos t$; $A(t)\in\left[-\frac{5}{3},\frac{5}{3}\right]$

Clearly $A'(0)=0$ has many zeroes.

So, infinitely many critical points

& $A(t)=0$ also for $t\in$ many real nos.

$A'(t)\subset\left[-\frac{5}{3},\frac{5}{3}\right]$

17. $f(x)=\int_{-10}^{x}2^{[t]}dt$

For $-10\le x<-9$

$f(x)=\int_{-10}^{x}2^{-10}dt$

$=2^{-10}(x+10)$; $-10\le x<-9$

For $-9\le x<-8$

$f(x)=\int_{-10}^{x}2^{[t]}dt$

$=\int_{-10}^{-9}2^{-10}dt+\int_{-9}^{x}2^{-9}dt$

$f(x)=2^{-10}(1)+2^{-9}(x+9)$; $-9\le x<-8$

For $-8\le x\le -7$

$f(x)=\int_{-10}^{x}2^{[t]}dt$

$=\int_{-10}^{-9}2^{-10}dt+\int_{-9}^{-8}2^{-9}dt+\int_{-8}^{x}2^{[t]}dt$

$=2^{-10}+2^{-9}+2^{-8}(x+8)$; $-8\le x\le -7$

Similarly all

$f(-9^-)=2^{-10}=f(-9^+)$

$f(-8^-)=2^{-10}+2^{-9}=f(-8^+)$

so no discontinuity

18. $\int_1^n[x]\{x\}dx$

$\Rightarrow\int_1^2[x]\{x\}dx+\int_2^3[x]\{x\}dx+...$

$...\int_{n-1}^{n}[x]\{x\}dx>2013$

$\Rightarrow\int_0^1 x(1+2+3+.....(n-1))dx>2013$

$\Rightarrow\frac{(n-1)n}{2}\left(\frac{x^2}{2}\right)_0^1>2013$

$\Rightarrow\frac{n(n-1)}{2}\times\frac{1}{2}>2013$

$\Rightarrow n(n-1)>8052$

so $n_{min}=91$

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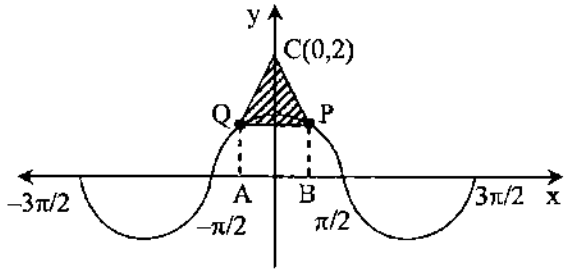
19.



$$\text{Area of trapezium ABPCQ} = \frac{1}{2}\left[\frac{\pi}{4}\right]\left[\frac{1}{\sqrt{2}}+2\right]$$

$$= \frac{1}{2}\times\frac{\pi}{4}\left(\frac{1+2\sqrt{2}}{\sqrt{2}}\right)$$

$$\text{Required area} = \frac{\pi}{8}\left(\frac{1+2\sqrt{2}}{\sqrt{2}}\right) - \int_0^{\pi/4}\cos x\,dx$$

$$= \frac{\pi}{8}\left(\frac{1+2\sqrt{2}}{\sqrt{2}}\right) - [\sin x]_0^{\pi/4}$$

$$= \frac{\pi}{8}\left[\frac{1+2\sqrt{2}}{\sqrt{2}}\right] - \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4}\left[\frac{1+2\sqrt{2}}{\sqrt{2}}\right] - \sqrt{2}$$

$$= \frac{\pi}{4}\left[\frac{8+2\sqrt{2}}{4}\right] - \sqrt{2}$$

20.

$$\int_1^{n+1}\frac{(\{x\})^{[x]}}{[x]}\,dx$$

$$= \int_1^2\{x\}\,dx + \int_2^3\frac{\{x\}^2}{2}\,dx + \int_3^4\frac{\{x\}^3}{3}\,dx + \ldots.$$

$$\ldots + \int_n^{n+1}\frac{\{x\}^n}{n}\,dx$$

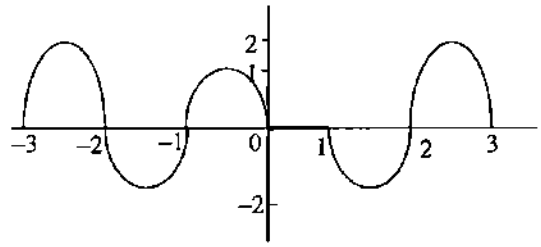
$$= \int_0^1\left(\{x\}+\frac{\{x\}^2}{2}+\frac{\{x\}^3}{3}+\ldots+\frac{\{x\}^n}{n}\right)dx$$

$$= \frac{1}{2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots.+\frac{1}{n.(n+1)}$$

$$= 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n}-\frac{1}{n+1}$$

$$= 1-\frac{1}{n+1} = \frac{n}{n+1}$$

21. $f(x) = [x]\sin\pi x$



Not differentiable for $\forall\ x \in R$.

Not symmetric about $x = 0$.

$$\int_{-3}^{3} f(x)dx \neq 0$$

$f(x) = \alpha$ will have ∞ Solution

22. $\because\ f(x) \geq 0$

$$\int_0^1 f(x)^2\,dx \leq 100 \text{ not necessarily true.}$$

because $(f(x))^2$ can take very high values then area bounded by $(f(x))^2$, x-axis & $x = 0$ to 1 may cross 100.

23. $$f(x) = x + \int_0^x f(t)dt$$

$$f'(x) = 1 + f(x)$$

$$\Rightarrow f'(x) - f(x) = 1$$

$$\Rightarrow e^{-x}f'(x) - f(x)\,e^{-x} = e^{-x}$$

$$\Rightarrow \frac{d}{dx}(f(x)\,e^{-x}) = e^{-x}$$

$$\Rightarrow f(x)\,e^{-x} = \frac{e^{-x}}{-1} + c$$

$$\Rightarrow f(x) = -1 + ce^x$$

$$f(0) = 0 = -1 + ce^0$$

$$\Rightarrow c = 1$$

$$f(x) = e^x - 1$$

$$f(x) + f(y) + f(x)\,f(y)$$

$$= e^x - 1 + e^y - 1 + (e^x - 1)(e^y - 1)$$

$$= e^x - 1 + e^y - 1 + e^x.e^y - e^y - e^x + 1$$

$$= e^x.e^y - 1$$

$$= e^{x+y} - 1$$

$$= f(x + y)$$

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24. $\int_0^n \cos(2\pi[x]\{x\})\,dx$

$= \int_0^1 \cos(0)\,dx + \int_1^2 \cos(2\pi(x-1))\,dx$

$+ \int_2^3 \cos(4\pi(x-2))\,dx + \ldots\ldots$

$\ldots + \int_{n-1}^n \cos(2\pi(n-1)(x-(n-1)))\,dx$

$= (1-0) + \int_1^2 \cos 2\pi x\,dx + \int_2^3 \cos 4\pi x\,dx + \ldots$

$\ldots + \int_{n-1}^n \cos(2\pi(n-1)x)\,dx$

$= 1 + \left.\frac{\sin 2\pi x}{2\pi}\right|_1^2 + \left.\frac{\sin 4\pi x}{4\pi}\right|_2^3 + \ldots$

$\ldots + \left.\frac{\sin 2\pi(n-1)x}{2\pi(n-1)}\right|_{n-1}^n$

$= 1 + 0 = 1$

25. $I_n = \int_0^{\pi/2} \underset{I}{x^n} \underset{II}{\cos x}\,dx$

$= x^n \sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} n x^{n-1} \sin x\,dx$

$= \left(\frac{\pi}{2}\right)^n - 0 - \left|n x^{n-1}(-\cos x)\right|_0^{\pi/2}$

$- \int_0^{\pi/2} n(n-1)x^{n-2}(-\cos x)\,dx$

$= \left(\frac{\pi}{2}\right)^n - 0 - n(n-1)\int_0^{\pi/2} x^{n-2}\cos x\,dx$

$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$

$\sum_{n=2}^{\infty}\left(\frac{I_n}{n!} + \frac{I_{n-2}}{(n-2)!}\right)$

$= \sum_{n=2}^{\infty}\left(\frac{\left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}}{n!} + \frac{I_{n-2}}{(n-2)!}\right)$

$= \sum_{n=2}^{\infty}\left(\left(\frac{\pi}{2}\right)^n \frac{1}{n!}\right)$

$= \left(\frac{\pi}{2}\right)^2 \frac{1}{2!} + \left(\frac{\pi}{2}\right)^3 \frac{1}{3!} + \left(\frac{\pi}{2}\right)^4 \frac{1}{4!} + \ldots.$

$= e^{\pi/2} - 1 - \left(\frac{\pi}{2}\right)$

26. Let $I = \int_1^n [x][\sqrt{x}]\,dx$

$1 \le x < 4 \qquad [\sqrt{x}] = 1$

$4 \le x < 9 \qquad [\sqrt{x}] = 2$

$9 \le x < 16 \qquad [\sqrt{x}] = 3$

Now,

$I = \int_1^2 dx + \int_2^3 2dx + \int_3^4 3dx + \int_4^5 8dx + \int_5^6 10dx + \int_6^7 12dx$

$+ \int_7^8 14dx + \int_8^9 16dx + \int_9^{10} 27dx + \int_{10}^{11} 30dx \ldots..$

$I = 1 + 2 + 3 + 8 + 10 + 12 + 14 + 16 = 66$

So $n = 9$

27. $f(x) = \max\{|x|, |x-1|, \ldots, |x-2n|\}$

$x \ge n$	$x < n$
$f(x) = \lvert x\rvert$	$\lvert x-2n\rvert$

$\int_0^{2n} f(x)dx = \int_0^n f(x)dx + \int_n^{2n} f(x)dx$

$= \int_0^n |x-2n|\,dx + \int_n^{2n} |x|\,dx$

$= \int_0^n (2n-x)\,dx + \int_n^{2n} x.dx$

$= \left[2nx - \frac{x^2}{2}\right]_0^n + \left[\frac{x^2}{2}\right]_n^{2n}$

$= \left(2n^2 - \frac{n^2}{2}\right) + \left(\frac{4n^2}{2} - \frac{n^2}{2}\right)$

$= \frac{3n^2}{2} + \frac{3n^2}{2} = 3n^2$

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28. Let $P(x) = ax^3 + bx^2 + cx + d$

$a + b + c + d = 3$

$d = 2$

$-a + b - c + d = 4$

$2b + 2d = 7$

$2b + 4 = 7$

$2b = 3$

$b = \frac{3}{2}$

$$\int_{-1}^{1}(ax^3 + bx^2 + cx + d)\,dx = 2\int_{0}^{1}(bx^2 + d)dx$$

$$= 2\left[b\frac{x^3}{3} + dx\right]_0^1 = 2\left(\frac{b}{3} + d\right)$$

$$= 2\left(\frac{1}{2} + 2\right) = 5$$

29. $f(x) = \int_0^\infty e^{-t}|x - t|dt$

$$f(x) = \int_0^x e^{-t}(x - t)dt + \int_x^\infty e^{-t}(t - x)dt$$

$$f'(x) = e^{-x}(x - x) - e^{-0}(x - 0).0 + \int_0^x e^{-t}(1)\,dt + 0 - e^{-x}(x - x).1 + \int_x^\infty -e^{-t}\,dt$$

$$= [-e^{-t}]_0^x + [e^{-t}]_x^\infty$$

$$= -e^{-x} + 1 + 0 - e^{-x}$$

$f'(x) = 1 - 2e^{-x}$

$dy = 1 - 2e^{-x}dx$

$y = x + 2e^{-x} + c$

$f(x) = x + 2e^{-x} + c$

$$f(0) = \int_0^\infty e^{-t}\,t\,dt$$

$$= [(-e^{-t})t]_0^\infty + \int_0^\infty e^{-t}dt$$

$$= [0 - e^{-t}]_0^\infty$$

$= 0 + 1$

$f(0) = 1$

$f(0) = 1 = 0 + 2e^{-0} + c \Rightarrow c = -1$

$f(x) = x + 2e^{-x} - 1$

30. $f(x) + \int_0^x t\,f(t)dt + x^2 = 0$

$\Rightarrow f'(x) + xf(x) + 2x = 0$

$$\Rightarrow \frac{f'(x)}{f(x) + 2} = -x$$

$$\Rightarrow \int\frac{f'(x)}{(f(x) + 2)}dx = -\int x\,dx$$

$$\Rightarrow \ell n\,(f(x) + 2) = -\frac{x^2}{2} + c$$

$$\Rightarrow f(x) + 2 = e^{-x^2/2} + c$$

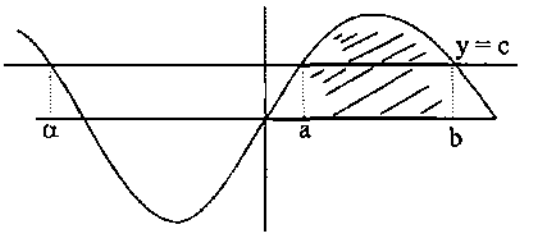
$\Rightarrow f(x) = k\,e^{-x^2/2} - 2$

where $x = 0 \quad f(x) = 0 \quad k = 2$

$f(x) = 2\,(e^{-x^2/2} - 1)$

clearly $\lim_{x \to -\infty} f(x) = -2$

31.



$$\int_a^b (2x - 4x^3)dx = 2\,(b - a)c$$

$(x^2 - x^4)_a^b = 2\,(b - a)\,c$

$(a + b)\,(1 - (a^2 + b^2)) = 2c$

$(a + b)\,(1 - (a + b)^2 + 2ab) = 2c \quad ...(1)$

again $2x - 4x^3 = c$

$4x^3 - 2x + c = 0$ ⟨ a, b, α

$a + b + \alpha = 0$

$\Rightarrow a + b = -\alpha \quad ...(2)$

$$ab + (a + b)\alpha = -\frac{1}{2}$$

$$\Rightarrow ab = \alpha^2 - \frac{1}{2} \quad ...(3)$$

$$ab\alpha = -\frac{c}{4}$$

$$\Rightarrow c = -4\alpha\left(\alpha^2 - \frac{1}{2}\right) \quad ...(4)$$

Put value of (a + b), ab, c from eq. (2), (3), (4) in equation (1) and solve it

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$$\left\{1-\alpha^2+2\left(\alpha^2-\frac{1}{2}\right)\right\}=-8\alpha\left(\alpha^2-\frac{1}{2}\right)$$

$\Rightarrow 1-\alpha^2+2\alpha^2-1=8\alpha^2-4$

$\Rightarrow \alpha^2=8\alpha^2-4$

$\Rightarrow 7\alpha^2=4$

$\Rightarrow \alpha=\frac{2}{\sqrt{7}}$

32. $A=\int_0^1 f(x)dx-\int_1^3 f(x)dx$

$=\int_0^1 (x^3-4x^2+3x)dx-\int_1^3 (x^3-4x^2+3x)dx$

$=\frac{37}{12}$

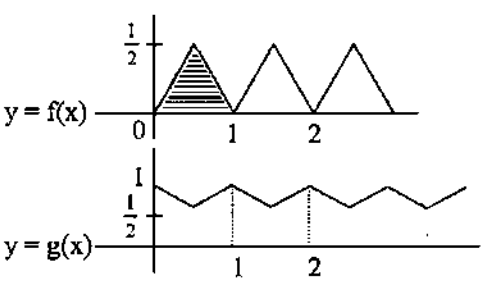
33. $\therefore$ f(x) is always positive for $x \in [0, 1]$

$\therefore f(x)<x^2 \quad \Rightarrow \int_0^1 f(x)dx<\int_0^1 x^2$

$$\boxed{I<\frac{1}{3}}$$

But it is given that $I=\frac{1}{3}$ which is not possible

34.



$\int_0^n g(x)dx=n\int_0^1 g(x)dx=\frac{3}{4}n$

$\int_0^n f(x)dx=\frac{n}{4}$

35. By Cauchy Schwarz inequality

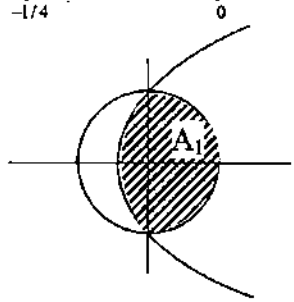
$$\left\{\int_a^b f(x)g(x)dx\right\}^2 \le \int_a^b (f(x))^2dx\int_a^b (g(x))^2dx$$

Here g(x) = 1

and equality holds only when $\frac{f(x)}{g(x)}=\lambda$

So, f(x) is constant

36. $A_1=2\int_{-1/4}^{0}\sqrt{4x+1}\,dx+2\int_0^1\sqrt{1-x^2}\,dx$



Solve $A_1=\frac{1}{3}+\frac{\pi}{2}$

$A_2=\pi(1)^2-A_1=\frac{\pi}{2}-\frac{1}{3}$

$|A_1-A_2|=\frac{2}{3}$

37. Given Integral can be distributed into

$\int_{1/2}^{1/\sqrt{3}}\frac{1}{x^2}.\,dx+\int_{1/\sqrt{3}}^{\sqrt{3}}3\,dx+\int_{\sqrt{3}}^{2}x^2.\,dx=\frac{14}{3}$

38. Definition can be break as

$g(x)=\int_0^1 f(x-y)\,dy$

$x-y=t\,;-dy\,.\,dt$

$g(x)=\int_{x-1}^{x} f(t)\,dt$

$$g(x)=\begin{cases}0 & x\le 0\\ x & 0<x<1\\ 2-x & 1\le x\le 2\\ 0 & x>2\end{cases}$$

Now, check yourself

39. $I_n=\int_0^1 e^{-y}y^n dy$

$I_n=-\frac{1}{e}+n\ I_{n-1}$ (by reduction formula)

$I_n-n\ I_{n-1}=-\frac{1}{e}.$

$\frac{I_n}{n!}-\frac{I_{n-1}}{n-1!}=-\frac{I}{e(n!)}$

$n=1 \quad \frac{I_1}{1!}-\frac{I_0}{0!}=-\frac{1}{e}$ ...(i)

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$n=2 \quad \frac{I_2}{2!}-\frac{I_1}{1!}=-\frac{1}{e(2!)}$ ...(ii)

$n=3 \quad \frac{I_3}{3!}-\frac{I_2}{2!}=-\frac{1}{e(3!)}$ ...(iii)

$n=4 \quad \frac{I_4}{4!}-\frac{I_3}{3!}=-\frac{1}{e(4!)}$ ...(iv)

eq.(1) + eq.(2) × 2 + eq.(3) × 3 + (eq. 4) × 4 + ...

$$-\left\{\frac{I_1}{1!}+\frac{I_2}{2!}+\frac{I_3}{3!}+\frac{I_4}{4!}\right\}-I_0$$

$$=-\frac{1}{e}\left\{1+1+\frac{1}{2!}+\frac{1}{3!}+...\right\}$$

Let $S=\sum_{n=1}^{\infty}\frac{I_n}{n!}$

$$-S-I_0=\frac{1}{e}\times e$$

$$S=1-I_0$$

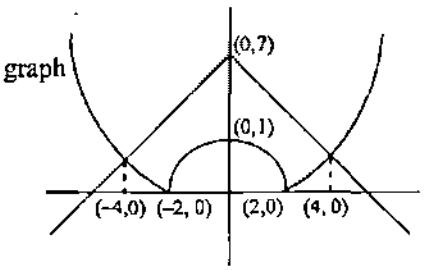
$$S=1-\int_0^1 e^{-y}dy$$

$$S=1+\left[e^{-y}\right]_0^1$$

$$S=1+\left[\frac{1}{e}-1\right]$$

$$S=\frac{1}{e}$$

40.



Required Area

$$A=2\left[\int_0^2\left((7-x)-\frac{1}{4}(4-x^2)\right)dx+\int_2^4\left((7-x)\ \frac{1}{4}(x^2-4)dx\right)\right]$$

solve this = 32

41. Apply L hospital rule

$$g'(x)=|x|^{1/2}\sin\left(\frac{1}{|x|^{3/4}}\right)$$

$$\lim_{x\to 0} g'(X)$$

$$\lim_{x\to 0}|x|^{1/2}\sin\left(\frac{1}{|x|^{3/4}}\right)=0$$

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Chapter 6

Vector & 3D

1. Let $\vec{u} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{v} = -3\hat{j} + 2\hat{k}$ be vectors in R^3 and $\vec{w}$ be a unit vector in the xy-plane. Then the maximum possible value of $|(\vec{u} \times \vec{v}).\vec{w}|$ is- [2010]

(A) $\sqrt{5}$ (B) $\sqrt{12}$

(C) $\sqrt{13}$ (D) $\sqrt{17}$

2. Let ABC be a triangle and P be a point inside ABC such that $\overrightarrow{PA} + 2\overrightarrow{PB} + 3\overrightarrow{PC} = \vec{0}$. The ratio of the area of triangle ABC to that of APC is- [2010]

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{5}{3}$ (D) 3

3. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors in the xyz space such that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$ If A, B, C are points with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively, then the number of possible positions of the centroid of triangle ABC is [2011]

(A) 1 (B) 2 (C) 3 (D) 6

4. Let H be the orthocenter of an acute - angled triangle ABC and O be its circumcenter. Then $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC}$ [2012]

(A) is equal to $\overrightarrow{HO}$

(B) is equal to $3\overrightarrow{HO}$

(C) is equal to $2\overrightarrow{HO}$

(D) is not a scalar multiple of $\overrightarrow{HO}$ in general

5. Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ be unit vectors in the xy-plane, one each in the interior of the four quadrants. Which of the following statements is necessarily true ? [2013]

(A) $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4 = 0$

(B) There exist i, j with $1 \le i < j \le 4$ such that $\vec{v}_i + \vec{v}_j$ is in the first quadrant

(C) There exist i, j with $1 \le i < j \le 4$ such that $\vec{v}_i . \vec{v}_j < 0$

(D) There exist i, j with $1 \le i < j \le 4$ such that $\vec{v}_i . \vec{v}_j > 0$

6. What is the angle subtended by an edge of a regular tetrahedron at its center? [2014]

(A) $\cos^{-1}\left(\frac{-1}{2}\right)$ (B) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(C) $\cos^{-1}\left(\frac{-1}{3}\right)$ (D) $\cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

7. The shortest distance from the origin to a variable point on the sphere $(x-2)^2 + (y-3)^2 + (z-6)^2 = 1$ is [2015]

(A) 5 (B) 6

(C) 7 (D) 8

8. Let $\vec{a} = 6\vec{i} - 3\vec{j} - 6\vec{k}$ and $\vec{d} = \vec{i} + \vec{j} + \vec{k}$. Suppose that $\vec{a} = \vec{b} + \vec{c}$ where $\vec{b}$ is parallel to $\vec{d}$ and $\vec{c}$ is perpendicular to $\vec{d}$. Then $\vec{c}$ is [2015]

(A) $5\vec{i} - 4\vec{j} - \vec{k}$

(B) $7\vec{i} - 2\vec{j} - 5\vec{k}$

(C) $4\vec{i} - 5\vec{j} + \vec{k}$

(D) $3\vec{i} + 6\vec{j} - 9\vec{k}$

9. Let $\vec{v}$ be a vector in the plane such that $|\vec{v} - \vec{i}| = |\vec{v} - 2\vec{i}| = |\vec{v} - \vec{j}|$. Then $|\vec{v}|$ lies in the interval - [2016]

(A) (0, 1] (B) (1, 2]

(C) (2, 3] (D) (3, 4]

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10. Let ABC be an acute scalene triangle, and O and H be its circumcentre and orthocenter respectively. Further let N be the midpoint of OH. The value of the vector sum $\vec{NA}+\vec{NB}+\vec{NC}$ is **[5 Nov, 2017]**

(A) $\vec{0}$ (zero vector) (B) $\overrightarrow{HO}$

(C) $\frac{1}{2}\overrightarrow{HO}$ (D) $\frac{1}{2}\overrightarrow{OH}$

11. A spherical ball is kept at the corner of a rectangular room such that the ball touches two (Perpendicular) walls and lies on the floor. If a point on the sphere is at distances of 9, 16, 25 from the two walls and the floor, then a possible radius of the sphere is **[19 Nov, 2017]**

(A) 13 (B) 15

(C) 26 (D) 36

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ANSWERS

1. (D) 2. (D) 3. (A) 4. (C) 5. (C) 6. (C) 7. (B)

8. (B) 9. (C) 10. (C) 11. (A)

SOLUTIONS

1. $\vec{u}\times\vec{v} = (2\hat{i}-\hat{j}+\hat{k})\times(-3\hat{j}+2\hat{k})$

$= -6\hat{k}-4\hat{j}-2\hat{i}+3\hat{i}$

$= \hat{i}-4\hat{j}-6\hat{k}$

Let $\vec{w} = a\hat{i}+b\hat{j}$

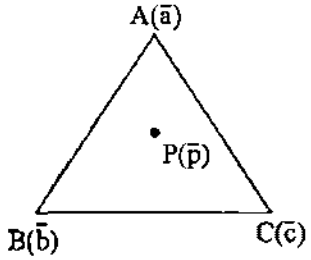
$a^2+b^2=1$

$a=\cos\theta$; $b=\sin\theta$

$\vec{u}\times\vec{v}.\vec{w} = a-4b = \cos\theta-4\sin\theta$

max. value $= \sqrt{1^2+(-4)^2} = \sqrt{17}$

2.



$\overrightarrow{PA}+2\overrightarrow{PB}+3\overrightarrow{PC}=0$

$(\vec{a}-\vec{p})+2(\vec{b}-\vec{p})+3(\vec{c}-\vec{p})=0$

$\vec{p}=\dfrac{\vec{a}+2\vec{b}+3\vec{c}}{6}$

$\dfrac{\text{Area }\Delta ABC}{\text{Area }\Delta APC} = \dfrac{\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{\frac{1}{2}|\vec{a}\times\vec{p}+\vec{p}\times\vec{c}+\vec{c}\times\vec{a}|}$

put $\vec{p}=\dfrac{\vec{a}+2\vec{b}+3\vec{c}}{6}$

ratio = 3

3. $\vec{a}\times\vec{b}+\vec{c}\times\vec{b}=0$ similarly $\vec{b}+\vec{c}=\lambda_2\vec{a}$

$\vec{a}+\vec{c}=\lambda_1\vec{b}$ $\vec{b}+\vec{a}=\lambda_3\vec{c}$

Hence $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

⇓

only 1 position of centroid

4. G is centroid

$G=\dfrac{A+B+C}{3}$; $G=\dfrac{2O+H}{3}$

$2O+H=3G$

$\overrightarrow{HA}+\overrightarrow{HB}+\overrightarrow{HC} = \vec{A}-\vec{H}+\vec{B}-\vec{H}+\vec{C}-\vec{H}$

$= \vec{A}+\vec{B}+\vec{C}-3\vec{H}$

$= 3\vec{G}-3\vec{H}$

$= 2\vec{O}+\vec{H}-3\vec{H}$

$= 2\vec{O}-2\vec{H}$

$= 2\overrightarrow{HO}$

5. $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ lies on unit circle

i.e. $v_1 = \cos\theta_1\hat{i}+\sin\theta_1\hat{j}$ θ_1 in I Quadrant

$v_2 = \cos\theta_2\hat{i}+\sin\theta_2\hat{j}$ θ_2 in II Quadrant

$v_3 = \cos\theta_3\hat{i}+\sin\theta_3\hat{j}$ θ_3 in III Quadrant

$v_4 = \cos\theta_4\hat{i}+\sin\theta_4\hat{j}$ θ_4 in IV Quadrant

(A) $\vec{v}_1+\vec{v}_2+\vec{v}_3+\vec{v}_4=0$ not necessarily true for all $v_1, v_2, v_3\ v_4$

(B) $\vec{v}_i+\vec{v}_j$ $(1\le i\le j\le 4)$

because

$\vec{v}_i+\vec{v}_j = (\cos\theta_i+\cos\theta_j)+i(\sin\theta_i+\sin\theta_j)$

If x coordinate is positive i.e.

θi is in I quadrant

θj is in IV quadrant

but in this case y coordinate is not necessarily positive.

(C) $\vec{v}_i.\vec{v}_j = \cos\theta i\cos\theta j+\sin\theta i\sin\theta j$

Let . θi is in I quadrant

θj is in III quadrant

$\vec{v}_i.\vec{v}_j<0$; $v_1.v_3<0$

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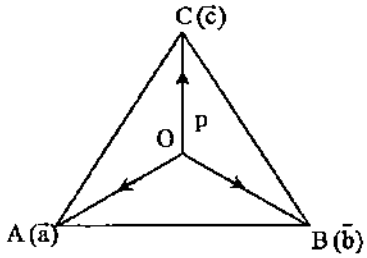
(D) $\bar{v}_i.\bar{v}_j = \cos\theta i \cos\theta j + \sin\theta i \sin\theta j$

$= \cos(\theta_i - \theta_j)$ is positive $0 < |\theta_i - \theta_j) < \frac{\pi}{2}$

$= \frac{3\pi}{2} < |\theta_i - \theta_j) < 2\pi$

Not positive for all values of v_1, v_2, v_3, v_4

6.



$\bar{a}, \bar{b}, \bar{c}$ are unit vectors

$\vec{a}\wedge\bar{b} = \bar{b}\wedge\bar{c} = \bar{c}\wedge\bar{a} = \frac{\pi}{3}$

Centre p $\left(\frac{\bar{o}+\vec{a}+\vec{b}+\vec{c}}{4}\right)$

Now angle between $\overrightarrow{AP}$ & $\overrightarrow{BP}$

$$\cos\theta = \frac{\overrightarrow{AP}\cdot\overrightarrow{BP}}{|\overrightarrow{AP}||\overrightarrow{BP}|}$$

$$= \frac{\left(\frac{\bar{a}+\bar{b}+\bar{c}}{4}-\bar{a}\right)\cdot\left(\frac{\bar{a}+\vec{b}+\vec{c}}{4}-\vec{b}\right)}{\left|\frac{\vec{a}+\bar{b}+\vec{c}}{4}-\vec{a}\right|\left|\frac{\vec{a}+\bar{b}+\vec{c}}{4}-\bar{b}\right|}$$

$$= \frac{(\vec{b}+\vec{c}-3\bar{a})\cdot(\bar{a}+\bar{c}-3\vec{b})}{|\vec{b}+\vec{c}-3\vec{a}|\cdot|\vec{a}+\vec{c}-3\vec{b}|}$$

$$= \frac{\bar{a}\cdot\vec{b}+\vec{b}\cdot\bar{c}-3\vec{b}^2+\bar{a}\cdot\bar{c}+\vec{c}^2-3\vec{b}\cdot\vec{c}-3\vec{a}^2 - 3\bar{a}\cdot\vec{c}+9\vec{a}\cdot\bar{b}}{(\bar{b}^2+\vec{c}^2+9\vec{a}^2+2\bar{b}\cdot\vec{c}-6\vec{a}\cdot\vec{c}-6\vec{a}\cdot\bar{b})}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}-3+\frac{1}{2}+1-\frac{3}{2}-3-\frac{3}{2}+\frac{9}{2}}{1+1+9+1-3-3}$$

$$= \frac{-5+3}{6} = -\frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

7. Sphere $x^2+y^2+z^2-4x-6x-12z+48=0$

Centre (2, 3, 6)

Radius $= \sqrt{4+9+36-48} = 1$

Distance between centre and origin

$= \sqrt{4+9+36} = 7$

Shortest distance $= 7-1$

$= 6$ (Origin lies outside the sphere)

8. $\vec{b} = \lambda(\hat{i}+\hat{j}+\hat{k}) \Rightarrow \vec{a} = \vec{b} + \vec{c}$

$\vec{c} = \vec{a} - \lambda(\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow \vec{c} = (6\hat{i}-3\hat{j}-6\hat{k}) - \lambda(\hat{i}+\hat{j}+\hat{k})$

$\vec{c} = (6-\lambda)\hat{i}+(-3-\lambda)\hat{j}+(-6-\lambda)\hat{k}$

$\vec{c}.\vec{\lambda} = 6-\lambda-3-\lambda-6-\lambda = 0$

$3\lambda = 3$

$\lambda = -1$

$\vec{c} = 7\hat{i}-2\hat{j}-5\hat{k}$

9. V is the circumcentre of $\Delta ABC \;\forall\; A \equiv (1, 0)$, $B \equiv (0, 1)$, $C(2, 0)$

Let V(x, y)

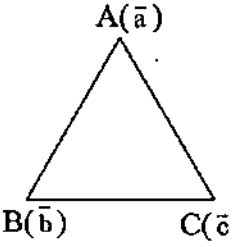
$VA = VB = VC$

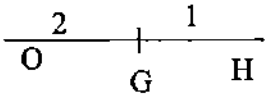
$(x-1)^2+y^2 = x^2+(y-1)^2 = (x-2)^2+y^2$

$(x, y) = \left(\frac{3}{2}, \frac{3}{2}\right) \Rightarrow V = \frac{3i+3j}{2}$

$|V| = \frac{3}{\sqrt{2}} \in (2,3)$

10. Circumcenter (origin O)





$(\bar{O}) = \left(\frac{\bar{a}+\vec{b}+\vec{c}}{3}\right)$; $H = \left(\frac{\vec{a}+\bar{b}+\vec{c}}{2}\right)$

$\bar{N} = \frac{1}{4}(\bar{a}+\bar{b}+\bar{c})$

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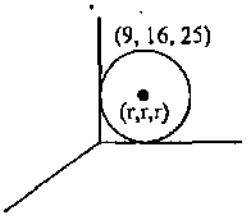
11.



$(r-9)^2+(r-16)^2+(r-25)^2=r^2$

solving this

$2r^2-100r+962=0$

$r^2-50r+481=0$

$r=37,\ r=13$

possible radius = 13

According to option

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Biology

[TOPIC- WISE KVPY [STREAM-SX] Questions with Solutions]

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Chapter 1

Diversity in the Living World, Structural Organisation in Plants and Animals

1. In 1670, Robert Boyle conducted an experiment where in he placed a viper (a poisonous snake) in a chamber and rapidly reduced the pressure in that chamber. Which of the following would be true ? [2010]
(A) Gas bubbles developed in the tissues of the snake
(B) The basal metabolic rate of the snake increased tremendously
(C) The venom of the snake was found to decrease in potency
(D) The venom of the snake was found to increase in potency

2 Bacteria can survive by absorbing soluble nutrients via their outer body surface, but animals cannot, because [2010]
(A) Bacteria cannot ingest particles but animals can
(B) Bacteria have cell walls and animals do not
(C) Animals have too small a surface area per unit volume as compared to bacteria
(D) Animals cannot metabolize soluble nutrients

3. Path-finding by ants is by means of- [2010]
(A) Visually observing landmarks
(B) Visually observing other ants
(C) Chemical signals between ants
(D) Using the earth's magnetic field

4. Bacteriochlorophylls are photosynthetic pigments found in phototrophic bacteria. Their function is distinct from the plant chlorophylls in that they- [2010]
(A) do not produce oxygen
(B) do not conduct photosynthesis
(C) absorb only blue light
(D) function without a light source

5. Four species of birds have different egg colors : [1] white with no markings, [2] pale brown with no markings. [3] grey-brown with dark streaks and spots, [4] pale blue with dark blue-green spots. Based on egg color, which species is most likely to nest in a deep tree hole? [2010]
(A) 1 (B) 2 (C) 3 (D) 4

6. Rodents can distinguish between many different types of odours. The basis for odour discrimination is that- [2011]
(A) they have a small number of odorant receptors that bind to many different odorant molecules
(B) the mechanoreceptors in the nasal cavity are activated by different odorant molecules found in the air passing through the nostrils
(C) the part of the brain that processes the sense of smell has many different receptors for odorant molecules
(D) a large number of different chemoreceptors are present in the nasal cavity that binds a variety of odorant molecules

7. Selection of lysine auxotroph (bacteria which requires lysine for growth) from a mixed population of bacteria can be done by growing the bacterial population in the presence of- [2011]
(A) lysine (B) penicillin
(C) lysine and penicillin (D) glucose

8. Increasing the number of measurements of an experimental variable will- [2011]
(A) increase the standard error of the sample
(B) increase the mean of the sample
(C) decrease the standard error of the sample
(D) result in all of the above

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9. Nocturnal animals have retinas that contain [2011]
(A) a high percentage of rods to increase sensitivity to low light conditions
(B) a high percentage of cones so that nocturnal color vision can be improved in low light conditions
(C) an equal number of rods and cones so that vision can be optimized
(D) retinas with the photoreceptor layer present in the front of the eye to increase light sensitivity

10. Some animals excrete uric acid in urine (uricotelic) as it requires very little water. This is an adaptation to conserve water loss. Which animals among the following are most likely to be uricotelic ? [2012]
(A) fishes (B) amphibians
(C) birds (D) mammals

11. Vibrio cholerae causes cholera in humans. Ganga water was once used successfully to combat the infection. The possible reason could be– [2012]
(A) High salt content of Ganga water
(B) Low salt content of Ganga water
(C) Presence of bacterophases in Ganga water
(D) Presence of antibiotics in Ganga Water

12. Fission yeasts are [2013]
(A) Archaebacteria (B) Eubacteria
(C) Prokaryotes (D) Eukaryotes

13. In orange and lemon, the edible part of the fruit is [2013]
(A) placenta
(B) thalamus
(C) hairs of the ovary wall
(D) succulent Mesocarp

14. Part of epidermis that keeps out unwanted particles is called' [2013]
(A) columnar epithelium
(B) squamous epithelium
(C) ciliated epithelium
(D) cuboidal epithelium

15. Which of the following results from conjugation in *Paramecium* ? [2014]
(A) Cell death (B) Cell division
(C) Budding (D) Recombination

16. A desert lizard (an ectotherm) and a mouse (an endotherm) are placed inside a chamber at 15 °C and their body temperature [T(L) for the lizard and T(M) for the mouse] and metabolic rates [M(L) for the lizard and M(M) for the mouse] are monitored. Which one of the following is correct - [2014]
(A) T(L) and M(L) will fall while T(M) and M(M) will increase
(B) T(L) and M(L) will increase while T(M) and M(M) will fall
(C) T(L) and M(L) will fall, T(M) will remain same and M(M) will increase
(D) T(L) and M(L) will remain same and T(M) and M(M) will decrease

17. Insectivorous plants such as Venus fly trap catch and digest insects in order to supplement the deficiency of [2016]
(A) Sulphur (B) Nitrogen
(C) Potassium (D) Phosphorus

18. Which of the following species makes the largest true flower in the world ? [2016]
(A) *Amorphophallus titanium*
(B) *Rafflesia arnoldii*
(C) *Nelumbo nucifera*
(D) *Helianthus annuus*

19. Match the type of cells given in Column I with organisms given in Column II. Choose the appropriate combination from the options below. [5 Nov, 2017]

Column I	Column II
(P) Flame cells	(i) Sponges
(Q) Collar cells	(ii) Hydra
(R) Stinging cells	(iii) Planaria

(A) P-iii, Q-i, R-ii (B) P-iii, Q-ii, R-i
(C) P-i, Q-ii, R-iii (D) P-ii, Q-iii, R-i

20. Amphimixis is [19 Nov, 2017]
(A) A fusion of pronuclei of male gametes.
(B) a fusion of pronuclei from male and female gametes
(C) a fusion of pronuclei of female gametes
(D) the development of a somatic cell into an embryo

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21. Botanical names of plants are given in Column-I, and the family/order name in Column-II. Choose the appropriate combination from the options below **[19 Nov, 2017]**

Column-I	**Column-II**
(P) *Tamarindus indica*	(i) Arecaceae
(Q) *Cocos nucifera*	(ii) Liliaceae
(R) *Colchicum automnale*	(iii) Solanaceae
(S) *Withania somnifera*	(iv) Papilionaceae

(A) P-iv, Q-i, R-ii, S-iii
(B) P-iv, Q-ii, R-iii, S-i
(C) P-i, Q-ii, R-iv, S-iii
(D) P-iv, Q-i, R-iii, S-ii

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ANSWERS

1. (A)	2. (C)	3. (C)	4. (A)	5. (A)	6. (D)	7. (D)
8. (C)	9. (A)	10. (C)	11. (C)	12. (D)	13. (C)	14. (C)
15. (D)	16. (C)	17. (B)	18. (B)	19. (A)	20. (B)	21. (A)

SOLUTIONS

10. Uricotelic organism excrete uric acid or its salts as a result of deamination. Uricotelic organisms included terrestrial arthropods, Lizards, Snakes and Birds.

11. Large number of bacteriophage are present in Ganga water which destroy bacteria.

12. Fission yeast eg. *Schizosaccharomyces octosporus* are ascomycetes member of fungus

13. Edible part of fruit in Orange and Lemon are juicy hairs.

14. Ciliated epithelium keeps the harmful particles out of the body by trapping them with their cilia. Example nasal epithelium.

15. Conjugation in paramecium results in recombination

16. Desert lizard is an ectotherm (poikilotherm) whose body temperature varies according to environmental temperature.
So at 15°C T(L) will fall due to fall in T(M)
While mouse is an endotherm (Homeotherm) whose body temperature remains constant always due to variation is metabolic rate
In homeotherms metabolic rate is inversely proportional to environmental temperature.
So at 15°C T(M) remain same and M((M) will increased.

17. Insectivorous plants grow in Nitrogen deficient soils and in order to compensate the deficiency they catch and digest insects and obtain N from chitin (NAG).

18. *Rafflesia arnoldii* is the total root parasite and forms the largest flower in angiosperms while *Amorphophallus titanium* is the largest inflorescence in the angiosperms

19. Flame cells are for secretion and osmoregulation in flatworms.
Collar cells line spongocoel and cavars in sponges and creat water amount.
Cnidocytes are stringing cells and help in defense, affiance and capture of prey these cells also help in attachment with substrate

20. Amphimixis → Formation of offspring due to fusion of ♂ & ♀ gametes

21. Fact based answer

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Chapter 2

Cell : Structure & Functions

1. Ribonucleic Acids (RNA) that catalyze enzymatic reactions are called ribozymes. Which one of the following acts as a ribozyme? **[2010]**
(A) Ribosome (B) Amylase
(C) tRNA (D) Riboflavin

2. Of the following combinations of cell biological processes which one is associated with embryogenesis ? **[2010]**
(A) Mitosis and Meiosis
(B) Mitosis and Differentiation
(C) Meiosis and Differentiation
(D) Differentiation and Reprogramming

3. The distance between two consecutive DNA base pairs is 0.34 nm. If the length of a chromosome is 1 mm, the number of base pairs in the chromosome is approximately- **[2010]**
(A) 3 million (B) 30 million
(C) 1.5 million (D) 6 million

4. A bust cell has intracellular bacteria symbionts. If the growth rate of the bacterial symbiont is always 10% higher than that of the host cell, after 10 generations of the host cell the density of bacteria in host cells will increase - **[2010]**
(A) by 10 % (B) two-fold
(C) ten-fold (D) hundred-fold

5. Which of the following correctly represents the results of an enzymatic reaction? Enzyme is E, substrate is S and products are P1 & P2. **[2010]**
(A) $P1 + S \Leftrightarrow P2 + E$
(B) $E + S \Leftrightarrow P1 + P2$
(C) $P1 + P2 + E \Leftrightarrow S$
(D) $E + S \Leftrightarrow P1 + P2 + E$

6. In which phase of the cell cycle are sister chromatids available as template for repair ? **[2011]**
(A) G1 phase (B) G2 phase
(C) S phase (D) M phase

7. If you fractionate all the organelles from the cytoplasm of a plant cell, in which one of the following sets of fractions will you find nucleic acids ? **[2011]**
(A) nucleus, mitochondria, chloroplast, cytoplasm
(B) nucleus, mitochondria, chloroplast, glyoxysome
(C) nucleus, chloroplast, cytoplasm and peroxisome
(D) nucleus, mitochondria, chloroplast, Golgi bodies

8. The Na^+/K^+ pump is present in the plasma membrane of mammalian cells where it- **[2011]**
(A) expels potassium from the cell
(B) expels sodium and potassium from the cell
(C) pumps sodium into the cell
(D) expels sodium from the cell

9. The process of cell death involving DNA cleavage in cells is known as **[2011]**
(A) necrosis (B) apoptosis
(C) cytokinesis (D) endocytosis

10. At which stage of Meiosis I does crossing over occur ? **[2011]**
(A) lepoptene (B) zygotene
(C) pachytene (D) diplotene

11. Human chromosomes undergo structural changes during the cell cycle. Chromosomal structure can be best visualized if a chromosome is isolated from a cell at **[2012]**
(A) G1 phase (B) S phase
(C) G2 phase (D) M phase

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12. A hormone molecule binds to a specific protein on the plasma membrane inducing a signal. The protein it binds to is called – **[2012]**
(A) ligand (B) antibody
(C) receptor (D) histone

13. Which one of the following is true about enzyme catalysis ? **[2012]**
(A) The enzyme changes at the end of the reaction
(B) The activation barrier of the process is lower in the presence of an enzyme
(C) The rate of the reaction is retarded in the presence of an enzyme
(D) The rate of the reaction is independent of substrate concentration

14. Nucleolus is an organelle responsible for the production of **[2013]**
(A) carbohydrates (B) messenger RNA
(C) lipids (D) ribosomal RNA

15. Genetic content of a cell reduces to half during **[2013]**
(A) meiotic prophase I
(B) mitotic prophase
(C) meiotic prophase II
(D) meiotic telophase

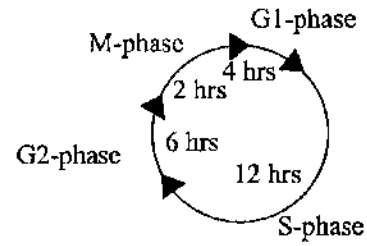
16. The two enzymatic activities associated with RUBISCO are **[2013]**
(A) oxidase and oxygenase
(B) oxygenase and carboxylase
(C) oxidase and carboxylase
(D) oxygenase and carbamylation

17 Which one of the following statements about nitrogenase is correct ? **[2013]**
(A) It is sensitive to CO_2 and therefore present in isolated nodules
(B) It requires O_2 and therefore functional during the day
(C) It is sensitive to O_2 and therefore is functional in anaerobic invironments
(D) It is sensitive to light and therefore function only in dark

18. Two tubes labelled 'P' and 'Q' contain food stuff. Tube 'P' gave positive test with Benedict's solution while tube 'Q' gave positive test with Nitric acid. Which of the following is correct ? **[2014]**
(A) Tube 'P' contains sugar; tube 'Q' contains protein
(B) Tube 'P' contains protein; tube 'Q' contains sugar
(C) Both, tube 'P' and tube 'Q' contain sugar
(D) Both, tube 'P' and tube 'Q' contain protein

19. A diploid plant has 14 chromosomes, but its egg cell has 6 chromosomes. which one of the following is the most likely explanation of this? **[2014]**
(A) Non-disjunction in meiosis I and II
(B) Non-disjunction in meiosis I
(C) Non-disjunction in mitosis
(D) Normal meiosis

20. Following the cell cycle scheme given below, what is the probability that a cell would be in M-phase at any given time ? **[2014]**



(A) 1/24 (B) 1/12 (C) 1/6 (D) 1/2

21. After meiosis-II, daughter cells differ from the parent cells and each other in their genotypes. This can occur because of which one of the following mechanism(s) ? **[2014]**
(A) Only synaptic crossing over
(B) Only crossing over and independent assortment of chromosomes
(C) Only crossing over and chromosomal segregation
(D) Crossing over, independent assortment and segregation of chromosomes

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22. Which one of the following has phosphoric acid anhydride bonds? [2015]
(A) Deoxy ribonucleic acid
(B) Ribonucleic acid
(C) dNTPs
(D) Phospholipids

23. Phospholipids are formed by the esterification of [2015]
(A) three ethanol molecules with three fatty acid molecules
(B) one glycerol and two fatty acid molecules
(C) one glycerol and three fatty acid molecules
(D) one ethylene glycol and two fatty acids molecules

24. Given the fact that histone binds DNA, it should be rich in [2015]
(A) arginine, lysine
(B) cysteine, methionine
(C) glutamate, aspartate
(D) isoleucine, leucine

25. Which one of the following can be used to detect amino acids? [2015]
(A) Iodine vapour (B) Ninhydrin
(C) Ethidium bromide (D) Bromophenol blue

26. Which of the following processes leads to DNA ladder formation? [2015]
(A) Necrosis (B) Plasmolysis
(C) Apoptosis (D) Mitosis

27. Co-enzymes are components of an enzyme complex which are necessary for its function. Which of these is a known co-enzyme? [2015]
(A) Zinc (B) Vitamin B_{12}
(C) Chlorophyll (D) Heme

28. The peptidoglycans of bacteria consist of [2015]
(A) sugars, D-amino acids and L-amino acids
(B) sugars and only D-amino acids
(C) sugars and only L-amino acids
(D) sugars and glycine

29. The mitochondrial inner membrane consists of a number of infoldings called cristae. The increased surface area due to cristae helps in : [2015]
(A) Increasing the volume of mitochondria
(B) Incorporating more of the protein complexes essential for electron transport chain
(C) Changing the pH
(D) Increasing diffusion of ions

30. Which one of the following pairs of molecules never forms a hydrogen bond between them ? [2016]
(A) Water and water (B) Water and glucose
(C) Water and ethanol (D) Water and octane

31. Lactase hydrolyses lactose into [2016]
(A) Glucose + glucose
(B) Glucose + galactose
(C) Galactose + galactose
(D) Galactose + fructose

32. Which of the following statements is incorrect regarding biological membrane ? [2016]
(A) It is composed of lipids and proteins
(B) Peripheral proteins are loosely associate with the membrane
(C) Integral proteins span the lipid bilayer
(D) Lipids and membrane proteins do not provide structural and functional asymmetry

33. Which of the following statements about nucleosome is true ? [2016]
(A) It consists of only DNA
(B) It is a nucleus-like structure found in prokaryotes
(C) It consists of DNA and proteins
(D) It consists of only histone proteins

34. Epithelial cells in animals are held by specialized junctions, one of them being "Gap junction". Function of a "Gap junction" is to [2016]
(A) Facilitate cell-cell communication by rapid transfer of small molecules
(B) Cement the neighbouring cells
(C) Stop substances from leaking
(D) Provide gaps in the tissue to facilitate uptake of small molecules across tissues

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35. Match the following organelles in Group I with the structures in Group II. Choose the correct combination. [2016]

Group I	Group II
P. Mitochondrion	i. Cisternae
Q. Golgi	ii. Cristae
R. Chloroplast	iii. Thylakoids
S. Centrosome	iv. Radial spokes

(A) P-ii , Q-i, R-iii, S-iv
(B) P-iii, Q-i, R-ii, S-iv
(C) P-iv, Q-i, R-ii, S-iii
(D) P-iv, Q-ii, R-i, S-iii

36. Which of the following best describes the DNA content and the number of chromosomes at the end of S and M phases of the cell cycle in mitosis, if the DNA content of the cell in the beginning of cell cycle (G1 phase) is considered as C and the number of chromosomes 2N ? [2016]
(A) 2C and 2N for S phase ; 2C and 2N for M phase
(B) 2C and N for S phase ; 2C and N for M phase
(C) 2C and 2N for S phase ; C and 2N for M phase
(D) C and N for S phase ; C and 2N for M phase

37. The rigidity of cellulose is due to [5 Nov, 2017]
(A) coiled structure of glucose polymer
(B) $\beta(1 \rightarrow 4)$ glycosidic linkage
(C) hydrogen bonding with adjacent glucose polymer
(D) cross-linking between glucose and peptides

38. Which ONE of the following statements is INCORRECT about restriction endonucleases? [5 Nov, 2017]
(A) They serve as primitive form of immune system in bacteria.
(B) They digest the DNA non-randomly.
(C) They digest the DNA at specific location.
(D) They digest the DNA from free ends.

39. Proteins P, Q, and R are associated with intact organellar membrane in a cell. If the intact organelle is treated with a high ionic strength buffer, only protein R remained associated with the membrane fraction. Based on this, one could conclude that [5 Nov, 2017]
(A) P and Q are peripheral membrane proteins.
(B) R is a peripheral membrane protein.
(C) P and Q are integral membrane bound proteins.
(D) P is peripheral and Q is integral membrane protein.

40. How many different proteins consisting of 100 amino acids can be formed from 20 different amino acids ? [5 Nov, 2017]
(A) 20^{100} (B) 100^{20}
(C) 2^{20} (D) 20×100

41. Choose the CORRECT combination of statements given below related to cysteine residue in proteins. [5 Nov, 2017]
i. Cysteine can be linked to tyrosine by S-O bond.
ii. Cysteine can be linked to another cysteine by S-S bond.
iii. Cysteine can complex with Zn^{2+}.
iv Cysteine can be linked to methionine by S-S bond
(A) i and ii (B) ii and iii
(C) iii and iv (D) i and iv

42. Which ONE of the following is a hetero-polysaccharide ? [19 Nov, 2017]
(A) Glycogen (B) Starch
(C) Cellulose (D) Hyaluronic acid

43. Bacterial plasmids are genetic entities that, [19 Nov, 2017]
(A) are non-transferable to the same bacterial species.
(B) are capable of independent replication.
(C) have RNA as genetic material.
(D) always require integration in the genome for their replication.

44. Which ONE of the following statements is INCORRECT for translation in cytoplasm ? [19 Nov, 2017]
(A) One codon codes for only one amino acid.
(B) One amino acid may be coded by many codons.
(C) More than one amino acids are coded by one specific condon.
(D) There are some codons that do not code for any amino acid.

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45. The protein P, the oligosaccharide O, and the oligonucleotide N are composed of 100 amino acid residues, 100 hexose residues, and 100 nucleotides, respectively. Which ONE of the following orders of molecular weights is CORRECT? **[19 Nov, 2017]**

(A) P > O > N (B) P > N > O

(C) N > O > P (D) O > P > N

46. An octapeptide (NH_2-Asn-Glu-Tyr-Lys-Trp-Met-Glu-Gly) is subjected to complete protease and chemical digestion. Based on the results obtained, choose the INCORRECT option from below. **[19 Nov, 2017]**

(A) Trypsin generates mixtures of dimer and trimer

(B) Trypsin generates tetramers only

(C) Cyanogen bromide generates a hexamer and a dimer

(D) Chymotrypsin generates mixture of dimer and trimers

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ANSWERS

1. (A)	2. (B)	3. (A)	4. (B)	5. (D)	6. (D)	7. (A)
8. (D)	9. (B)	10. (C)	11. (D)	12. (C)	13. (B)	14. (D)
15. (D)	16. (B)	17. (C)	18. (A)	19. (B)	20. (B)	21. (D)
22. (C)	23. (B)	24. (A)	25. (B)	26. (C)	27. (B)	28. (A)
29. (B)	30. (D)	31. (B)	32. (D)	33. (C)	34. (A)	35. (A)
36. (C)	37. (C)	38. (D)	39. (A)	40. (A)	41. (B)	42. (D)
43. (B)	44. (C)	45. (C)	46. (A)			

SOLUTIONS

11. Chromosome structure is best seen at metaphase which is a sub stage of M-phase of the cell cycle.

12. Hormone receptors for protein hormone are present on the surface of plasma membrane of cell.

14. Nucleolus are also termed as ribosomal factories and synthesizes rRNA.

15. DNA content reduces to half each time during meiotic telophase i.e. during separation of homologous chromosome and also during separation of sister chromatids.

16. RUBISCO → Ribulose-1, 5-bis phosphate carboxylase/oxygenase

17. Nitrogenase enzyme is sensitive to oxygen and so requires anaerobic environment for its functioning.

18. → Nitric acid reacts with proteins to form yellow nitrated products.
→ Benedict test is used to test the presence of monosaccharide and reducing sugar.

19. Egg cell is formed by meiosis. Less number of chromosome indicates non-disjunction in meiosis I.

20. Total time for cell cycle = 24 hrs.
Time for M-phase = 2 hrs
So, probability of cell in M-phase at any given time is $\frac{2}{24} \Rightarrow \frac{1}{12}$

21. After meiosis-II, daughter cells differ from the parent cells and each other in their genotypes due to Crossing over, independent assortment and segregation of chromosomes

30. Hydrogen bond is a weak bond between two molecules resulting from an electrostatic attraction between a proton in one molecule and an electronegative atom in another molecule. This is not possible in case of water and octane

31. Lactose $\xrightarrow{\text{Lactase}}$ Glucose + Galactose

32. Proteins provide asymmetry to plasma membrane, as they are of 2 types i.e. peripheral and integral.

33. Nucleosome is the smallest unit of DNA packaging containing 200 nitrogen bases and four types of histone proteins i.e. H_2A, H_2B, H_3 and H_4. H_1 type of histone is used in plugging.

34. Gap junctions are cytoplasmic communications between two epithelial cells for rapid transfer of some ions and small molecule

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35. Cristae are invaginations of inner mitochondrial membrane. Cisternae is unit of golgi body where glycosidation and glycosylation takes place. Grana of chloroplast are composed of thylakoids. Centrosome contains centrioles with radial spokes.

36.

	G_1	$\longrightarrow$ S	$\longrightarrow G_2$	$\longrightarrow$ M
DNA	C	2C	2C	$2C \rightarrow C$
Content No. of Chromosome	2N	2N	2N	$2N \rightarrow 4N \rightarrow 2N$

37. Bundle of cellulose fibre are rigid to H-bonds.

38. Endonucleases do not cut from free ends.

39. Peripheral proteins can be detached eaily

40. 20^{100}

41. Fact base answer

42. Hyaluronic acid is polymer of D-Guluconic acid, N-Acetyl, D-Glucose amine. So heteropolysaccharide.

43. It is extra chromosomal genetic material capable of independent replication.

44. One specific codon codes for only one aminoacid.

45. Monomer of protein is amino acid oligosaccharide is sugar & oligonucleotide is nucleotides order of its molecular weight is Nucleotide > Monosaccharide > Amino acid

46. **Cleavage site** :- for Trypsin-After Lys & Arg
For chymotrypsin - After Phe, Trp, or Tyr
For cyanogen bromide- After met

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Chapter 3

Plant Physiology

1. If the total number of photons falling per unit area of a leaf per minute is kept constant, then which of the following will result in maximum photosynthesis ? [2010]
(A) Shining green light
(B) Shining sunlight
(C) Shining blue light
(D) Shining ultraviolet light

2. Fruit wrapped in paper ripens faster than when kept in open air because- [2010]
(A) Heat of respiration is retained better
(B) A chemical in the paper helps fruit ripening
(C) A volatile substance produced by the fruit is retained better and helps in ripening
(D) The fruit is cut off from the ambient oxygen which is an inhibitor to fruit repening

3. If you dip a sack full of paddy seeds in water overnight and then keep it out for a couple of days, it feels warm. What generates this heat ? [2010]
(A) Imbibation
(B) Exothermic reaction between water and seed coats
(C) Friction among seeds due to swelling
(D) Respiration

4. You marked two ink-spots along the height at the base of a coconut tree and also at the top of the tree. When you examine the spots next year when the tree has grown taller, your will see- [2010]
(A) the two spots at the top have grown more apart than the two spots at the bottom
(B) the top two spots have grown less apart then the bottom two spots
(C) both sets of spots have grown apart to the same extent
(D) both sets of spots remain un-altered

5. A water molecule can from a maximum of ..... hydrogen bonds. [2011]
(A) 1 (B) 2 (C) 3 (D) 4

6. Which sequence of events gives rise to flaccid guard cells and stomatal closure at night ? [2011]
(A) low [Glucose] $\Rightarrow$ low osmotic pressure $\Rightarrow$ low pH $\Rightarrow$ high pCO_2
(B) low pH $\Rightarrow$ high pCO_2 $\Rightarrow$ low [Glucose] $\Rightarrow$ low osmotic pressure
(C) low osmotic pressure $\Rightarrow$ high pCO_2 $\Rightarrow$ low pH $\Rightarrow$ low [Glucose]
(D) high pCO_2 $\Rightarrow$ low pH $\Rightarrow$ low [Glucose] $\Rightarrow$ low osmotic pressure

7. A ripe mango, kept with unripe mangoes causes their ripening. This is due to the release of a gaseous plant hormone– [2012]
(A) auxin (B) gibberlin
(C) cytokinine (D) ethylene

8. Which of the following is true for TCA cycle in eukaryotes ? [2012]
(A) takes place in mitochondrion
(B) produces no ATP
(C) takes place in golgi complex
(D) independent of electron transport chain

9. Plant roots are usually devoid of chlorophyll and cannot perform photosynthesis. However, there are exceptions. Which of the following plant root can perform photosynthesis ? [2012]
(A) Arabidopsis (B) Tinospora
(C) Rice (D) Hibiscus

10. Insectivorous plants that mostly grow on swampy soil use insects as a source of– [2012]
(A) Carbon (B) Nitrogen
(C) Phosporous (D) Magnesium

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11. In green leaves, the light and dark reaction occur in [2013]
(A) stroma and grana respectively
(B) grana and stroma respectively
(C) cristae and matrix respectively
(D) both occur in cytoplasm

12. In an experiment investigating photoperiodic response, the leaves of a plant are removed. What is the most likely outcome ? [2014]
(A) Photoperiodism is not affected
(B) Photoperiodic response does not occur
(C) The plant starts flowering
(D) The plant starts to grow taller

13. Name the terminal acceptor of electrons in the mitochondrial electron transport chain [2014]
(A) Nitrate (B) Fumarate
(C) Succinate (D) Oxygen

14. If the humidity of the atmosphere suddenly increases substantially, the water flow in the xylem will - [2014]
(A) increase
(B) decrease
(C) remain unaltered
(D) increase sharply and then reduce slowly to the preexisting level

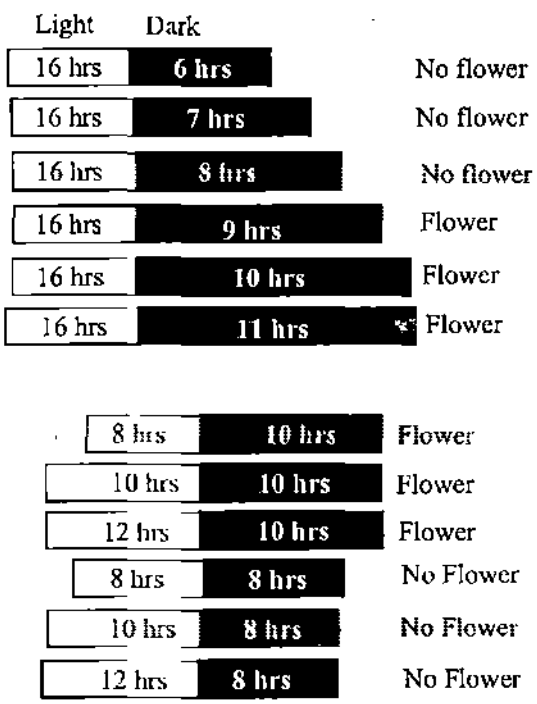
15. If mitochondria isolated from a cell are first placed without carbon source in a buffer at pH 8.0 and then transferred to a buffer at pH4 , it will lead to - [2014]
(A) an increase in intra-mitochondrial acidity
(B) a decrease in intra-mitochondrial acidity
(C) blockage of ATP synthesis
(D) synthesis of ATP

16. Which of the following molecules is a primary acceptor of CO_2 in photosynthesis ? [2016]
(A) Pyruvate
(B) 3-phosphoglycerate
(C) Phosphoenol pyruvate
(D) Oxaloacetate

17. The percentage of sunlight captured by plants is [2016]
(A) 2-10% (B) 10-20%
(C) 60-80% (D) 100%

18. If $^{14}CO_2$ is added to a suspension of photosynthesizing chloroplasts, which of the following will be the first compound to be radioactive ? [2016]
(A) ATP (B) NADPH
(C) NADH (D) 3-phosphoglycerate

19. In an experiment represented in the schematic below, a plant species was grown in different day and night cycles and its photoperiodic flowering behaviour was noted. This species is a [2016]


Light
Dark
16 hrs 6 hrs No flower
16 hrs 7 hrs No flower
16 hrs 8 hrs No flower
16 hrs 9 hrs Flower
16 hrs 10 hrs Flower
16 hrs 11 hrs Flower
8 hrs 10 hrs Flower
10 hrs 10 hrs Flower
12 hrs 10 hrs Flower
8 hrs 8 hrs No Flower
10 hrs 8 hrs No Flower
12 hrs 8 hrs No Flower

(A) short day plant and actually measures day length to flower
(B) short day plant and actually measures night length to flower
(C) long day plant and actually measures night length to flower
(D) long day plant and actually measures day length to flower

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20. In a chemical reaction, enzymes catalyze the reaction by **[5 Nov, 2017]**
(A) lowering the activation energy.
(B) increasing the activation energy.
(C) decreasing the free energy change between reactants and products.
(D) increasing the free energy change between reactants and products.

21. The number of net ATP molecules produced from 1 glucose molecule during glycolysis is **[5 Nov, 2017]**
(A) 1 (B) 2
(C) 3 (D) 4

22. Which ONE of the following coenzymes is required for the conversion of L-alanine to a racemic mixture of D-and L-alanine ? **[5 Nov, 2017]**
(A) Pyridoxal-6-phosphate
(B) Thiamine pyrophosphate
(C) Coenzyme A
(D) Flavin adenine dinucleotide

23. The cyclic electron flow during photosynthesis generates **[5 Nov, 2017]**
(A) NADPH alone.
(B) ATP and NADPH.
(C) ATP alone.
(D) ATP, NADPH and O_2.

24. In photosynthesis, oxygen is produced by **[5 Nov, 2017]**
(A) photosystem I from carbon dioxide.
(B) photosystem II from carbon dioxide.
(C) photosystem I from water.
(D) photosystem II from water.

25. Which ONE of the following molecules serves as a substrate for direct synthesis of ATP ? **[19 Nov, 2017]**
(A) 1, 3-bisphosphoglycerate
(B) Glucose 6-phosphate
(C) Pyruvate
(D) Fructose 1,6-bisphosphate

26. If a pure chlorophyll solution is illuminated with ultraviolet light, the solution appears **[19 Nov, 2017]**
(A) green (B) violet
(C) red (D) black

27. Nitrogen fixation is inhibited by oxygen. However, in aerobic nitrogen fixing bacteria, nitrogen is fixed in the presence of oxygen. Nitrogenase in such organisms is protected by which ONE o the following mechanisms **[19 Nov, 2017]**
(A) channelizing oxygen to form ozone
(B) removal of oxygen by metabolic activity
(C) utilizing oxygen for membrane remodelling
(D) utilizing oxygen for synthesis of pentapeptide chain in peptidoglycan

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ANSWERS

1. (C)	2. (C)	3. (D)	4. (A)	5. (D)	6. (D)	7. (D)
8. (A)	9. (B)	10. (B)	11. (B)	12. (B)	13. (D)	14. (B)
15. (D)	16. (C)	17. (A)	18. (D)	19. (B)	20. (A)	21. (B)
22. (A)	23. (C)	24. (B)	25. (A)	26. (C)	27. (B)	

SOLUTIONS

7. The only gaseous hormone out of these four options is ethylene. Its main function is ripening.

8. TCA (Tricarboxylic acid cycle), also known as Krebs' cycle, takes place in matrix of mitochondrion.

9. Tinospora and Trapa are plants which have assimilatory or photosynthetic roots.

10. Insectivorous plants are grown in nitrogen deficient soil and they get nitrogen by trapping insect e.g. *Utricularia, Dionea, Nepenthes.*

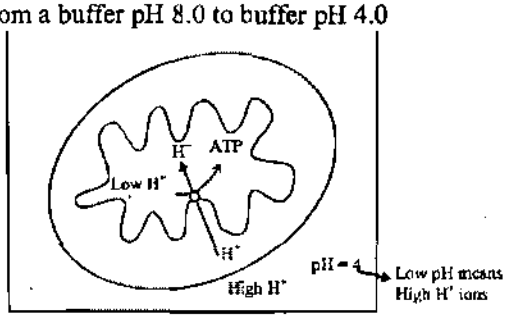
11. Grana → light reaction
Stroma → Dark/Light independent reaction

12. Leaves are the site for photoperiodic perception.

13. Oxygen is the terminal acceptor of electrons in mitochondrial ETS.

14. Humidity of atmosphere is inversely proportional to transpiration (water flow). Increase in humidity will decrease the water flow in the xylem.

15. Mitochondria synthesizes ATP based on chemiosmosis when mitochondria is transferred from a buffer pH 8.0 to buffer pH 4.0



16. In C_4 plants the primary acceptor of CO_2 is phosphoenol pyruvate, a 3C compound. e.g. In Maize, Sugarcane etc.

17. Plants capture 2-10 % of PAR

18. During Calvin cycle, the $^{14}CO_2$ is incorporated with a 5C compound Ribulose 1-5 Biphosphate and forms 2 molecules of 3C compound 3-phosphoglycerate.

19. SDP (Short day plants) or LNP (Long Night Plants) flowers only when photoperiod is below critical day length (Critical photoperiod) or they are responsible to night length and flower when night length is above critical dark period.
In this experiment plant flowers when dark period is above 8 hrs.
So, it is SDP and actually measures night length to flower.

20. Enzyme decreases activation energy in reaction.

21. In glycolysis net ATP produced
= 4 – 2 = 2 ATP

22. Information based question

23. In cyclic photophosphorylation only ATP is formed.

24. photosynthesis at Ps II in lumen of thylakoid

25. 1,3-biPGA → 3 PGA
ADP → ATP

26. Fact based answer

27. Excess O_2 is used for metabolic activity.

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Chapter 4

Human Physiology

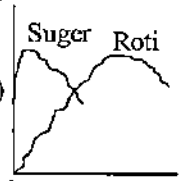
1. When a person is suffering from high fever, it is sometimes observed that the skin has a reddish tinge. Why does this happen ? **[2010]**
(A) Red colour of the skin radiates more heat
(B) Fever causes the release of a red pigment in the skin
(C) There is more blood circulation to the skin to keep the body warm
(D) There is more blood circulation to the skin to release heat from the body

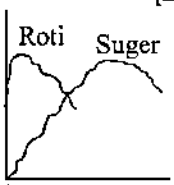
2. Athletes often experience muscle cramps. Which of the following statements is true muscle cramps ? **[2010]**
(A) Muscle cramp is caused due to conversion of pyruvic acid into lactic acid in the cytoplasm
(B) Muscle cramp is caused due to conversion of pyruvic acid into lactic acid in the mitochondria
(C) Muscle cramp is caused due to nonconversion of glucose to pyruvate in the cytoplasm
(D) Muscle cramp is caused due to conversion of pyruvic acid into ethanol in the cytoplasm

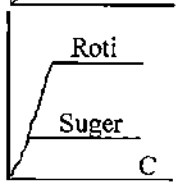
3. The fluid part of blood flows in and out of capillaries in tissue to exchange nutrients and waste materials. Under which of the following conditions will fluid flow out from the capillaries into the surrounding tissue ? **[2010]**
(A) When arterial blood pressure exceeds blood osmotic pressure
(B) When arterial blood pressure is less than blood osmotic pressure
(C) When arterial blood pressure is equal to blood osmotic pressure
(D) Arterial blood pressure and blood osmotic pressure have nothing to do with the outflow of fluid from capilleries

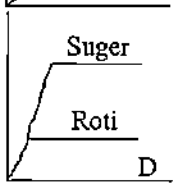
4. Estimate the order of the speed of propagation of an action potential or nerve impulse - **[2010]**
(A) nm/s (B) micron/s
(C) cm/s (D) m/s

5. Which of the following graphs accurately represents the insulin levels (Y-axis) in the body as a function of time (X-axis) after eating sugar and bread/roti ? **[2010]**

(A) 

(B) 

(C) 

(D) 

6. The major constituents of neurofilaments are- **[2011]**
(A) microtubules
(B) intermediate filaments
(C) actin filaments
(D) protofilaments

7. A person has difficulty in breathing at higher altitudes because- **[2011]**
(A) oxygen is likely to diffuse from lungs to blood
(B) oxygen is likely to diffuse from blood to lungs
(C) partial pressure of O_2 is lower than partial pressure of CO_2
(D) overall intake of O_2 by the blood becomes low

8. The CO_2 in the blood is mostly carried- **[2011]**
(A) by hemoglobin in RBCs
(B) in the cytoplasm of WBCs
(C) in the plasma as bicarbonate ions
(D) by plasma proteins

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9. Circadian Rhythm is an endogenously driven cycle for biochemical, physiological and behavioral processes. In humans, the approximate duration of this 'biological clock' is- [2011]
(A) 1 Hour (B) 6 Hours
(C) 12 Hours (D) 24 Hours

10. An electrode is placed in the axioplasm of a mammalian axon and another electrode is placed just outside the axon. The potential difference measured will be- [2011]
(A) 0 (B) –70 mV
(C) –70 μV (D) +70 μV

11. Although blood flows through large arteries at high pressure, when the blood reaches small capillaries the pressure decreases because- [2011]
(A) the valves in the arteries regulate the rate of blood flow into the capillaries
(B) the volume of blood in the capillaries is much lesser than that in the arteries
(C) the total cross-sectional area of capillaries arising from an artery is much greater than that of the artery
(D) elastin fibers in the capillaries help to reduce the arterial pressure

12. Peptic ulcers are caused by – [2012]
(A) a fungus, Candida albicans
(B) a virus, cytomegalovirus
(C) a parasite, Trypanosoma brucei
(D) a bacterium, Helicobacter pylori

13. By which of the following mechanisms is glucose reabsorbed from the glomerular filtrate by the kidney tubule ? [2012]
(A) osmosis (B) diffusion
(C) active transport (D) passive transport

14. In mammals, the hormones secreted by the pituitary, the master gland, is itself regulated by [2012]
(A) hypothalamus (B) median cortex
(C) pineal gland2 (D) cerebrum

15. Vitamin A deficiency leads to night-blindness. Which of the following is the reason for the disease ? [2012]
(A) rod cells are not converted to cone cells
(B) rhodopsin pigment of rod cells is defective
(C) melanin pigment is not synthesized in cone cells
(D) cornea of eye gets dried

16. A reflex action is a quick involuntary response to stimulus. Which of the following is an example of BOTH, unconditioned and conditioned reflex ? [2012]
(A) knee Jerk reflex
(B) secretion of saliva in response to the aroma of food
(C) sneezing reflex
(D) contraction of the pupil in response to bright light

17. Why hydrogen peroxide is applied on the wound as a disinfectant, there is frothing at the site of injury, which is due to the presence of an enzyme in the skin that used hydrogen peroxide as a substrate to produce– [2012]
(A) Hydrogen
(B) Carbon Dioxide
(C) Water
(D) Oxygen

18. Persons suffering from hypertension (high blood pressure) are advised a low-salt diet because– [2012]
(A) More salt is absorbed in the body of a patient with hypertension
(B) High salt leads to water retention in the blood that further increases the blood pressure
(C) High salt increases nerve conduction and increases blood pressure
(D) High salt causes adrenaline release that increases blood pressure

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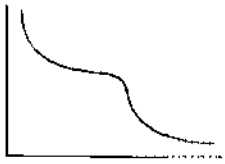
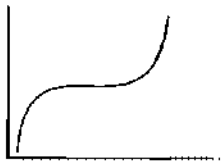
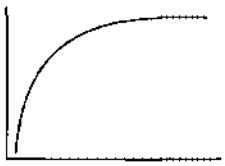
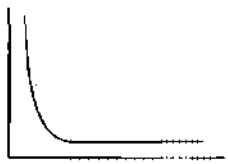
19. When a person beings to fast, after some time glycongen stored in the liver is mobilized as a source of glucose. Which of the following graphs best represents the change of glucose level (y axis) in his blood, starting from the time (x axis) when the beings to fast ? [2012]

(A) Glucose

Time

(B) Glucose

Time

(C) Glucose

Time

(D) Glucose

Time

20. Human fetal haemoglobin differs from the adult haemoglobin in that it has- [2013]
(A) higher affinity for oxygen
(B) lower affinity for oxygen
(C) two subunits only
(D) is glycosylated

21. Stroke could be prevented/treated with [2013]
(A) balanced diet (B) clotting factors
(C) insulin (D) blood thinners

22. In vertebrates 'glycogen' is stored chiefly in [2013]
(A) heart and blood
(B) spleen and stomach
(C) bones and lymph
(D) liver and muscles

23. Brown fat is a specialized adipose tissue with abundant mitochondria and rich blood supply. Brown fat [2013]
(A) insulates animals that are acclimatized to cold.
(B) is the major source of heat production in birds.
(C) provides energy tomuscles.
(D) produces heat without producing ATP.

24. In which of the following types of glands is the secretion collected inside the cell and discharged by disintegration of the entire gland? [2014]
(A) Apocrine (B) Merocrine
(C) Holocrine (D) Epicrine

25. Where is the third ventricle of the brain located? [2014]
(A) Cerebrum (B) Cerebellum
(C) Pons varoli (D) Diencephalon

26. Which of the following is secreted at the ends of an axon ? [2014]
(A) Ascorbic acid (B) Acetic acid
(C) Acetyl choline (D) Acetyl CoA

27. The two components of autonomous nervous system have antagonistic actions. But in certain cases their effects are mutually helpful. Which of the following statement is correct? [2015]
(A) At rest, the control of heart beat is not by the vagus nerve
(B) During exercise the sympathetic control decreases
(C) During exercise the parasympathetic control decreases
(D) Stimulation of sympathetic system results in constriction of the pupil

28. A person has 400 million alveoli per lung with an average radius of 0.1 mm for each alveolus. Considering the alveoli are spherical in shape, the total respiratory surface of that person is closest to [2015]
(A) 500 mm^2 (B) 200 mm^2
(C) 100 mm^2 (D) 1000 mm^2

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29. Which of the following statements is true about glandular epithelium in salivary gland ?**[2016]**
(A) It consists of isolated single cells
(B) It consists of mutlicellular cluster of cells
(C) Its secretions are endocrine
(D) It consists of squamous epithelial cells

30. Which one of the following ion pairs is involved in nerve impulses ? **[2016]**
(A) Na^+, K^+ (B) Na^+, Cl^-
(C) K^+, Cl^- (D) K^+, Ca^{2+}

31. Which of the following hormones that controls blood pressure is secreted by human heart ? **[2016]**
(A) Erythropoietin
(B) Atrial natriuretic factor
(C) ACTH
(D) Glucocorticoid

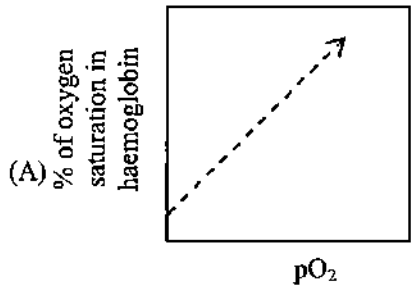
32. Oxytocin and vasopressin are synthesized in **[2016]**
(A) Hypothalamus (B) Adrenal gland
(C) Pituitary gland (D) Ovary

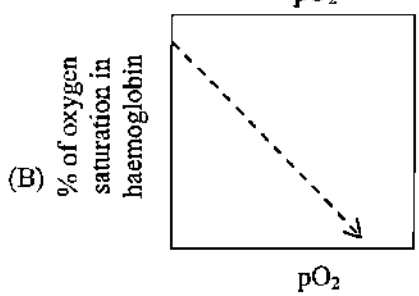
33. If you exhale multiple times into a conical flask containing lime water through a single inlet fixed through a stop cork, lime water will ? **[2016]**
(A) Become cooler
(B) Turn milky
(C) Remain unchanged
(D) Turn yellow

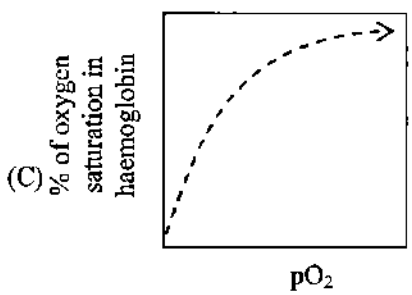
34. The path of passage of stimulus when you accidentally touch a hotplate is **[2016]**
(A) Receptor → Brain → Muscles
(B) Muscles → Spinal cord → Receptor
(C) Muscles → Brain → Receptor
(D) Receptor → Spinal cord → Muscles

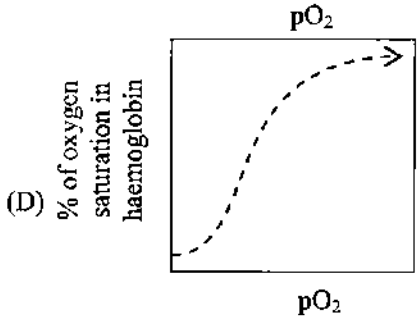
35. Which of the following anions neutralize the acidic pH of the chyme that enters into the duodenum from the stomach ? **[2016]**
(A) $H_2PO_4^-$ (B) HSO_4^-
(C) HCO_3^- (D) CH_3COO^-

36. Which of the following graphs best describes the oxygen dissociation curve where pO_2 is the partial pressure of oxygen ? **[2016]**

(A) 

(B) 

(C) 

(D) 

37. Leydig cells synthesize **[5 Nov, 2017]**
(A) insulin (B) growth hormone
(C) testosterone (D) estrogen

38. Glucagon increases the blood glucose concentration by **[5 Nov, 2017]**
(A) promoting glycogenolysis.
(B) increasing the concentration of fructose 2,–6-bisphosphate.
(C) increasing the concentration of pyruvate kinase.
(D) inhibiting gluconeogenesis.

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39. Concentration of Na^+ ions outside a nerve cell is ~100 times more than inside. The concentration of K^+ ions is more inside the cells. The levels of Na^+ ions and K^+ ions are maintained by **[5 Nov, 2017]**

(A) free diffusion of Na^+ ions and pumping of K^- ions across the membrane.
(B) Na^+ and K^+ pumps in the membrane.
(C) free diffusion of K^+ ions and pumping of Na^+ ions across the membrane.
(D) water channels formed by lipids in the membrane.

40. Compared to the atmospheric air, the alveolar air has **[5 Nov, 2017]**

(A) more pO_2 and less pCO_2
(B) less pO_2 and pCO_2
(C) more pO_2 and more pCO_2
(D) less pO_2 and less pCO_2

41. Resting membrane potential of a neuron is approximately **[19 Nov, 2017]**

(A) – 70 mV (B) + 70 mV
(C) – 0.7 V (D) + 0.7 V

42. Activation of sympathetic nervous system **[19 Nov, 2017]**

(A) decreases blood pressure.
(B) causes pupil contraction.
(C) increases heart rate.
(D) causes bronchoconstriction.

43. At physiological temperature, sterols in biological membranes **[19 Nov, 2017]**

(A) increase their fluidity.
(B) decrease their fluidity.
(C) increase their permeability to water.
(D) decrease their permeability to water.

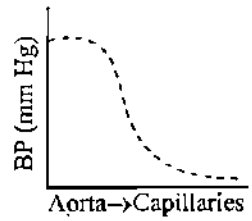
44. In diabetic patients, the pH of blood plasma can decrease leading to acidosis. This is because tissues catabolise **[19 Nov, 2017]**

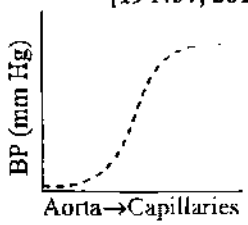
(A) amino acids leading to loss of buffering capacity of the blood
(B) stored glycogen leading to the accumulation of pyruvic acid
(C) stored fatty acids leading to the accumulation of beta hydroxybutyric acid and acetoacetic acid
(D) nucleic acid pool leading to decrease in blood pH

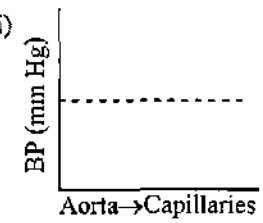
45 If the number of alveoli in an individual is doubled without changing the total alveolar volume, the gas exchange capacity of the lungs will **[19 Nov, 2017]**

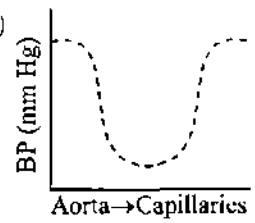
(A) increase for both O_2 and CO_2
(B) decrease for both O_2 and CO_2
(C) remain unaltered for both O_2 and CO_2
(D) increase for O_2 and decrease for CO_2

46. Which ONE of the following graphs best describes the blood pressure (BP) change when blood moves from aorta to capillaries ? **[19 Nov, 2017]**

(i)


(ii)


(iii)


(iv)


(A) (i)
(B) (ii)
(C) (iii)
(D) (iv)

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ANSWERS

1. (D)	2. (A)	3. (A)	4. (D)	5. (A)	6. (D)	7. (C)
8. (C)	9. (D)	10. (B)	11. (C)	12. (D)	13. (C)	14. (A)
15. (B)	16. (B)	17. (D)	18. (B)	19. (A)	20. (A)	21. (D)
22. (D)	23. (D)	24. (C)	25. (D)	26. (C)	27. (C)	28. (D)
29. (B)	30. (A)	31. (B)	32. (A)	33. (B)	34. (D)	35. (C)
36. (D)	37. (C)	38. (A)	39. (B)	40. (B)	41. (A)	42. (C)
43. (A)	44. (C)	45. (A)	46. (A)			

SOLUTIONS

12. Peptic ulcer is mucosal erosion equal to or greater than 0.5 cm of GIT. 70 - 90% of peptic ulcer are associated with helicobacter pylori spiral shaped bacterium that lives in the acidic environment of stomach.

13. Glucose from glomerular filtrate is reabsorbed from proximal convulated tubule via active transport.

15. Aldehyde form of vitamin A (Retinal) is required for synthesis of rhodopsin pigments of rod cells. deficiency of vitamin A will lead to defective formation of rhodopsin.

20. Fetal haemoglobin has has highr affinity with oxygen as compound to adult haemoglobin.

22. Glycogen is stored primarily in liner & muscles.

23. Brown fat stored in adipocyte cells acts as thermogenic tissue & fat metabolism produces heat rather than producing ATP

24. Holocrine gland distingrate completely for discharge of secretion.

25. Cavity of Diancephelon is called as diocoel or third ventricle.

26. Synaptic bulbs of axon have vesicles which are filled with acetylcholine.

27. Parasympathetic control decreased during exercise.

28. Radius of spherical alveoli = 0.1 mm
Total No. of Alveoli = $400 \times 2 = 800$ million
Surface area of sphere = $4\pi R^2$
Total respiratory surface Area
$= 4\pi R^2 \times 800$ million
$= 1000\ mm^2$ (approx)

29. Salivary gland is multicellular exocrine gland, made up cuboidal epithelium.

30. $Na^+ - K^+$ pump is essential for impulse conduction

31. ANF [Anti-Natriuretic Factor] is antagonist of Renin Hormone and act as Vasodilator to reduce blood pressure. ANF is secreted from heart.

32. Oxytocin and Vasopressin synthesised in hypothalamus then comes into neurohypophysis of pituitary gland to release in blood
[Given answer by KVPY is C]

33. Exhaled air has CO_2 in it.
$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$
Lime water Exhaled air Milky white ppt
So lime water become milky white.

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34. Reflex arch
Receptor → Sensory Neuron → Spinal cord → Motor neuron → Muscle [Effector]

35. Duodenum receives Bile and Pancreatic juice alkaline in nature due to high amount of HCO_3^- anion which helps in neutralization of acidity of chyme.

36. Oxyhaemoglobin dissociation curve is sigmoid shaped.

37. Also know as interstitial cells present in testis between somniferous tubule.

38. Promote breakdown of Glycogen in liver.

39. Energy derived pumps are present in plasma membrane.

40. Pressure gradient is essential for movement of gases in and out of the body.

41. Resting membrane potential is potential deference across the plasma membrane when neuron is at rest.

42. Sympathetic nervous system stimulate Sino-Atrial Node.

43. Cholesterol in eukaryote & hapanoid in prokaryote decreases membranes fluidity

44. Lack of blood gulcose CEAO to break down of Fat which produce acetoacetic Acid and β-hydroxy butyric acid which decrease pH of blood.

45. Surface area will increase

46. Blood pressure decrease as it channelise in numerous fine blood vessel.

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Chapter 5

Reproduction, Genetics and Evolution

1. A horse has 64 chromosomes and a donkey has 62. Mules result from crossing a horse and a donkey. State which of the following is INCORRECT ? **[2010]**
(A) Mules can have either 64, 63 or 62 chromosomes
(B) Mules are infertile
(C) Mules have well defined gender (male/female)
(D) Mules have 63 chromosomes

2. A couple went to a doctor and reported that both of them are "carriers" for a particular disorder, their first child is suffering from that disorder and that they are expecting their second child. What is the probability that the new child would be affected by the same disorder ? **[2010]**
(A) 100 % (B) 50 %
(C) 25 % (D) 75 %

3. Restriction endonucleases are enzymes that cleave DNA molecules into smaller fragments. Which type of bond do they act on ? **[2010]**
(A) N-glycosidic Bond
(B) Hydrogen bond
(C) Phosphodiester bond
(D) Disulfide bond

4. In a diploid organism, there are three different alleles for a particular gene. Of these three alleles one is recessive and the other two alleles exhibit co-dominance. How many phenotypes are possible with this set of alleles ? **[2010]**
(A) 3 (B) 6 (C) 4 (D) 2

5. Two students are given two different double stranded DNA molecules of equal length. They are asked to denature the DNA molecules by heating. The DNA given to student A has the following composition of bases (A:G:T:C:35:15:35:15) while that given to student B is (A:G:T:C::12:38:12:38). Which of the following statements is true ? **[2010]**
(A) Both the DNA molecules would denature at the same rate
(B) The information given is insufficient to draw any conclusion
(C) DNA molecule given to student B would denature faster than that of student A
(D) DNA molecule given to student A would denature faster than that given to student B

6. The amino acid sequences of a bacterial protein and a human protein carrying out similar function are found to be 60% identical. However the DNA sequences of the genes coding for these proteins are only 45% identical. This is possible because- **[2010]**
(A) Protein sequence does not depend on DNA sequence
(B) DNA codons having different nucleotides in the third position can code for the same amino acids
(C) DNA codons having different nucleotides in the second position can code for the same amino acids
(D) Same DNA codons can code for multiple amino acids

7. The following DNA sequence (5' → 3') specifies part of a protein coding sequence, starting from position 1. Which of the following mutations will give rise to a protein that is shorter than the full-length protein ? **[2010]**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	T	G	C	A	A	G	A	T	A	T	A	G	C	T

(A) Deletion of nucleotide 13
(B) Deletion of nucleotide 8
(C) Insertion of a single nucleotide between 3 and 4
(D) Insertion of a single nucleotide between 10 and 11

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8. Consider a locus with two alleles, A and a. If the frequency of AA is 0.25, what is the frequency of A under Hardy-Weinberg equilibrium ? **[2010]**
(A) 1 (B) 0.25 (C) 0.5 (D) 0

9. In humans, the composition of a zygote that will develop into a female is- **[2011]**
(A) 44 A + XX (B) 44 A + XY
(C) 22 + X (D) 23 A

10. A protein with 100 amino acid residues has been translated based on triplet genetic code. Had the genetic code been quadruplet, the gene that codes for the protein would have been- **[2011]**
(A) same in size
(B) longer in size by 25 %
(C) longer in size by 100 %
(D) shorter in size

11. If the sequence of bases in DNA is 5'-ATGTATCTCAAT-3', then the sequence of bases in its transcript will be- **[2011]**
(A) 5'-TACATAGAGTTA-3'
(B) 5'-UACAUAGAGUUA-3'
(C) 5'-AUGUAUCUCAAU-3'
(D) 5'-AUUGAGAUACAU-3'

12. Modern evolutionary theory consists of the concepts of Darwin modified by knowledge concerning- **[2011]**
(A) population statistics
(B) Mendel's laws
(C) the idea of the survival of the fittest
(D) competition

13. Soon after the three germ layers are formed in a developing embryo, the process of organogenesis starts. The human brain is formed from the- **[2011]**
(A) ectoderm
(B) endoderm
(C) mesoderm
(D) partly endoderm and partly mesoderm

14. Puffs in the polytene chromosomes of *Drosophila melanogaster* salivary glands represent- **[2011]**
(A) transcriptionally active genes
(B) transcriptionally inactive genes
(C) heterochromatin
(D) housekeeping genes

15. According to the original model of DNA, as proposed by Watson & Crick in 1953, DNA is a- **[2011]**
(A) left handed helix
(B) helix that makes a full turn every 70 nm
(C) helix where one turn of DNA contains 20 basepairs
(D) two stranded helix where each strand has opposite polarity

16. Rice has a diploid genome with 2n = 24. If crossing-over is stopped in a rice plant and then selfed seeds are collected, will all the offsprings be genetically identical to the parent plant ? **[2011]**
(A) yes, because crossing-over is the only source of genetic variation
(B) no, because stopping of crossing-over automatically increases rate of point mutation
(C) yes, only if the parent plant was a completely inbred line
(D) yes, only if the parent plant was a hybrid between two pure-bred lines

17. *E.coli* about to replicate was pulsed with tritiated thymidine for 5 min and then transferred to normal medium. After one cell division which one of the following observations would be correct ? **[2011]**
(A) both the strands of DNA will be radioactive
(B) one strand of DNA will be radioactive
(C) none of the strands will be radioactive
(D) half of one strand of DNA will be radioactive

18. For a human male what is the probability that all the maternal chromosomes will end up in the same gamete ? **[2011]**
(A) 1/23 (B) 2^{23} (C) 2^{46} (D) $(1/2)^{23}$

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19. The length of one complete turn of a DNA double helix is- **[2011]**
(A) 34 Å (B) 34 nm (C) 3.4 Å (D) 3.4 μm

20. Fossils are most often found in which kind of rocks ? **[2012]**
(A) meteorites (B) sedimentary rocks
(C) igneous rocks (D) metamorphic rocks

21. Transfer RNA (tRNA) – **[2012]**
(A) is present in the ribosomes and provides structural integrity
(B) usually has clover leaf-like structure
(C) carries genetic information form DNA to ribosomes
(D) codes for proteins

22. DNA mutations that do not cause any functional change in the protein product are known as – **[2012]**
(A) nonsense mutations
(B) missense mutations
(C) deletion mutations
(D) silent mutations

23. If the sequence of bases in sense strand of DNA is 5'-GTTCATCG-3', then the sequence of bases in its RNA transcript would be – **[2012]**
(A) 5'-GTTCATCG-3'
(B) 5'-GUUCAUCG-3'
(C) 5'-CAAGTAGC-3'
(D) 5'-CAAGUAGC-3'

24. In cattle, the coat colour red and white are two dominant traits, which express equally F_1 to produce roan (red and white colour in equal proportion). If F_1 progeny are selfbred, the resulting progency in F_2 will have phenotypic ration (red : roan: white) is – **[2012]**
(A) 1: 1:1 (B) 3 : 9 : 3
(C) 1 : 2 : 1 (D) 3 : 9 : 4

25. The restriction endonuclease EcoR-I recognizes and cleaves DNA sequence as shown below

5' -G A A T T C-3'
3' -C T T A A G-5'

What is the probable number of cleavage sites that can occur in a 10 kb long random DNA sequence ? **[2012]**
(A) 10 (B) 2 (C) 100 (D) 50

26. The following sequence contains the open reading frame of a polypeptide. How many amino acids will the polypeptide consists of ? **[2012]**

5'- AGCATATGATCGTTT
CTCTGCTTTGAACT-3'

(A) 4 (B) 2
(C) 10 (D) 7

27. The sequences of four DNA molecules are given below: **[2013]**
(i) TATATATATATATA
(ii) TTTCCCGGGAA ATATATATATATAT
AAAGGGCCCTTT
(iii) TTGCGTTGCGCC
(iv) GCCGGATCCGGC AACGCAACGCGG
CGGCCTAGGCCG
Which one of these DNA molecules will have the highest melting temperature (T_m) ?
(A) i (B) ii
(C) iii (D) iv

28. If DNA codons are ATG GAA, insertion of thymine after the first codon results in, **[2013]**
(A) non-sense mutation
(B) mis-sense mutation
(C) frameshift mutation
(D) silent mutation

29. According to mendel,.....segregate and.....assort independently. **[2013]**
(A) alleles of a gene; alleles of different genes
(B) alleles of different genes; alleles of a gene
(C) dominant traits; recessive traits
(D) recessive traits; recessive traits

30. Morphogenetic movements take place predominantly during the following embryonic stage **[2013]**
(A) Blastula (B) Morula
(C) Gastrula (D) Fertilized eggs

31. The only organ which is capable of producing Fructose in humans is- **[2013]**
(A) liver (B) pancreas
(C) seminal vesicles (D) muscle

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32. In a large isolated population, alleles p and q at a locus are at Hardy Weinberg equilibrium. The frequencies are p = 0.6 and q = 0.4. The proportion of the heterozygous genotype in the population is: **[2013]**
(A) 0.24 (B) 1 (C) 0.48 (D) 0.12

33. Watson and Crick model of DNA is **[2013]**
(A) B-form DNA with spiral length of 34 Å and a diameter of 20 Å
(B) A-form DNA with a spiral length of 15 Å and a diameter of 20 Å
(C) Z-form DNA with spiral length of 34 Å and a diameter of 20 Å
(D) B-form DNA with spiral length of 28 Å and a diameter of 14 Å

34. From an early amphibian embryo the cells that would give rise to skin adults were transplanted into the developing brain region of another embryo. The transplanted cells developed into brain tissue in the recipient embryo. What do you infer from this experiment ? **[2013]**
(A) Cell fate is permanently determined during early embryonic development
(B) Developmental fate of donor cells is influenced by the surrounding cells
(C) Developmental fate of donor cells is not influenced by recipient cells
(D) Any cell which is transplanted into another embryo always develops into a brain.

35. Which of the following is the final product of a gene ? **[2014]**
(A) a polypeptide only
(B) an RNA only
(C) either polypeptide or RNA
(D) a nucleotide only

36. Forelimbs of whales, bats, humans and cheetah are examples of which of the following processes ? **[2014]**
(A) Divergent evolution
(B) Convergent evolution
(C) Adaptation
(D) Saltation

37. Testosterone is secreted by which endocrine part of testis ? **[2014]**
(A) Leydig cells
(B) Seminiferous tubules
(C) Tunica albugenia
(D) Sertoli cells

38. The mutation of a purine to a pyrimidine is known as **[2014]**
(A) transition (B) frame shift
(C) nonsense (D) transversion

39. Which one of the following is the complementary sequence for the DNA with 5′-CGTACTA-3′ **[2014]**
(A) 5′-TAGTACG-3′ (B) 5′-ATCATGC-3′
(C) 5′-UTCUTGC-3′ (D) 5′-GCUAGCA-3′

40. A flower with Tt genotype is cross-pollinated by TT pollens. What will the genotypes of the resulting endosperm and embryo, respectively, be ? **[2014]**
(A) TTT, (TT + Tt) (B) (TTT + TTt), TT
(C) TTt, Tt (D) TTt, (TT + Tt)

41. A new life form discovered on a distant planet has a genetic code consisting of five unique nucleotides and only one stop codon. If each codon has four bases, what is the maximum number of unique amino acids this life form can use ? **[2014]**
(A) 624 (B) 20 (C) 124 (D) 3124

42. A spontaneous mutation results in a couple having only female progeny. When the daughter marries and has children, none of them are males. However, in the third generation there are few male offspring. What is the most likely explanation of this observation - **[2014]**
(A) The mutation reverses spontaneously in the third generation
(B) The mutation occurs on the X chromosome and is both recessive and lethal
(C) The mutation occurs on the X chromosome and is both recessive and dominant
(D) The mutation occurs on an autosome and is dominant

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43. In Griffith's experiments mice died when injected with - **[2014]**
(A) heat killed S-strain
(B) heat killed S-strain combined with R-strain
(C) heat killed R-strain
(D) live R-strain

44. Which one of these substances will repress the *lac* operon? **[2015]**
(A) Arabinose (B) Glucose
(C) Lactose (D) Tryptophan

45. Assume a spherical mammalian cell has a diameter of 27 microns. If a polypeptide chain with alpha helical conformation has to stretch across the cell, how many amino acids should it be comprised of? **[2015]**
(A) 18000 (B) 1800 (C) 27000 (D) 12000

46. In a random DNA sequence, what is the lowest frequency of encountering a stop codon? **[2015]**
(A) 1 in 20 (B) 1 in 3 (C) 1 in 64 (D) 1 in 10

47. The two alleles that determine the blood group AB of an individual are located on **[2015]**
(A) two different autosomes
(B) the same autosome
(C) two different sex chromosomes
(D) one on sex chromosome and the other on an autosome

48. Spermatids are formed after the second meiotic division from secondary spermatocytes. The ploidy of the secondary spermatocytes is **[2015]**
(A) n (B) 2n (C) 3n (D) 4n

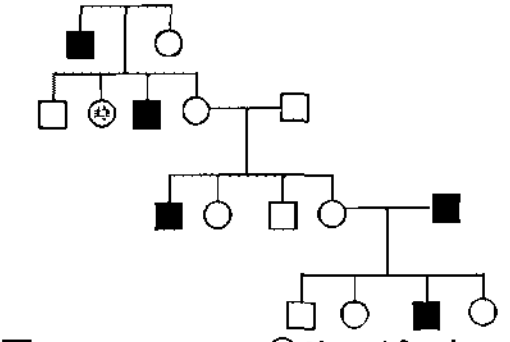
49. If molecular weight of a polypeptide is 15.3 kDa, what would be the minimum number of nucleotides in the mRNA that codes for this polypeptide? Assume that molecular weight of each amino acid is 90 Da. **[2015]**
(A) 510 (B) 663 (C) 123 (D) 170

50. Melting temperature for double stranded DNA is the temperature at which 50% of the double stranded molecules are converted into single stranded molecules. Which one of the following DNA will have the highest melting temperature? **[2015]**
(A) DNA with 15% guanine
(B) DNA with 30% cytosine
(C) DNA with 40% thymine
(D) DNA with 50% adenine

51. Mutation in a single gene can lead to changes in multiple traits. This is an example of **[2015]**
(A) Heterotrophy (B) Co-dominance
(C) Penetrance (D) Pleiotropy

52. Which one of the following is used to treat cancers? **[2015]**
(A) Albumin (B) Cyclosporin A
(C) Antibodies (D) Growth hormone

53. Consider the following pedigree over four generations and mark the correct answer below about the inheritance of haemophilia. **[2015]**



(A) Haemophilia is X-linked dominant
(B) Haemophilia is autosomal dominant
(C) Haemophilia is X-linked recessive
(D) Haemophilia is Y-linked dominant

54. If a double stranded DNA has 15% cytosine, what is the % of adenine in the DNA? **[2015]**
(A) 15% (B) 70% (C) 35% (D) 30%

55. The activity of a certain protein is dependent on its phosphorylation. A mutation in its gene changed a single amino acid which affected the function of the molecule. Which amino acid change is most likely to account for this observation? **[2015]**
(A) Tyrosine to Tryptophan
(B) Lysine to valine
(C) Leucine to isoleucine
(D) Valine to alanine

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56. Enzyme X catalyzes hydrolysis of GTP into GDP. The GTP-bound form of X transmits a signal that leads to cell proliferation. The GDP – bound form does not transmit any such signal. Mutations in X are found in many cancers. Which of the following alterations of X are most likely to contribute to cancer? **[2015]**

(A) Mutations that increase the affinity of X for GDP

(B) Mutations that decrease the affinity of X for GTP

(C) Mutations that decrease the rate of GTP hydrolysis

(D) Mutations that prevent expression of enzyme X.

57. The hard outer layer of pollens, named exine, is made of **[2016]**

(A) cellulose (B) tapetum

(C) sporopollenin (D) pectin

58. In the presence of glucose and lactose, *Escherichia coli* utilizes glucose. However, lactose also enters the cells because **[2016]**

(A) low level of permease is always present in the cell

(B) it uses the same transporter as glucose

(C) if diffuses through the bacterial cell membrane

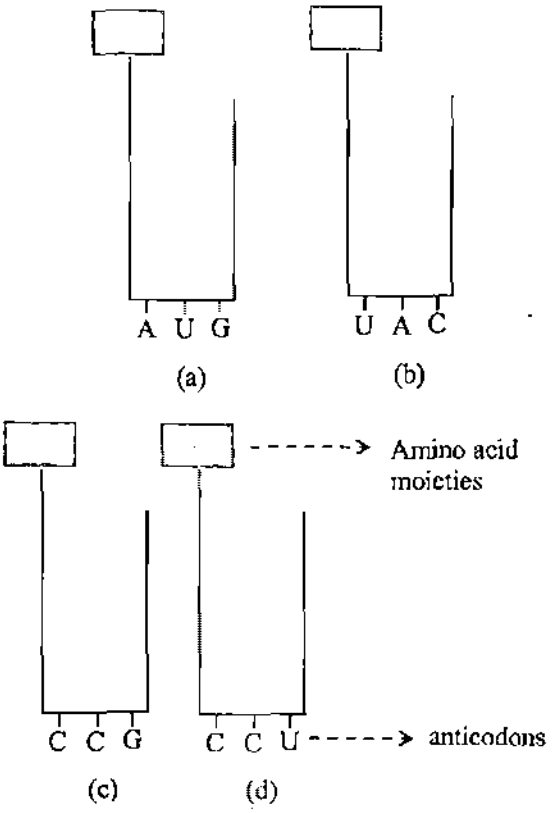
(D) it is transported through porins

59. A human population containing 200 individuals has two alleles at the 'T' locus, named *T* and *t*. *T*, which produces tall individuals, is dominant over *t*, which produces short individuals. If the population has 90 *TT*, 40 *Tt* and 70 *tt* genotypes, what will be the frequencies of these two alleles in this population ? **[2016]**

(A) *T*, 0.50 ; *t*, 0.50 (B) *T*, 0.55 ; *t*, 0.45

(C) *T*, 0.45 ; *t*, 0.35 (D) *T*, 0.90 ; *t*, 0.10

60. An mRNA is transcribed from a DNA segment having the base sequence 3'-TACATGGGTCCG-5'. What will be the correct order of binding of the four amino acyl-tRNA complexes given below during translation of this mRNA ? **[2016]**



(A) a, b, c, d (B) b, a, c, d

(C) c, d, a, b (D) b, a, d, c

61. If the initial number of template DNA molecules in a PCR reaction is 1000, the number of product DNA molecules at the end of 20 cycles will be closest to **[2016]**

(A) 10^3 (B) 10^6

(C) 10^9 (D) 10^{12}

62. The allele for black hair (B) is dominant over brown hair (b) and the allele for brown eye (E) is dominant over blue eye (e). Out of the offsprings obtained upon mating a black-haired and brown-eyed individual (BbEe) with a brown-haired and brown-eyed individual (bbEE), the ratio of brown-haired and brown-eyed individuals to black- haired and brown-eyed individuals is **[2016]**

(A) 2 : 1 (B) 3 : 1

(C) 1 : 1 (D) 1 : 2

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63. For a particular gene that determines the coat color in a diploid organism, there are three different alleles that are codominant. How many different skin colors are possible in such an organism? **[5 Nov, 2017]**

(A) 9 (B) 6
(C) 4 (D) 3

64. Two genetic loci controlling two different traits are linked. During the inheritance of these traits, the Mendelian laws that would be affected is/are **[5 Nov, 2017]**

(A) Law of dominance, law of segregation and law of independent assortment
(B) Law of segregation and Law of independent assortment
(C) Only Law of independent assortment
(D) Only Law of segregation

65. Which ONE of the following statements is INCORRECT? **[5 Nov, 2017]**

(A) Alleles are different forms of the same gene.
(B) Alleles are present at the same locus.
(C) Alleles code for different isoforms of a protein.
(D) Alleles are non-heritable.

66. The genetic distance between genes A and B is 10 cm. An organism with Ab combination of the alleles is crossed with the organism with aB combination of alleles. What will be the percentage of the gametes with AB allele combination by an F1 individual ? **[5 Nov, 2017]**

(A) 1 (B) 5
(C) 10 (D) 50

67. The minimum number of plants to be screened to obtain a plant of the genotype AabbCcDd from a cross between plants of genotypes AaBbCcDd and AABbCCDd is **[5 Nov, 2017]**

(A) 8 (B) 16
(C) 32 (D) 64

68. When a pure bred, red flower-producing plant of genotype *RR* is crossed with a pure bred, white flower-producing plant of genotype *rr*, all the F_1 plants produced pink flowers If all the plants in each generation from F_1 to F_6 are selfed, what will be the percentage of plants with red and white flowers in the final population consisting of a large number of individuals ? (Consider that flower colour has no effect on reproduction and survival.)

[5 Nov, 2017]

(A) 3 – 4 (B) 12 –13
(C) 49 – 51 (D) 97 – 100

69. Two homozygous parents harboring two different alleles of a gene, exhibiting incomplete dominance for flower colour were used for a genetic experiment. Which ONE of the following statements is INCORRECT ?

[19 Nov, 2017]

(A) The F_2 generation will consist of plants of three different flower colours
(B) The genotypic and phenotypic ratios obtained in the F_2 generation will be different
(C) The F_1 generation will be of a different flower colour compared to both the parents
(D) The genotypic ratio obtained in the F_2 generation will be the same irrespective of whether it is complete dominance or incomplete dominance

70. Which ONE of the following is an essential condition for a population to be at Hardy-Weinberg equilibrium ? **[19 Nov, 2017]**

(A) Random mating
(B) Immigration
(C) Emigration
(D) Geographical isolation

71. Inbreeding in a population leads to **[19 Nov, 2017]**

(A) decrease in recessive disorders
(B) heterosis
(C) increase in homozygosity
(D) increase in heterozygosity

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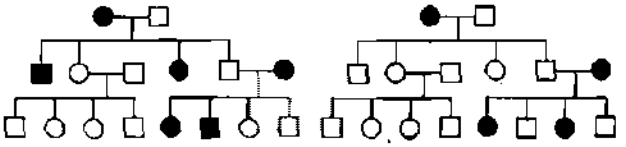
72. Suppose the three non-linked autosomal genes A, B and C control coat color in an animal and the dominants alleles A, B and C are responsible for dark color and the recessive alleles a, b and c are responsible for light color. If a cross between a male of AABBCC genotype and a female of aabbcc genotype produce 640 off springs in the F_2 generation, how many of them are likely to be of the parental genotype ? **[19 Nov, 2017]**

(A) 10 (B) 20

(C) 160 (D) 640

73. The following two pedigrees describe the autosomal genetic disorders P and Q in Family 1 and Family 2, respectively **[19 Nov, 2017]**

Family 1 Family 2



Choose the CORRECT statement from the following options.

(A) Both P and Q are dominant traits

(B) P is a dominant trait and Q is a recessive trait

(C) Both P and Q are recessive traits

(D) P is a recessive trait and Q is a dominant trait

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ANSWERS

1. (A)	2. (C)	3. (C)	4. (C)	5. (D)	6. (B)	7. (B)
8. (C)	9. (A)	10. (B)	11. (D)	12. (C)	13. (A)	14. (A)
15. (D)	16. (C)	17. (B)	18. (D)	19. (A)	20. (B)	21. (B)
22. (D)	23. (B)	24. (C)	25. (B)	26. (D)	27. (D)	28. (C)
29. (A)	30. (C)	31. (C)	32. (C)	33. (A)	34. (B)	35. (C)
36. (A)	37. (A)	38. (D)	39. (A)	40. (A)	41. (A)	42. (B)
43. (B)	44. (B)	45. (A)	46. (A)	47. (B)	48. (A)	49. (A)
50. (B)	51. (D)	52. (C)	53. (C)	54. (C)	55. (A)	56. (C)
57. (C)	58. (A)	59. (B)	60. (Bonus)	61. (C)	62. (C)	63. (B)
64. (C)	65. (D)	66. (B)	67. (C)	68. (D)	69. (B)	70. (A)
71. (C)	72. (B)	73. (B)				

SOLUTIONS

12. Propounded by Herbert Spencer for natural selection process. This explains that adaptability has the genetic basis which proves fitness of an organism and nature selects organism for its fitness and allow to produce its progeny in large number.

13. Organogenesis begins with the process of neurulation. This neurulation begin with the formation of primitive streak in epiblast (Ectoderm) which leads to formation of neural tube (nerve cord)

20. Sedimentary rocks are type of rock that are formed by deposition of material at the earth's surface and within bodies of water. It forms only 8% of total volume of crust fossils are mostly found in these.

21. The clover leaf model is the 2-D model. Given by Holley. Its 3-D model is a L-shaped model.

22. Silent mutations also called same - sense mutations are not lethal because in these mutations, the amino acid do not get changed.

23. The direction of RNA sequence is also from 5'-3'. The sequence of sense strand is - 5'-GTTCATCG-3'
We know the sequence of m-RNA is similar to sense strand. Only uracil is present instead of Thymine.
Hence the m-RNA sequence will be - 5'-GUUCAUCG-3'

24. This is an example of Co-dominance. (Result is 1 Red: 2 Roan: 1 white).

25. Eco RI is an example of six-cutter restriction endonuclease. It usually cleaves once in every 4096 bp.

$$\left(\frac{1}{4}\right)^6 = \frac{1}{4096}$$

Given length of DNA fragment = 10 Kb
= 10000 bp
Hence,

Probable no. of cleaving sites = $\frac{10000}{4096} = 2.44$

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26. 5' AGCAT A TG
(1)
A TC G TT T CT C TG C TT
(2) (3) (4) (5) (6)
T GA T GAA CT' 3'
(7)
5' AGC
AU A U G A U C G U U U C U
(1) (2) (3) (4)
C U G C U U U G A ACU 3'
(5) (6) (Stop)
So this sequence is present in mRNA and translation start from start codon 'AUG' so 6 Amino acid will be formed close to 7
So answer can (D)

27. More the hydrogen bonds, more will be the melting temperature (T_m)
(iv) DNA molecules has highest number of C ≡ G bonds

28. Insertion or deletion of a single nucleotide causes shifting in complete framework of amino acids

29. Segregation is related to separation of alleles of same gene whereas independent assortment is related to separation of different genes.

30. Morphogenic movements takes place during gastrulation that results in formation of 3 germ layers ectoderm, mesoderm & endoderm.

31. Fructose is produced by seminal vesicle, it helps in providing energy to sperms.

32. p = 0.6 q = 0.4
Proportion of heterozygous genotype in population = 2pq
2pq = 2 × 0.6 × 0.4 = 0.48

33. Watson and Crick model was based on common form of DNA i.e. B-form which has spiral turn = 34Å
Diameter = 20Å

34. The fate of cells during embryonic life depends on their surrounding cells

35. Gene is a segment of genetic material which produces either polypeptide or a RNA [rRNA or tRNA]

36. Fore limbs of whale, bat, human & cheetah are homologous organ which represents divergent evolution.

37. Leydig cells or Interstitial cells of testes secretes testosteron hormone.

38. Mutation of purine to purimidine is known as transversion. 8 possible transversion can occurs.

39. Double stranded DNA has antiparalllel strands and complementary N-bases
so, 5′ CGTACTA 3′
3′ GCATGAT 5′
←
thus, answer is 5′TAGTACG3′

40. Tt × TT (Pollen means male plant)
Endosperm → 2 polar nuclei (female) (should be same)
+ 1 male nuclei
So, TTT
Embryo → Egg cell (female) + 1 male nuclei
So, either TT or Tt

41. No. of unique nucleotide = 5
No. of bases in codon = 4
so, total combinations ⇒ 5^4 ⇒ 625
No. of stop codons = 1
so, unique amino acids (maximum) = 625 – 1 ⇒ 624

42. The mutation that occurred is on x-chromosome and is both recessive and lethal since, female is carrier it survives but due to hemizygous condition male is unable to survive.
XX^m → carrier (survives)
X^mY → dies

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While in third generation, the possibility of male child occurs.

43. In Griffith experiment, mice died when injected with combination

heat killed s-strain + Live R strain

which resulted in transformation of R II into S III form.

44. Presence of glucose inhibit the lac operon

45. No. of amino acid in one term in α helix = 3.6

Pitch length for α Helix = 5.4 A°

$\therefore$ The No. of Amino acid in polypeptide

$$= \frac{3.6}{5.4\times10^{-10}} \times 27\times10^{-6}$$

$= 1.8 \times 10^4$ Amino acid

= 18000 amino acid

46. Total no. of codon = 64

Functional codon = 61

Stop codon = 3

$\therefore$ frequency of encountering stop codon

$$= \frac{3}{61} \approx \frac{1}{20}$$

47. Allele of the same gene are present at same gene locus on homologous chromosome

48. Secondary spermatocyte formed by meiosis-I .

49. Molecular weight of polypeptide = 15.3 kda

Molecular weight of amino acid = 90 Da

$\therefore$ No. of Amino acid in polypeptide

$$= \frac{15.3\times10^3}{90} = 170$$

$\because$ One amino acid is coded by = 3 Nitrogen base

$\therefore$ 170 amino acid would be coded by

$= 170 \times 3 = 510$ Nitrogen base

50. Melting temperature of DNA $\propto$ GC content

51. Pleiotrophy : A single gene affects more than one phenotype.

52. Monoclonal antibodies are used for treatment of cancer.

53.

	X^h	Y
X^h	X^hX^h	X^hY
X	X^hX	XY

Female parent is carrier and male and female offspring is affected.

54. Cytosine = 15%

According to chargaff principle

A = T & G = C

A + T + G + C = 100

Thus A will be 35%

55. Serine, tyrosine, threonine and histidine are most common target for phosphorylation in eukaryotes.

If Tyrosine get mutated in tryptophan than phosphorylation of gene does not occur and gene can not be activated

56. GTP –X Complex ⟶ Active cell division

GDP –X Complex ⟶ No cell division

and GTP $\xrightarrow[\text{X enzyme}]{\text{Hydrolysis}}$ GDP

Mutation cause the decrease of rate of hydrolysis of GTP. Thus GTP –X complex remain present for long time and cell division become uncontrolled.

57. Sporopollenin forms the exine of pollen grain which is resistant to acids, high temperature and radiations.

58. The preferred molecule is glucose first, then lactose is used by *E.coli* because ultimately lactose is also broken down into galactose and glucose. Permease is a carrier protein which helps in facilitated diffusion. Lac operon is always operational at low levels.

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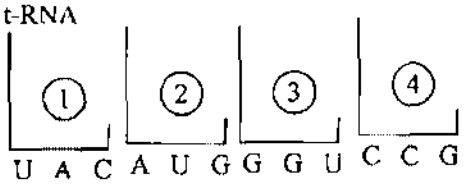
59. A population with 200 individual has 90TT, 40 Tt and 70 tt genotypes
i.e. dominant allele (T) is TT + Tt
i.e. 90 + 90 + 40
$\Rightarrow$ 220
Recessive allele (t) is Tt + tt
i.e. 40 + 70 + 70
$\Rightarrow$ 180
Total allele = 400

Dominant allele (T) frequency = $\frac{220}{400} = 0.55$

Recessive allele (t) frequency = $\frac{180}{400} = 0.45$

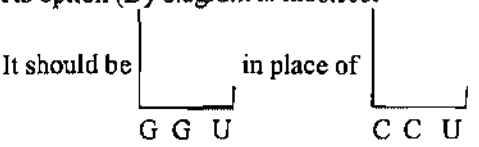
59. DNA sequence

60. 3' TAC ATG GGT CCG 5'
mRNA
5' AUG UAC CCA GGC 3'
t-RNA



5' AUG UAC CCA GC 3'
mRNA
Direction of translation

None of the option is correct
As option (D) diagram is incorrect



61. During PCR, the number of DNA molecules increases by 2^n. Where 'n' is number of divisions

62. Parents – BbEe × bbEE
Black haired Brown eyed × Brown haired Brown eyed
↓ ↓
Gametes –

BE	Be	bE	be	
BbEE	BbEe	bbEE	bbEe	bE

Ratio of Brown haired and Brown eyed to Black haired and Brown eyed is 2 : 2 or 1 : 1

63. $\frac{n(n+1)}{2} = \frac{3(3+1)}{2} = 6$

64. Linkage in exception of law of independent assortment

65. Alleles are alternate form of gives of one character present on same locus.

66. Recombinants formed = 10% (AB 5% & ab 5%)

67. AaBbCcDd × AaBbCCDd
↓
AabbCcDd
$\frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$

68. After selfing homozygous indivisiual produces All white & red offspring and heterozygous also produces 50%. Red & White flower.

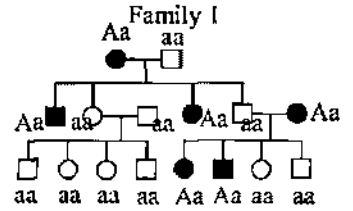
69. Both genotype & phenotype ratio same 1 : 2 : 1

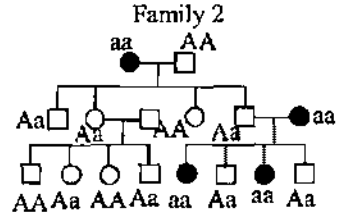
70. For Random mixing of alleles.

71. Inbreeding is the production of offspring from the mating or breeding of individuals or organisms that are closely related genetically

72. $\frac{2}{64} \times 640 = 20$ (Trihybrid cross)

73.





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Chapter 6

Biology in Human Welfare and Biotechnology

1. Sometimes urea is fed to ruminants to improve their health. It works by- [2010]
(A) Helping growth of gut microbes that break down cellulose
(B) Killing harmful microorganisms in their gut
(C) Increasing salt content in the gut
(D) Directly stimulating blood cell proliferation

2. Conversion of the Bt protoxin produced by *Bacillus thuringiensis* to its active form in the gut of the insects is mediated by- [2010]
(A) acidic pH of the gut
(B) alkaline pH of the gut
(C) lipid modification of the protein
(D) cleavage by chymotrypsin

3. Patients who have undergone organ transplants are given anti-rejection medications to- [2011]
(A) minimize infection
(B) stimulate B-macrophage cell interaction
(C) prevent T-lymphocyte proliferation
(D) adopt the HLA of donor

4. Saline drip is given to a Cholera patient because- [2011]
(A) NaCl kills *Vibrio cholera*
(B) NaCl generates ATP
(C) Na^+ ions stops nerve impulse and hence sensation of pain
(D) Na^+ ions help in retention of water in body tissue

5. The disorders that arise when the immune system destroys self cells are called autoimmune disorders. Which of the following would be classified under this? [2012]
(A) rheumatoid arthritis
(B) asthma
(C) rhinitis
(D) eczema

6. When of the following class of immunoglobulins can trigger the complement cascade ? [2012]
(A) IgA (B) IgM
(C) IgD (D) IgE

7. Diabetes insipidus is due to -- [2012]
(A) hypersecretion of vasopressin
(B) hyposecretion of insulin
(C) hypersecretion of insulin
(D) hyposecretion of vasopressin

8. In Dengue virus infection, patients often develop haemorrhagic fever due to internal bleeding. This happens due to the reduction of [2012]
(A) platelets (B) RBCs
(C) WBCs (D) lymphocytes

9. Which one of the following techniques is used for the detection of proteins ? [2013]
(A) Northern blotting
(B) Western blotting
(C) Southern blotting
(D) In-situ hybridization

10. 10^9 bacteria were spread on a agar plate containing penicillin. After incubation overnight at 37°C, 10 bacterial colonies were observed on the plate. That the colonies are likely to be resistant to penicillin can be tested by [2013]
(A) measuring their growth rate
(B) observing the colour of the colonies
(C) checking their ability to grow on another plate containing penicillin
(D) checking their ability to cause disease

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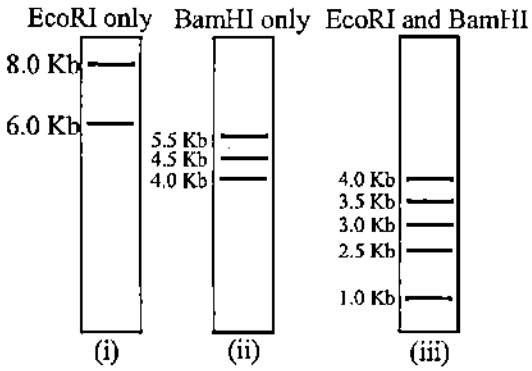
11. Eco RI and Rsa I restriction endonucleases require 6 and 4 bp sequences respectively for cleavage. In a 10 kb DNA fragment how many probable cleavage sites are present for these enzymes **[2013]**
(A) 0 Eco RI and 10 Rsa I
(B) 1 Eco RI and 29 Rsa I
(C) 4 Eco RI and 69 Rsa I
(D) 2 Eco RI and 39 Rsa I

12. Presence of plastids in Plasmodium suggests **[2013]**
(A) it is a plant species
(B) it is a parasite with a cynobacterium as an endosymbiont
(C) it is a parasite with an archebacterium as an endosymbiont
(D) it is a plant species with an archebacterium as an endosymbiont

13. A scientist has cloned an 8 Kb fragment of a mouse gene into the Eco RI site of a vector of 6 Kb size. The cloned DNA has no other Eco RI site within. Digestions of the cloned DA is shown below.
Which one of the following sets of DNA fragments generated by digestion with both Eco RI and Bam HI as shown in (iii) is from the gene ? **[2013]**



(A) 1 Kb and 4 Kb (B) 1 Kb and 2.5 Kb
(C) 1 Kb and 3 Kb (D) 1 Kb and 3.5 Kb

14. A bacterial colony is produced from **[2014]**
(A) a single bacterium by its repetitive division
(B) multiple bacterium without replication
(C) clumping of two to three bacteria
(D) a single bacterium without cell division

15. Rhinoviruses are the causative agents of **[2014]**
(A) Diarrhoea (B) AIDS
(C) Dengue (D) Common cold

16. What is the genetic material of Ebola virus ? **[2014]**
(A) Single-stranded DNA
(B) Double-stranded RNA
(C) Single-stranded RNA
(D) Double-stranded DNA

17. How many linear DNA fragments will be produced when a circular plasmid is digested with a restriction enzyme having 3 sites ? **[2014]**
(A) 4 (B) 5
(C) 3 (D) 2

18. A circular plasmid of 10,000 base pairs (bp) is digested with two restriction enzymes, A and B, to produce a 3000 bp and a 2000 bp bands when visualized on an agarose gel. When digested with one enzyme at a time, only one band is visible at 5000 bp. If the first site for enzyme A (A1) is present at the 100th base, the order in which the remaining sites (A2, B1 and B2) are present is - **[2014]**
(A) 3100, 5100, 8100 (B) 8100, 3100, 5100
(C) 5100, 3100, 8100 (D) 8100, 5100,. 3100

19. Immunosuppressive drugs like cyclosporin delay the rejection of graft post organ transplantation by **[2015]**
(A) inhibiting T cell infiltration
(B) killing B cells
(C) killing macrophages
(D) killing dendrite cells

20. In biotechnology applications, a selectable marker is incorporated in a plasmid **[2015]**
(A) to increase its copy number
(B) to increase the transformation efficiency
(C) to eliminate the non-transformants
(D) to increase the expression of the gene of interest

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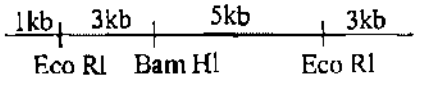
21. Following are the types of immunoglobulin and their functions. Which one of the following is INCORRECTLY paired? **[2015]**
(A) IgD : viral pathogen
(B) IgG : phagocytosis
(C) IgE : allergic reaction
(D) IgM : complement fixation

22. How many bands are seen when immunoglobulin G molecules are analysed on a sodium dodecyl sulphate polyacrylamide gel electrophoresis (SDS – PAGE) under reducing conditions? **[2015]**
(A) 6 (B) 1 (C) 2 (D) 4

23. In a mixed culture of slow and fast growing bacteria, penicillin will **[2015]**
(A) kill the fast growing bacteria more than the slow growing
(B) kill slow growing bacteria more than the fast growing
(C) kill both the fast and slow growing bacteria equally
(D) will not kill bacteria at all

24. A mixture of equal numbers of fast and slow dividing cells is cultured in a medium containing a trace amount of radioactively labeled thymidine for one hour. The cells are then transferred to regular (unlabelled) medium. After 24hrs of growth in regular media **[2015]**
(A) fast dividing cells will have maximum radioactivity
(B) slow dividing cells will have maximum radioactivity
(C) both will have same amount of radioactivity
(D) there will be no radioactivity in either types of cells

25. Consider the linear double stranded DNA shown below. The restriction enzyme sites and the lengths demarcated are shown. This DNA is completely digested with both *Eco*RI and *Bam*HI restriction enzymes. If the product is analyzed by gel electrophoresis, how many distinct bands would be observed? **[2015]**



(A) 5 (B) 2 (C) 3 (D) 4

26. Passive immunization is achieved by administering **[2016]**
(A) Heat killed vaccines
(B) Toxoids
(C) Live attenuated vaccines
(D) Antibodies

27. Match the human disorders shown in Group I with the biochemical processes in Group II. Choose the correct combination **[2016]**

Group I	Group II
P. Phenylketonuria	i. Melanin synthesis
Q. Albinism	ii. Conversion of Phenylalanine to Tyrosine
R. Homocystinuria	iii. Tyrosine degradation
S. Argininemia	iv. Methionine metabolism
	v. Urea Synthesis

(A) P-ii, Q-i, R-iv, S-v
(B) P-i, Q-iv, R-ii, S-v
(C) P-ii, Q-i, R-v, S-iii
(D) P-v, Q-iii, R-i, S-ii

28. Interferons combat viral infection by **[5 Nov, 2017]**
(A) inhibiting viral packaging directly.
(B) increasing the binding of antibodies to viruses.
(C) binding to the virus and agglutinating them.
(D) restricting viral spread to the neighboring cells.

29. Which ONE of the following is NOT essential for Polymerase Chain Reaction (PCR) ? **[5 Nov, 2017]**
(A) Restriction enzyme
(B) Denaturation of DNA
(C) Primers
(D) DNA polymerase

30. Antigen-angtibody reactions **[5 Nov, 2017]**
(A) always result in precipitation of the complex
(B) depend only on covalent interactions.
(C) are irreversible.
(D) depend on ionic and hydrophobic interactions.

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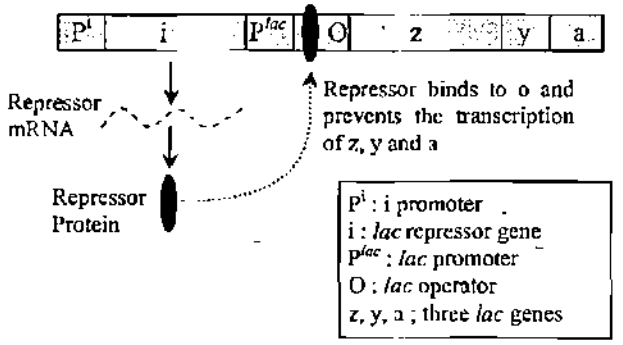
31. Which ONE of the following combinations of molecular masses of polypeptides are obtained from purified human IgM when analysed on sodium dodecyl suplhate polyacrylamide gel electrophoresis (SDS-PAGE) under reducing conditions? **[5 Nov, 2017]**

(A) 55 kDa, 15 kDa
(B) 70 kDa, 25 kDa, 15 kDa
(C) 55 kDa, 25 kDa
(D) 155 kDa

32. Molecular weight of *E. Coli* DNA is 3.1×10^9 g/mol. Average molecular weight of nucleotide pair is 660 g/mol and each nucleotide pair contributes to 0.34 nm to the length of DNA. The length of *E. coli* DNA molecule will be approximately **[5 Nov, 2017]**

(A) 0.8 nm (B) 1.6 nm
(C) 1.6 μm (D) 1.6 mm

33. The schematic below describes the status of *lac* operon in the absence of lactose. Which ONE of the following happens when lactose is present in the cell ? **[5 Nov, 2017]**



(A) Lactose binds to P^I and stops the transcription of *i*.
(B) Lactose is converted to allolactose, which binds to P^{lac} and results in the displacements of the repressor from *O*.
(C) Lactose is converted to allolactose, which binds to the repressor protein and prevents its interaction with *O*.
(D) Lactose has no effect on the status of the *lac* operon.

34. Skin-prick test on the forearm is conducted to identify the responsible allergen. This is because **[19 Nov, 2017]**

(A) of the presence of mast cells under the skin.
(B) lymphocytes migrate rapidly from the blood to the skin.
(C) hair follicles can enhance the reaction.
(D) Neutrophils migrate rapidly from the blood to the skin.

35. Which ONE of the following processes in *E coli* does NOT directly involve RNA ? **[19 Nov, 2017]**

(A) DNA replication (B) Transcription
(C) Translation (D) DNA repair

36. Frederick Griffith performed an experiment where mice were killed when injected with a mixture of killed S-type *Streptococcus* (HKS) and live R-type *Streptococcus* (LRS) but not with HKS or LRS separately. Mice were killed because **[19 Nov, 2017]**

(A) lipids from HKS made LRS virulent
(B) RNA from HKS transformed LRS and made it virulent
(C) proteins from HKS made LRS virulent
(D) DNA from HKS transformed LRS and made it virulent

37. In an experiment, bacteria were infected with ^{32}P labelled virus in a ratio of 5 : 1. The culture was rigorously shaken followed by centrifugation. Radioactivity was **[19 Nov, 2017]**

(A) lost due to metabolic activity
(B) detected in supernatant as inorganic phosphate
(C) detected in the supernatant in association with viral capsid
(D) detected in bacterial cell pellet

38. As indicated in the gel image, lanes X and Y represent samples obtained from a circular plasmid DNA after complete digestion using restriction enzyme X or Y with different sites, respectively. How many sites for X and Y are present in the plasmid (sizes of the bands in kilo base pairs (kb) is shown) ? **[19 Nov, 2017]**

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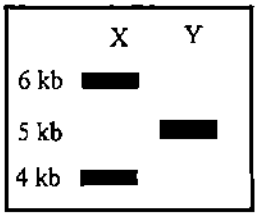

(A) 1 for X, 1 for Y (B) 2 for X, 1 for Y
(C) 1 for X, 2 for Y (D) 2 for X, 2 for Y

39. Matthew Meselson and Franklin Stahl grew *E.coli* (doubling time is 20 min) in medium containing $^{15}NH_4Cl$ for many generations. Then the *E.coli* was transferred to medium containing $^{14}NH_4Cl$. After 40 minutes, the cells were harvested and DNA was extracted and subjected to cesium chloride density gradient centrifugation. The proportion of light and hybrid DNA densities will be **[19 Nov, 2017]**

(A) 50% light and 50% hybrid DNA
(B) 100% light DNA
(C) 100% hybrid DNA
(D) 25% light and 75% hybrid DNA

40. Match the enzymes in column-I with their respective biochemical reactions in column-II. Choose the CORRECT combination from below **[19 Nov, 2017]**

Column-I	Column-II
(P)Transaminases acid	(i) removal of phosphoryl group from a specific amino
(Q) Protein Kinases acid	(ii) removal of α-amino group from a specific amino
(R) Protein Phosphatases acid	(iii) addition of phosphoryl group to a specific amino
(S) Dehydrogenases	(iv) interconversion of optical isomers
	(v) oxidation and reduction of substrates

(A) P-iv, Q-ii, R-iii, S-v
(B) P-ii, Q-i, R-ii, S-iv
(C) P-ii, Q-iii, R-i, S-v
(D) P-v, Q-ii, R-iii, S-i

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ANSWERS

1. (A)	2. (B)	3. (C)	4. (D)	5. (A)	6. (B)	7. (D)
8. (A)	9. (B)	10. (C)	11. (D)	12. (B)	13. (A&C)	14. A)
15. (D)	16. (C)	17. (C)	18. (C)	19. (A)	20. (C)	21. (A)
22. (C)	23. (A)	24. (A)	25. (C)	26. (D)	27. (A)	28. (D)
29. (A)	30. (D)	31. (B)	32. (C)	33. (C)	34. (A)	35. (D)
36. (D)	37. (D)	38. (D)	39. (A)	40. (C)		

SOLUTIONS

5. Rheumatoid arthritis is a chronic, systemic inflammatory disorder that may affect many tissues and organs, but principally attacks flexible joints. RA is considered a systemic autoimmune disease arises from an inappropriate immune response of the body against substances and tissues normally present in the body.

6. The complement system helps the ability of antibodies and phagocytic cells to clear pathogens from an organism. it si a part of innate immune system. it is operational via classical pathways. it is triggered by antigen bound antibody molecule preferably IgG or IgM. Although IgM is move effective

7. Diabetes incipidus (DI) is a condition characterized by excessive thirst and excretion of large amounts of severely diluted urine. DI is caused by a deficiency of vasopressin also known as antidiuretic hormone.

8. Dengue fever also known as break bone fever is an infections tropical disease caused by dengue virus. This results in bleeding, low levels of blood platelets and blood plams leakage.

9. Western blotting technique is used to transfer proteins and detection there after.

10. The resistant bacterial colonies will grow on other agar plates containing penicillin.

11. Eco RI → 6 bp sequence

So, probability of occurance of site

$4^n = 4^6 = 4096$

Rsa I → 4 bp sequence

So, probability of occurrence of site

$4^n = 4^4 = 256$

Total DNA length = 10 Kb = 10,000 bp

So, probable cleavage would be

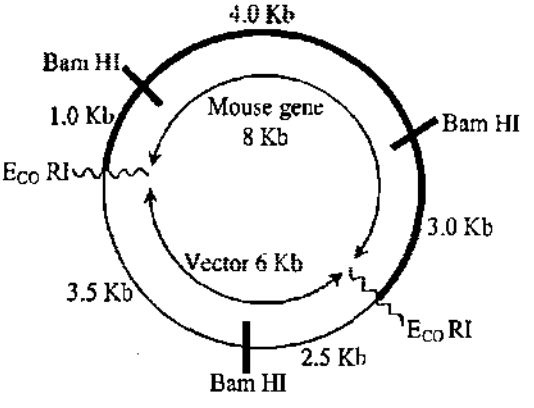
$$\text{Eco RI} = \frac{10000}{4096} = 2.44$$

$$\text{Rsa I} = \frac{10000}{256} = 39.06$$

12. Plasmodium is a parasite.

Presence of plastid indicates endo-symbiotic presence (Plastid is believed to have originated from cyanobacteria)

13.



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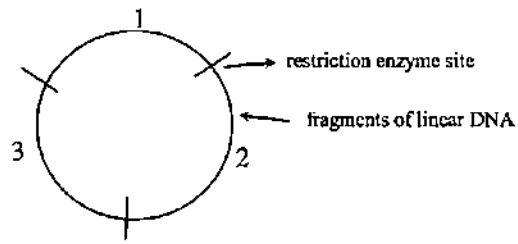
14. A bacterial colony on culture media is formed on repetitive division of bacterium.

15. Rhinovirus are the primary cause of common cold.

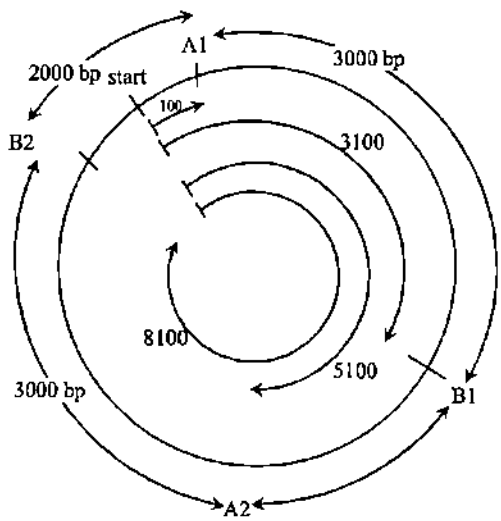
16. Ebola virus consist of ss RNA.

17.



Plasmid

18.



Question asks the position in A2 → B1 → B2 so, 5100, 3100, 8100

19. Immunosuppressive Drug inhibits T-cell infiltration

20. Selectable marker is used to eliminate the non transformant from the transformants

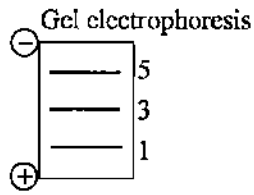
21. IgD activates , B- lymphocyte

22. On SDS – PAGE of IgG two principal Bands are visible

23. Penicillin inhibit the cell wall formation in bacteria thus it kill rapidly growing bacteria more than the slow growing bacteria.

24. Radioactivity is mostly incorporated in rapidly dividing cell during S phase. Thus after 24 hour it will be mainly present in few rapidly dividing cell

25.



There are 4 fragment but 2 fragment of 3 Kb will appear as one band so three band will appear

26. Administration of already prepared antibodies is called passive immunisation

27. Phenylketonuria is due to non-conversion of phenylalanine into tyrosine. Albinism is non-synthesis of melanin pigment. Homocystinuria is associated with methionine metabolism. Argininemia is associated with urea synthesis.

28. In a typical scenario, a virus defected cell will release interferons causing near by cells to heighten their anti-viral defense

29. RE is not required in PCR

30. Antibody- antigen interaction is essentially Non-Covalent, Electrostatic interaction, Hydrogen Bonds, Vander walls forces and Hydrophobic interactions are all known to be involved depending on the interaction sites.

32. $\frac{3.1\times10^{9}}{660}\times 0.34$

33. Lac operon is inducible operon here.

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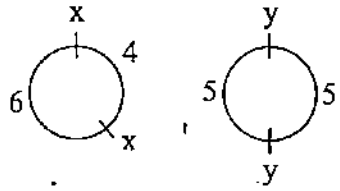
34. Mast Cells release histamine.

35. RNA primers are involved in DNA replication

36. Transformation occurs when DNA is taken up by R-strain form dead S-strain.

37. Bacteriophage infected bacteria found at the bottom containing viral DNA i.e. radioactive.

38.



39. DNA replication is semiconservative

40. Fact based

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Chapter 7

Ecology

1. If you compare adults of two herbivore species of different sizes, but from the same geographical area, the amount of faeces produced per kg body weight would be- [2010]
(A) More in the smaller one than the larger one
(B) More in the larger one than the smaller one
(C) Roughly the same amount in both
(D) Not possible to predict which would be more

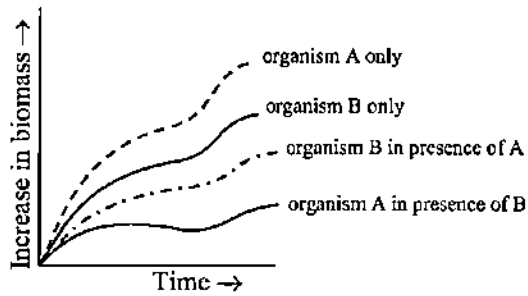
2. In a food chain such as grass → deer → lion, the energy cost of respiration as a proportion of total assimilated energy at each level would be [2012]
(A) 60% - 30 % - 20% (B) 20% - 30 % - 60%
(C) 20% - 60 % - 30% (D) 30% - 30 % - 30%

3. Insects constitute the largest animal group on earth. About 25-30% of the insect species are known to be herbivores. In spite of such huge herbivore pressure, globally, green plants have persisted. one possible reason for this persistence is – [2012]
(A) Food preference of insects has tended to change with time
(B) Herbivore insects have become inefficient feeders of green plants
(C) Herbivore population has been kept in control by predators
(D) Decline in reproduction of herbivores with time

4. Chlorofluorocarbons (CFCs) are believed to be associated with cancers because, [2013]
(A) CFCs react with DNA and cause mutations
(B) CFCs react with proteins involved in DNA repair
(C) CFCs destroy the ozone layer and permit harmful UV rays to reach the earth
(D) CFCs react with DNA polymerase and reduce fidelity of DNA replication

5. Species that are most effective at colonizing new habitats show [2013]
(A) low reproductive ability
(B) high dispersal ability
(C) slow growth and maturation
(D) high competitive ability

6. The figure below demonstrates the growth curves of two organisms A and B growing in the same area. What kind of relation exists between A and B ? [2013]



(A) Competition (B) Symbiosis
(C) Commensalisms (D) Mutualism

7. In some species, individuals forego reproduction and help bring up another individual's offspring. Such altruistic behaviour CANNOT be explained by which of the following ? [2013]
(A) An individual helps relatives only and gets indirect genetic benefits.
(B) The individual benefits because is can later inherit the breeding position.
(C) The individual benefits because it gets access to resources, such as food and security from predators, in return.
(D) The species benefits from a reduction in competition among offspring.

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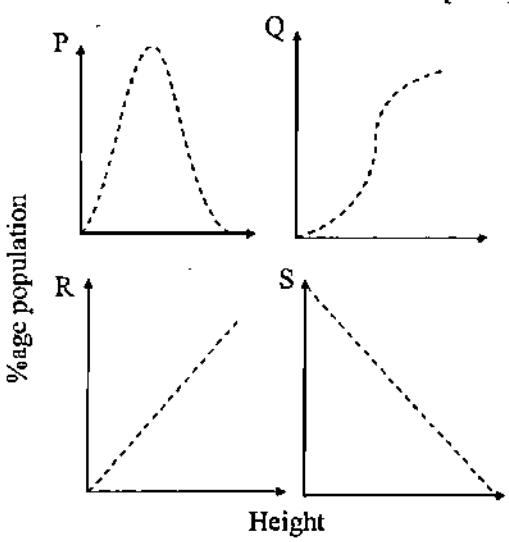
8. Lions in India are currently restricted to Gir, Gujarat. Efforts are being made to move them to other parts of the country. This is because they are MOST susceptible to extinction due to infectious diseases under the following conditions when present as- **[2013]**
(A) several small, isolated populations
(B) one large population
(C) several large, connected populations
(D) several large, isolated populations

9. Which one of the following interactions doest **NOT** promote coevolution ? **[2014]**
(A) Commensalism
(B) Mutualism
(C) Parasitism
(D) Interspecific competition

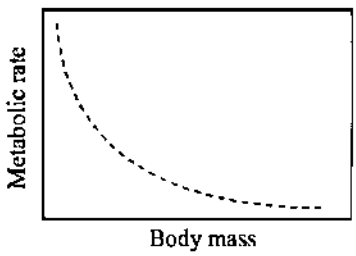
10. Stratification is more common in which of the following ? **[2014]**
(A) Deciduous forest (B) Tropical rain forest
(C) Temperate forest (D) Tropical savannah

11. Human height is a multigenic character. If the heights of all the individuals living in a metropolis are measured and the percentages of the population belonging to a specific heat are plotted as shown below, which of the plots would represent the most realistic distribution - **[2014]**



(A) P (B) Q (C) R (D) S

12. Study the following graph of metabolic rate of various terrestrial mammals as a function of their body mass and choose the correct option below. **[2016]**



(A) Animals are distributed throughout the curve with the smaller animals towards the left and progressively bigger animals towards the right
(B) The smaller animals below a certain critical mass cluster at the left end of the curve and the larger animals above the critical mass cluster on the right end
(C) Animals are distributed throughout the curve with the larger animals towards the left and progressively smaller animals towards the right
(D) The larger animals above a certain critical mass cluster at the left end of the curve and the smaller animals below the critical mass cluster on the right end

13. CO_2 acts as a greenhouse gas because **[5 Nov, 2017]**
(A) it is transparent to heat but traps sunlight.
(B) it is transparent to sunlight but traps heat.
(C) it is transparent to both sunlight and heat.
(D) it traps both sunlight and heat.

14. A graph of species richness *vs* area on log-log axes is **[5 Nov, 2017]**
(A) linear (B) sigmoidal
(C) oscillatory (D) parabolic

15. Which ONE of the following options is TRUE with respect to Emigration ? **[5 Nov, 2017]**
(A) It is the difference between the births and deaths in a population.
(B) It is the difference between individuals who have come to a habitat and who have left the habitat.
(C) It involves individuals of different species coming to a habitat from elsewhere during the period under consideration
(D) It involves individuals of a population leaving a habitat during the time period under consideration.

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16. In a population of families having three children each, the percentage of population of families having both boys and girls is

[19 Nov, 2017]

(A) 10 (B) 25

(C) 50 (D) 75

17. In a population interaction between the species X and the species Y, which ONE of the following statements is CORRECT ?

[19 Nov, 2017]

(A) When X benefits and Y is disadvantaged, it is Competition

(B) When both X and Y benefit, it is Mutualism

(C) When both X and Y are disadvantaged, it is Predation

(D) When both X and Y are disadvantaged, it is Parasitism

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ANSWERS

1. (A)	2. (B)	3. (C)	4. (C)	5. (D)	6. (A)	7. (D)
8. (C)	9. (D)	10. (B)	11. (A)	12. (A)	13. (B)	14. (A)
15. (D)	16. (D)	17. (B)				

SOLUTIONS

2. Actually around one half of the energy is lost through respiration.
Hence best option is 20% - 30 % - 60%

4. Chlorine of CFCs destroys Ozone layer causing harmful UV rays (mainly UV-B) to reach to earth surface which causes cancer.

5. Species which has the capacity to establish itself at new area has the power to struggle with challenges such as availability of food & shelter

6. Graph represents competition as the organism A and B individually shows increase in biomass.

7. Altruistic behaviour helps the species in its survival but the species showing it does not benefit from reduction in competition among offspring.

8. There are chances that the lions may get infected in a large interconnected population.

9. Interspecific competition doesn't lead to co-evolution

10. Tropical rain forest shows vertical zonation i.e. stratification.

11. P → plot
→ Height of human is multigenic character and shows bell shape curve as the occurance of extreme height will be low but medium height will be maximum.
→ Maximum % of population will have average medium height

12. The metabolic theory of Ecology (MTE) is an extension of Kleiber's law and states that the metabolic rate of organism is the fundamental biological rate that governs most observed patterns in ecology

13. Green house effect increases earth's temperature by trapping heat.

14. $\log S = \log c + z \log A$

15. Emigration going out from one population

16. Probability of 2 Boy & 1 Girl $= \frac{3}{8}$

Probability of 1 Girl & 2 Boys $= \frac{3}{8}$

Total probability $= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = 75\%$

17. Both species are benefited mutualism

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Model Test Paper

[KVPY [STREAM-SX] 3 Practice Test]

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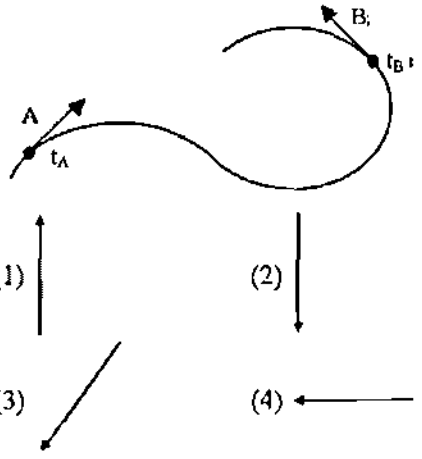
KVPY (Stream SX) Model Test Papers
Practice Test -1

PART- I
One - Mark Questions

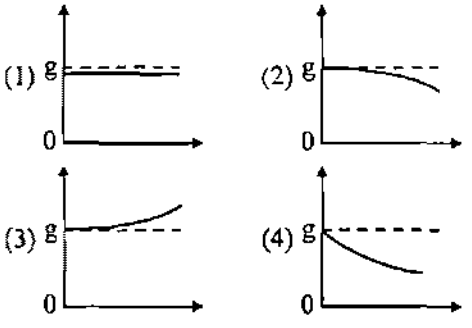
Questions 1 to 80 are Single correct answer type questions. Each correct answer carries 1 mark. 0.25 marks will be deducted for each wrong answer.

PHYSICS

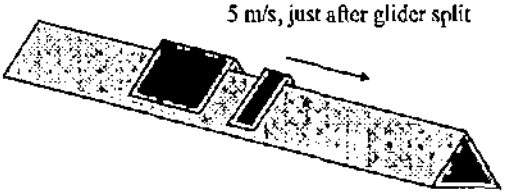
Q.1 An object travels along a path shown with changing velocity as indicated by vectors A and B. Which vector best represents the net acceleration of the object from time t_A to t_B?



Q.2 An object is dropped and accelerates downwards. As it falls it is affected by air friction, but never reaches terminal velocity during the course of its fall. The graph that could indicate the magnitude of the object's acceleration as a function of time is :

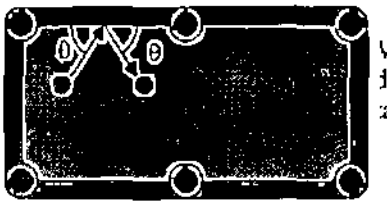


Q.3 An air track glider of mass M is built, consisting of two smaller connected gliders with a small explosive charge located between them. The glider is traveling along a frictionless rail at 2 m/s to the right when the charge is detonated, causing the smaller glider with mass $\frac{1}{4}$M, to move off to the right at 5 m/s. What is the final velocity of the second small glider ?

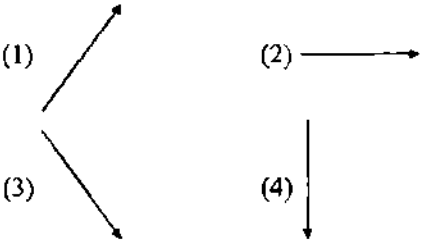


(1) 4 m/s to the left (4 m/s)
(2) 2 m/s to the left (2 m/s)
(3) 1 m/s to the left (1 m/s)
(4) 1 m/s to the right (1 m/s)

Q.4 A billiard ball hits the side of a pool table at an angle θ as shown in the top view above, and bounces away at the same angle, and with the same speed. Which vector indicates the direction of the net change in momentum of the billiard ball?



View looking down on pool table



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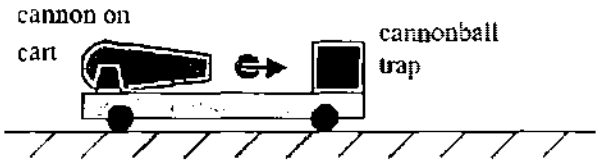
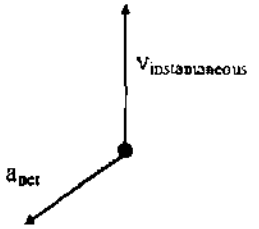
Q.5 A large cannon is mounted on a cart with frictionless wheels that is initially at rest on a horizontal surface. The cannon fires a large cannonball to the right with a speed $v_{cannonball}$, which is then caught by a trap firmly attached to the cart. What is the final speed v of the cannon-cart cannonball system?



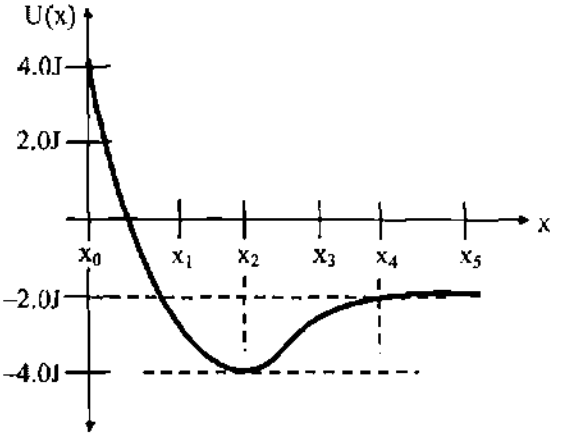
(1) $v > v_{cannonball}$, to the left ($v > v_{cannonball}$)
(2) $v > v_{cannonball}$, to the right ($v > v_{cannonball}$)
(3) 0
(4) $v < v_{cannonball}$, to the left ($v < v_{cannonball}$)

Q.6 The instantaneous velocity and net acceleration for an object moving in a circular path are shown. At this moment in time, the object is :



(1) speeding up in a clockwise circle
(2) slowing down in a clockwise circle
(3) speeding up in a counterclockwise circle
(4) slowing down in a counterclockwise circle

Q.7 The potential energy function U(x) is associated with a conservative force F and described by the graph given here. If a particle being acted upon by this force has a kinetic energy of 1.0 J at position x_0, what is the particle's kinetic energy at position x_4?



(1) 6.0 J (2) 7.0 J
(3) 2.0 J (4) –2.0 J

Q.8 A certain star, of mass m and radius r, is rotating with a rotational velocity ω. After the star collapses, it has the same mass but with a much smaller radius. Which statement below is true?

(1) The star's moment of inertia I has decreased, and its angular momentum L has increased
(2) The star's moment of inertia I has decreased, and its angular velocity ω has decreased
(3) The star's moment of inertia I remains constant, and its angular momentum L has increased
(4) The star's angular momentum L remains constant, and its rotational kinetic energy has increased

Q.9 As shown below, Lily is using the rope through a fixed pulley to move a box with constant speed v.
The kinetic friction coefficient between the box and the ground is $\mu < 1$; assume that the fixed pulley is massless and there is no friction between the rope and the fixed pulley. Then, while the box is moving, which of the following statements is correct?

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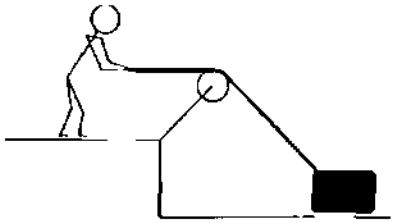
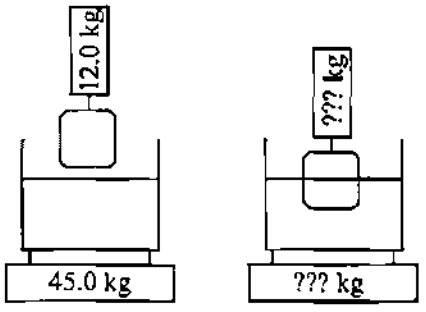


(1) The magnitude of the force on the rope is constant
(2) The magnitude of friction between the ground and the box is decreasing
(3) The magnitude of the normal force of the ground on the box is increasing
(4) The pressure of the box on the ground is constant

Q.10 A container of water is sitting on a scale. Originally, the scale reads $M_1 = 45$ kg. A block of wood is suspended from a second scale; originally the scale read $M_2 = 12$ kg. The density of wood is 0.60 g/cm^3 ; the density of the water is 1.00 g/cm^3. The block of wood is lowered into the water until half of the block is beneath the surface. What is the resulting reading on the scales?



(1) $M_1 = 45$ kg and $M_2 = 2$kg
(2) $M = 45$ kg and $M_2 = 6$ kg
(3) $M_1 = 45$ kg and $M_2 = 10$ kg
(4) $M_1 = 55$ kg and $M_2 = 2$kg

Q.11 A helium nucleus makes a full revolution in a circle of radius 0.8 m in 2 s. The magnetic field at the centre of the circle will be –
(1) $10^{-19}/\mu_0$ (2) $10^{-19}\,\mu_0$
(3) $2 \times 10^{-19}\mu_0$ (4) $2 \times 10^{-19}/\mu_0$

Q.12 A vertical straight conductor carries a current vertically upwards. A point P lies to the east of it at a small distance and another point Q lies to the west of it at the same distance. The magnetic field at P is -
(1) greater than at Q
(2) same as at Q
(3) less than at Q
(4) greater or less than at Q depending upon the strength of the current

Q.13 A wire of length ℓ and carrying a current i is placed along X-axis in a magnetic field given by $\vec{B} = B_0(\hat{i}+\hat{j}+\hat{k})$. The magnitude of the force acting on the wire is –
(1) $B_0\, i\, \ell$ (2) $\sqrt{2}\ B_0\, i\, \ell$
(3) $2\ B_0\, i\, \ell$ (4) $B_0\, i\, \ell/\sqrt{2}$

Q.14 A charged particle projected perpendicularly into a magnetic field of $(7\hat{i}\ \ 3\hat{j}) \times 10^{-3}$ tesla acquires an acceleration of $(x\hat{i}+7\hat{j})$ ms^{-2}. The value of x is –
(1) 2m (2) 3m (3) 4m (4) 7m

Q.15 Two circular, identical coaxial loops carry same current i in the same direction. If the loops are brought close to each other, then -
(1) current will increase in each loop
(2) current will decrease in each loop
(3) current will remain same in each loop
(4) current will increase in one and will decrease in the other

Q.16 The voltage and the current of an a.c. circuit are V = 100 sin(100 t) volt and i = 100 sin(100 t + π/3) mA respectively. The power dissipated in the circuit is –
(1) 10^4 W (2) 10 W (3) 2.5 W (4) 5.0 W

Q.17 Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is : [Take hc = 1240 eV nm]

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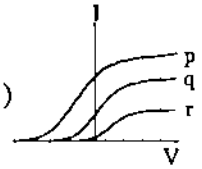
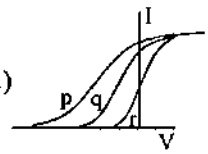
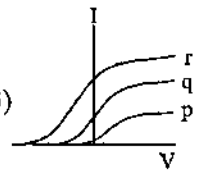
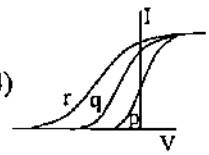
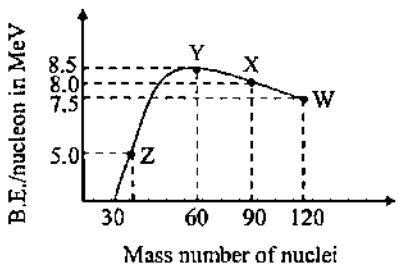
(1) 

(2) 

(3) 

(4) 

Q.18 Binding energy per nucleon verses mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is-



(1) $Y \rightarrow 2Z$ (2) $W \rightarrow X + Z$

(3) $W \rightarrow 2Y$ (4) $X \rightarrow Y + Z$

Q.19 Two radioactive nuclei P and Q, in a given sample decay into a stable nucleus R. At time $t = 0$, number of P species are $4\ N_0$ and that of Q are N_0. Half-life of P (for conversion to R) is 1 minute where as that of Q is 2 minutes. Initially there are no nuclei of R present in the sample. When number of nuclei of P and Q are equal, the number of nuclei of R present in the sample would be

(1) $\frac{5N_0}{2}$ (2) $2N_0$

(3) $3N_0$ (4) $\frac{9N_0}{2}$

Q.20 Which feedback is used in oscillator -

(1) Positive (2) Negative

(3) Both (4) None of thes

CHEMISTRY

Q.21 Friedel-Crafts acylation is-

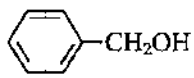
(1) α-acylation of a carbonyl compound

(2) acylation of phenols to generate esters

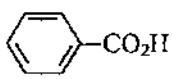
(3) acylation of aliphatic olefins

(4) acylation of aromatic ring

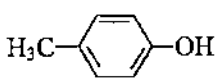
Q.22 The order of acidity of compounds I-IV, is-



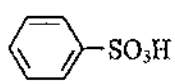
(I)



(II)



(III)

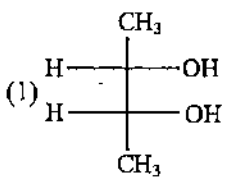


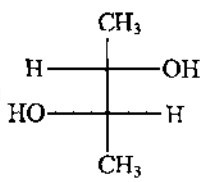
(IV)

(1) I < III < II < IV (2) IV < I < II < III

(3) III < I < II < IV (4) II < IV < III < I

Q.23 The most stable isomer for 2,3-butane diol –

(1) 

(2) 

(3) Both are same (4) None of these

Q.24 Identify Z in the following reaction sequence

$$CH_3I \xrightarrow[\text{Ether}]{Mg} X \xrightarrow[\text{ice}]{Dry} Y \xrightarrow[\text{Red P}]{Cl_2} Z -$$

(1) CH_3COOH (2) CH_3MgI

(3) CH_3COCl (4) $ClCH_2COOH$

Q.25 The rate of esterfication of HCOOH (I), CH_3COOH (II), $(CH_3)_2CHCOOH$ (III) and $(CH_3)_3CCOOH$ (IV) with ethanol follows in the order –

(1) IV > III > II > I

(2) I > II > III > IV

(3) II > I > IV > III

(4) III > IV > I > II

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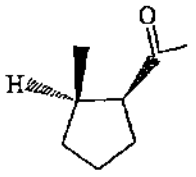
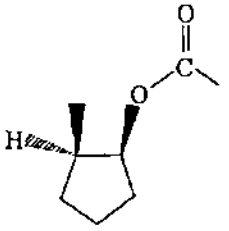
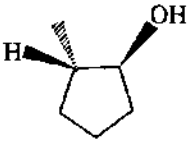
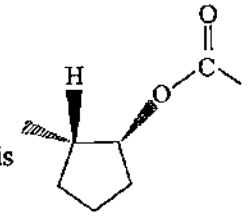
Q.26

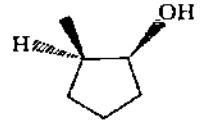


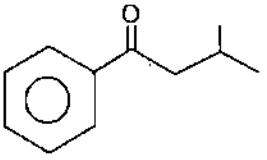
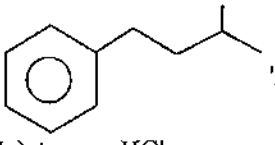
$\xrightarrow{RCO_3H}$ A $\xrightarrow{NaOH}$ B.

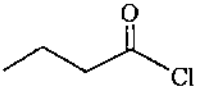
(1) A is 

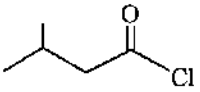
(2) B is 

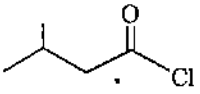
(3) A is 

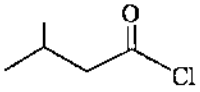
(4) B is 

Q.27 Benzene on reaction with 'A' forms  which on reaction with 'B' forms  'A' and 'B' are

(1) Zn (Hg) + conc. HCl, 

(2)  , $LiAlH_4$

(3)  , $NaBH_4$

(4)  , Zn (Hg) + conc. HCl

Q.28 Select incorrect statement about the compound $NO[BF_4]$:

(1) It has 5σ and 2π bond

(2) It is a diamagnetic species

(3) Nitrogen oxygen bond length is higher than nitric oxide (NO)

(4) B–F bond length in this compound is higher than in BF_3

Q.29 Which of the following equilibria would have highest value of K_P at a common temperature ?

(1) $BeCO_3 \rightleftharpoons BeO + CO_2$

(2) $CaCO_3 \rightleftharpoons CaO + CO_2$

(3) $SrCO_3 \rightleftharpoons SrO + CO_2$

(4) $BaCO_3 \rightleftharpoons BaO + CO_2$

Q.30 Which of the following will show optical isomers?

(I) cis-$[Co(NH_3)_2(en)_2]^{+3}$

(II) trans-$[IrCl_2(C_2O_4)_2]^{-3}$

(III) $[Rh(en)_3]^{+3}$

(IV) cis-$[Ir(H_2O)_3Cl_3]$

(1) I, III (2) II, IV (3) I, III, IV (4) III

Q.31 The magnetic moment of $[NiX_4]^{-2}$ ion is found to be zero. Then the hybridisation of Ni is :
(X = monodentate anionic ligand)

(1) sp^3 (2) spd^2 (3) dsp^2 (4) d^2sp

Q.32 In which of the following pair of metals both are commercially extracted from their respective ores by carbon reduction method ?

(1) Zn, Cu (2) Fe, Cu

(3) Sn, Zn (4) Al, Ag

Q.33 Formation of metallic copper from the sulphide ore in the commercial metallurgical process involves :

(1) $Cu_2S + \frac{3}{2}O_2 \rightarrow Cu_2O + SO_2$

; $Cu_2O + C \rightarrow 2Cu + CO$

(2) $Cu_2S + \frac{3}{2}O_2 \rightarrow Cu_2O + SO_2$

; $2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

(3) $Cu_2S + 2O_2 \rightarrow CuSO_4$

; $CuSO_4 + Cu_2S \rightarrow 3Cu + 2SO_2$

(4) $Cu_2S + \frac{3}{2}O_2 \rightarrow Cu_2O + SO_2$

; $Cu_2O + CO \rightarrow 2Cu + CO_2$

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Q.34 Heating of ammonium dichromate produces :
(1) NH_3, Cr_2O_3 and H_2O
(2) N_2, Cr_2O_3 and H_2O
(3) NO, CrO_3 and H_2O
(4) N_2O, CrO_3 and H_2O

Q.35 A compound is made up of atoms A and B. Atoms A are present at the corners of the cube as well as at the centre of each face while atoms B are present at 50% of octahedral voids and 25% tetrahedral voids. If the crystal cuts in such a way that all face centred atoms from one of the axis are removed the new formula of the compound is :
(1) AB (2) A_4B_3 (3) A_3B_4 (4) A_2B_3

Q.36 For A → B, $\Delta H = 4$ kcal mol^{-1}, $\Delta S = 10$ cal K^{-1} mol^{-1} reaction is spontaneous when temperature can be :
(1) 400 K (2) 300 K
(3) 500 K (4) None of these

Q.37 What mass of $CuCl_2$ to be dissolved in 100 g of water to raise the boiling point by 0.16 degree? [Consider complete ionisation. the molar mass of $CuCl_2 = 134.5$ g mol^{-1} & K_b of water $= 0.52$ Km^{-1}]
(1) 1.345 g (2) 2.69 g
(3) 4.035 g (4) 0.4035 g

Q.38 When pressure is applied to the equilibrium system ice $\rightleftharpoons$ water. Which of the following phenomenon will happen ?
(1) More ice will be formed
(2) Water will evaporate
(3) More water will be formed
(4) Equilibrium will not be formed

Q.39 The solubility product of AgCl is 4.0×10^{-10} at 298 K. The solubility of AgCl in 0.04 M $CaCl_2$ will be :
(1) 5.0×10^{-9} M (2) 2.0×10^{-5} M
(3) 2.2×10^{-4} M (4) 1.0×10^{-4} M

Q.40 What is the reduction electrode potential (in volts) of copper electrode when $[Cu^{2+}]$ in a solution is 0.01 M at 25°C? ($E°$ of Cu^{2+}/Cu electrode is + 0.34 V)
(1) 0.3991 (2) 0.2809
(3) 0.3105 (4) 0.3695

MATHEMATICS

Q.41 Let $\int \tan^{-1}\left(\frac{\tan x}{2}\right)dx = \alpha$ then the value of $\int \tan^{-1}\left(\frac{\tan x - 2\cot x}{3}\right)dx; \left(0 < x < \frac{\pi}{2}\right)$ equals to:
(1) $\alpha + \frac{\pi x}{2} + \frac{x^2}{2} + c$ (2) $\alpha - \frac{\pi x}{2} + \frac{x^2}{2} + c$
(3) $-\alpha - \frac{\pi x}{2} + \frac{x^2}{2} + c$ (4) $-\alpha + \frac{\pi x}{2} - \frac{x^2}{2} + c$

Q.42 $\sum_{r=1}^{n} \frac{r}{r^4 + r^2 + 1}$ is equal to
(1) $\frac{n^2+n}{2(n^2+n+1)}$ (2) $\frac{n^2+2n}{2(n^2+n+1)}$
(3) $\frac{2n^2+n}{2(n^2+n+1)}$ (4) $\frac{n^2+n}{(n^2+n+1)}$

Q.43 In the expansion of $(x + y + z)^9$, which one of the following is false :
(1) every term is of the form ${}^9C_r \cdot {}^rC_k \cdot x^{9-r} \cdot y^{r-k} \cdot z^k$
(2) coefficient of $x^4y^7z^3$ is 10
(3) the number of terms is 55
(4) coefficient of $x^2y^3z^4$ is 1260

Q.44 The function f (x) = 0 has eight distinct real solutions and f also satisfy f (4 + x) = f (4 − x). The sum of all the eight solutions of f (x) = 0 is
(1) 12 (2) 32
(3) 16 (4) 15

Q.45 Let the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ $(a > 0)$ at any point on it cuts the coordinate axes at P and Q. Then OP + OQ, where O is origin is equal to-
(1) a (2) 2a
(3) a^2 (4) $\sqrt{a}$

Q.46 If $|z - 4 + 3i| \le 1$ and α and β be the least and greatest values of $|z|$ and k be the least value of $\frac{x^4 + x^2 + 4}{x}$ on the interval $(0, \infty)$, then k is equal to-
(1) α (2) β
(3) $\alpha + \beta$ (4) None of these

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Q.47 If $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ...+ a_{2n}x^{2n}$, then $\begin{vmatrix} a_{n-3} & a_{n-1} & a_{n+1} \\ a_{n-6} & a_{n-3} & a_{n+3} \\ a_{n-14} & a_{n-7} & a_{n+7} \end{vmatrix}$ is:

(1) 1 (2) 2
(3) 0 (4) None of these

Q.48 If the ellipse $\frac{x^2}{4}+\frac{y^2}{1} = 1$ meet the ellipse $\frac{x^2}{1}+\frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 10b + 25$ then the value of b does not satisfy-

(1) $(-\infty, 2)$ (2) $(2, 3)$
(3) $(6, \infty)$ (4) $(4, 6)$

Q.49 If $\log_\alpha 8 = \gamma$, $\log_\beta \alpha = -1$ and $\log_{1/4} \beta = -1$ then $\left(\frac{1}{\alpha}+1\right)^{\log_{\sqrt{3}}(\beta^2+4\gamma^2)}$ is equal to-

(1) $\sqrt{5}$ (2) 5 (3) 25 (4) 625

Q.50 If A, B and C are n × n matrices and det (A) = 2, det (B) = 3 and det (C) = 5, then the value of the det$(A^2 BC^{-1})$ is equal to:

(1) $\frac{6}{5}$ (2) $\frac{12}{5}$ (3) $\frac{18}{5}$ (4) $\frac{24}{5}$

Q.51 The letters of word ZENITH are written in all possible ways. If all these words are written in the order of a dictionary, then the rank of the word ZENITH is-

(1) 716 (2) 692 (3) 698 (4) 616

Q.52 Arg $[i^{-53} + \{i^n + i^{n+1} + i^{n+2} + i^{n+3} + 4\}i]$ is

(1) 0 (2) π (3) $\frac{\pi}{2}$ (4) $-\frac{\pi}{2}$

Q.53 The perpendicular distance between the lines represented by $x^2 - 4xy + 4y^2 + x - 2y - 6 = 0$ is-

(1) $\frac{1}{\sqrt{5}}$ (2) $\sqrt{5}$ (3) $2\sqrt{5}$ (4) $3\sqrt{5}$

Q.54 $\sin\left(\frac{1}{5}\cos^{-1} x\right) = 1$ has -

(1) one solution (2) two solutions
(3) three solutions (4) none of these

Q.55 If the product of the intercepts made by the line $x \tan\alpha + y \sec\alpha = 1$ on the co-ordinate axes is $\sin\alpha$ then α is equal to –

(1) $\pi/4$ (2) $\pi/2$ (3) π (4) 0

Q.56 There are twenty bags each containing 10 bulbs and it is known that no bag contains more than 5 defective bulbs and 3 bags have 5 defective bulbs. 4 bags have atleast 4 defective bulbs, 5 bags have atleast 3 defective bulbs, 6 bags have atleast 2 defective bulbs and 7 bags have atleast 1 defective bulb. Then the ratio of total defective bulbs is to non defective bulbs is

(1) $\frac{4}{7}$ (2) $\frac{3}{7}$ (3) $\frac{2}{7}$ (4) $\frac{1}{7}$

Q.57 If $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, $g(x) = |\sin x| - |\cos x|$ and $\phi(x) = f(x)\, g(x)$ (where [.] denotes the greatest integer function) then the respective fundamental periods of f(x), g(x) and $\phi(x)$ are

(1) π, π, π (2) $\pi, 2\pi, \pi$
(3) $\pi, \pi, \frac{\pi}{2}$ (4) $\pi, \frac{\pi}{2}, \pi$

Q.58 For every $x \in R$ the value of the expression $y = \frac{x^2}{8} + x\cos x + \cos 2x$ is never less than

(1) –1 (2) 0 (3) 1 (4) 2

Q.59 If $\text{Re}\left(\frac{z-2i}{z+1}\right) = 1$, then the locus of the point representing z in the complex plane is -

(1) circle (2) a straight line
(3) parabola (4) none of these

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Q.60 If the pairs of lines $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have exactly one line in common then the joint equation of the other two lines can be-
(1) $3x^2 + 8xy - 3y^2 = 0$
(2) $3x^2 + 10xy + 3y^2 = 0$
(3) $y^2 + 2xy - 3x^2 = 0$
(4) $x^2 + 2xy - 3y^2 = 0$

BIOLOGY

Q.61 Development of egg without fertilization is called :
(1) oogenesis (2) metagenesis
(3) gametogenesis (4) parthenogenesis

Q.62 If the ovary is inferior, the outermost layer of fruit produced by this ovary will be formed by :
(1) Epicarp (2) Mesocarp
(3) Pericarp (4) Thalamus

Q.63 Ovulation occurs under the influence of :
(1) LH (2) Estrogens
(3) FSH (4) Progesterone

Q.64 Progesterone hormone is active during :
(1) follicular phase
(2) secretory phase
(3) menstrual phase
(4) proliferative phase

Q.65 Saheli, a new oral contraceptive for the females developed by Indian scientists is a :
(1) steroidal preparation
(2) hormonal preparation
(3) non-steroidal preparation
(4) toxic preparation to kill sperms

Q.66 In a cross 45 tall and 14 dwarf plants were obtained. Genotypes of parents are :
(1) Tt × tT (2) TT × tt
(3) TT × Tt (4) TT × TT

Q.67 Phenotypic ratio of dihybrid test cross is :
(1) 15 : 1 (2) 3 : 1
(3) 9 : 3 : 3 : 1 (4) 1 : 1 : 1 : 1

Q.68 Crossing over occurs between :
(1) two nonsister chromatids of same bivalent
(2) two sister chromatids of same chromosome
(3) two nonsister chromatids of different bivalents
(4) none of the above

Q.69 Mongolism is also known as :
(1) Down syndrome
(2) Turner syndrome
(3) Klinefelter syndrome
(4) Hypothalamic syndrome

Q.70 Which type of coiling is found in DNA ?
(1) Zig-zag (2) Opposite
(3) Left-handed (4) Right-handed

Q.71 Evolution of diversified species due to environmental changes is called :
(1) divergent evolution
(2) evolutionary inertia
(3) convergent evolution
(4) none of these

Q.72 Which of the following sets are vestigial structures in man ?
(1) Hair, olecranon process, coccyx and vermiform appendix
(2) Wisdom tooth, mammary glands, coccyx and patella
(3) Coccyx, nictitating membrane, appendix and ear muscles
(4) Hair, ear ossicles, patella and atlas

Q.73 DPT vaccine is given for :
(1) tetanus, polio, plague
(2) diphtheria, pneumonia, tetanus
(3) diphtheria, whooping cough, tetanus
(4) diphtheria, whooping cough and leprosy

Q.74 Lysis of foreign cell is mediated through :
(1) IgM only (2) IgA only
(3) IgM and IgG (4) IgD and IgE

Q.75 Green revolution means :
(1) increase in greenary everywhere
(2) increase in production of food crops
(3) increase in growth of green vegetation to develop ecological balance
(4) none of the above

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Q.76 In which country, the 'DOGS' were once worshipped as 'GODS'?
(1) Italy (2) Egypt
(3) Greece (4) Mangolia

Q.77 In genetic engineering, recombinant DNA means :
(1) DNA with a piece of RNA
(2) DNA with a piece of foreign DNA
(3) DNA which takes part in recombination
(4) DNA not associated with recombination

Q.78 Which of the following cut DNA at specific sites ?
(1) Ligase
(2) Exonuclease
(3) Alkaline phosphatase
(4) Restriction endonuclease

Q.79 A lake with an inflow of domestic sewage rich in organic waste may result in :
(1) Death of fish due to lack of oxygen
(2) Drying of the lake very soon due to algal bloom
(3) An increased production of fish due to a lot of nutrients
(4) Increased population of aquatic food web organisms

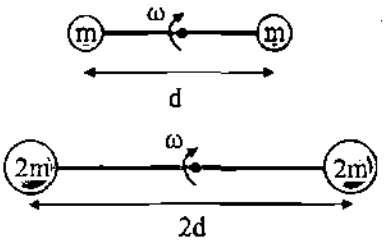
Q.80 Effect of pollution is observed first on :
(1) food crops (2) herbivores
(3) microorganisms (4) green vegetation

PART- II

Questions given below are single correct type Questions. Each correct answer carries +2. 0.5 marks will be deducted for each wrong answers.

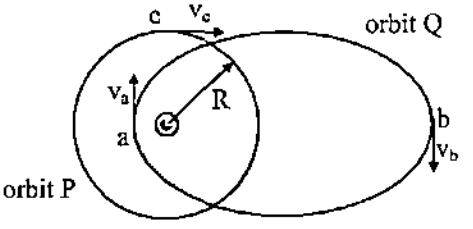
PHYSICS

Q.81 A dumbbell consists of two masses m connected by a rigid rod of negligible mass and length d. A physics student takes the dumbbell and rotates it about its center of mass with an angular velocity ω, giving it an angular momentum L_1. The student then takes a second dumbbell, with masses 2m and length 2d, and rotates them with the same angular velocity ω. What is the angular momentum L_2 of this second dumbbell?



(1) $2L_1$ (2) $4L_1$ (3) $6L_1$ (4) $8L_1$

Q.82 Consider the two orbits around the sun shown below. Orbit P is circular with radius R, orbit Q is elliptical such that the farthest point b is between 2R and 3R, and the nearest point a is between R/3 and R/2. Consider the magnitudes of the velocity of the circular orbit v_c, the velocity of the comet in the elliptical orbit at the farthest point v_b, and the velocity of the comet in the elliptical orbit at the nearest point v_a. Which of the following rankings is correct?



(1) $v_b > v_c > 2v_a$ (2) $2v_c > v_b > v_a$
(3) $10v_b > v_a > v_c$ (4) $v_c > v_a > 4v_b$

Q.83 In Young's double slit experiment, the introduction of a thin transparent film reduces the intensity at centre of screen by 75%. Then (μ = refractive index of film, t = thickness of film and λ = wavelength of light used)

(1) $\mu = \frac{5}{2}$ if $\lambda = 2t$

(2) $\mu = \frac{5}{3}$ if $\lambda = t$

(3) $\mu = \frac{5}{3}$ if $\lambda = 3t$

(4) $\mu = \frac{4}{3}$ for any value of λ

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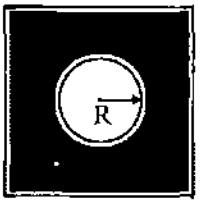
Q.84 A certain mass of gas undergoes a process given by $dU = \frac{dW}{2}$. If the molar heat capacity of the gas for this process is $\frac{15}{2}R$, then the gas is :

(1) monoatomic (2) polyatomic
(3) diatomic (4) data insufficient

Q.85 There is rectangular wire frame having a thin film of soap solution. A massless thin wire of radius R and area of cross section A is placed on the surface of film and inside portion of the film is pricked. If surface tension of soap solution is S and Young's modulus of wire is Y then change in radius of the wire is :



(1) $\frac{SR^2}{AY}$ (2) $\frac{2SR^2}{AY}$
(3) $\frac{SR^2}{3AY}$ (4) none of these

Q.86 In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be
(1) $v\,(3/4)^{1/2}$
(2) $v\,(4/3)^{1/2}$
(3) Less than $v\,(4/3)^{1/2}$
(4) Greater than $v\,(4/3)^{1/2}$

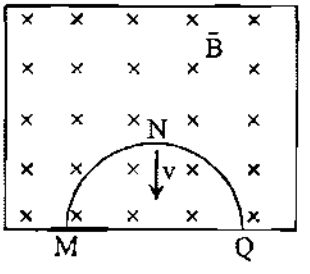
Q.87 Select the correct alternative (s) :
H^+, He^+ and O^{++} having masses 1 amu, 4 amu and 16 amu respectively, have same kinetic energy and pass through a uniform magnetic field perpendicular to their velocity. Then -
(a) H^+ will be deflected most
(b) O^{++} will be deflected most
(c) He^+ and O^{++} will be deflected equally
(d) all will be deflected equally
(1) a, c (2) a, b
(3) a, d (4) None of these

Q.88 Two coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. In both of them currents are increased at the same constant rate. At a certain instant the power given to the two coils is the same. If at that instant, i_1, V_1, U_1 and i_2, V_2, U_2 be the currents, induced voltages and energies stored in the two coils respectively, then

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$
(c) $\frac{U_2}{U_1} = 4$ (d) $\frac{V_2}{V_1} = \frac{1}{4}$
(1) a, b, c (2) a, b, d
(3) a, c, d (4) b, c, d

Q.89 Select the correct alternative(s) : An electron-beam is deflected into a circular path in a uniform magnetic field which is perpendicular to the beam at every point. Then :
(a) electrons do not acquire energy in the magnetic field
(b) the momentum of electrons is proportional to the radius of their circular path
(c) on increasing the magnetic field sufficiently, the path may become parabolic
(d) on applying an electric field parallel to the magnetic field, the beam may again adopt its original path
(1) a, c (2) a, b (3) a, d (4) None

Q.90 A thin semi-circular conducting ring of radius R is falling its plane vertical in a horizontal magnetic induction $\bar{B}$. At the position MNQ the speed of the ring is v and potential difference developed across the ring is –



(1) zero
(2) $B\,v\,\pi R^2/2$ and M is at higher potential
(3) $\pi\,R\,B\,v$ and Q is at higher potential
(4) $2\,R\,B\,v$ and Q is at higher potential

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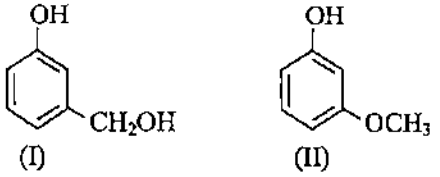
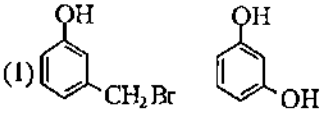
CHEMISTRY

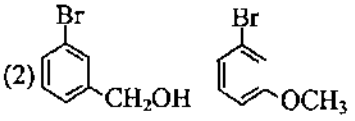
Q.91 Two isomeric compounds I and II are heated with HBr :

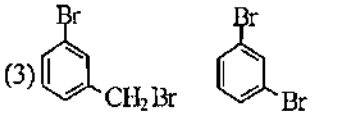


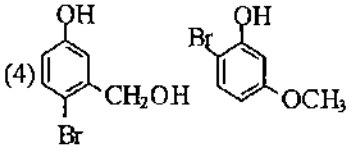
(I) (II)

The products obtained are -

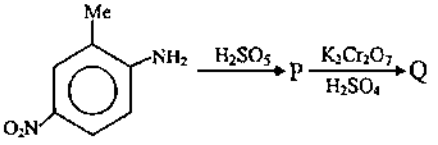
(1) 

(2) 

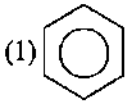
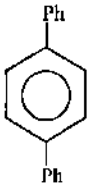
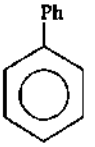
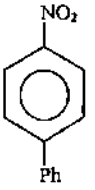
(3) 

(4) 

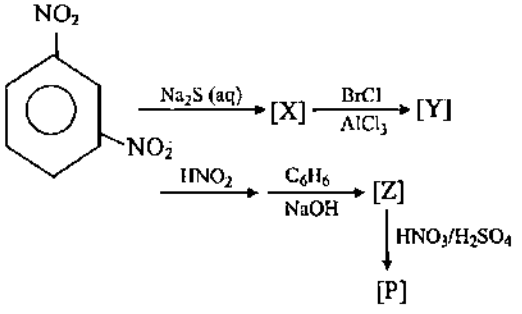
Q.92



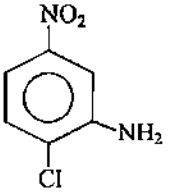
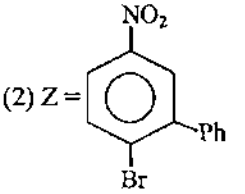
U is

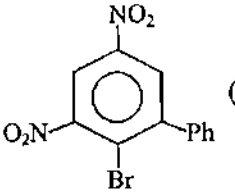
(1)  (2)  (3)  (4) 

Q.93



The correct statement is -

(1) Y =  (2) Z = 

(3) P =  (4) None of these

Q.94 What is the quantity of electricity (in columobs) required to deposit all the silver from 250 ml of a 1 M $AgNO_3$ solution ?

(1) 2412.5 (2) 24125
(3) 4825.0 (4) 48250

Q.95 Which of the following equation is incorrectly written?

(1) $P_4 + 20\ HNO_3 \rightarrow 4H_3PO_4 + 20NO_2 + 4H_2O$
(2) $I_2 + 10\ HNO_3 \rightarrow 2HIO_3 + 10\ NO_2 + 4H_2O$
(3) $S + 6\ HNO_3 \rightarrow H_2SO_4 + 6NO_2 + 2H_2O$
(4) None of these

Q.96

$AgNO_3 \xrightarrow{\Delta} (W) + (X) + O_2$
$(X) + H_2O \longrightarrow HNO_3 + HNO_2$
$(W) + HNO_3 \longrightarrow (Y) + NO + H_2O$
$(Y) + Na_2S_2O_3(\text{excess}) \longrightarrow (Z) + NaNO_3$

(1) W = Ag X = N_2O
Y = $AgNO_3$ Z = $Na_2[Ag(S_2O_3)_2]$
(2) W = Ag_2O X = NO
Y = $AgNO_3$ Z = $Na_3[Ag(S_2O_3)_2]$
(3) W = Ag X = NO_2
Y = $AgNO_3$ Z = $Na_3[Ag(S_2O_3)_2]$
(4) W = Ag_2O X = N_2
Y = $AgNO_3$ Z = $Na[Ag(S_2O_3)_2]$

Q.97 $(X) + K_2CO_3 + \text{air} \xrightarrow{\Delta} (Y)$

$(Y) + Cl_2 \longrightarrow \underset{\text{pink}}{(Z)}$

Which of the following is correct ?

(1) X = black, MnO_2
Y = Blue , K_2CrO_4 Z = $KMnO_4$
(2) X = green , Cr_2O_3
Y = Yellow, K_2CrO_4 Z = $K_2Cr_2O_7$
(3) X = black, MnO_2
Y = green, K_2MnO_4 Z = $KMnO_4$
(4) X = black, Bi_2O_3
Y = colourless, $KBiO_2$ Z = $KBiO_3$

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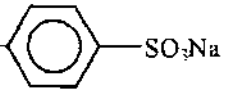
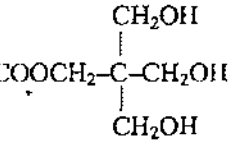
Q.98 In the reaction
$CH_3COCH_3(g) \longrightarrow C_2H_4(g) + H_2(g) + CO(g)$
the initial pressure was found to be 0.40 atm and after 10 min it was 0.58 atm. The rate constant for first order reaction is
[log 4 = 0.6021, log 3.5 = 0.5441]
(1) 0.0133 min^{-1} (2) 0.4 s^{-1}
(3) 10 s^{-1} (4) 0.6 min^{-1}

Q.99 Which of the following is non ionic detergent ?
(1) $C_{15}H_{31}COONa$
(2) 
(3) $[C_{17}H_{35}N(CH_3)_3]Br$
(4) 

Q.100 10.5 ml of a mixture of CH_4 and CO was mixed with 25 ml of oxygen and exploded. When the resultant cooled gas was passed through KOH solution, all the CO_2 got absorbed and the volume of remaining gas was found to be 16 ml. The volume of CH_4 in the mixture is :
(1) 2.5 ml (2) 3.5 ml
(3) 7.0 ml (4) 8.0 ml

MATHEMATICS

Q.101 If the imaginary part of the expression $\frac{e^{i\theta}}{z-2} + \frac{z-2}{e^{i\theta}}$ is zero, then the locus of z is
(where θ is a constant such that $\theta \neq \arg(z-2)$)
(1) a straight line parallel to x-axis
(2) a parabola
(3) a circle of radius 1 and centre (2, 0)
(4) a straight line parallel to y-axis

Q.102 The range of $f(x) = \cot^{-1}(-x) - \tan^{-1} x + \sec^{-1} x$ is :
(1) $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (2) $\left[\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right]$
(3) $\left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$ (4) $\left(\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

Q.103 $\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^{\alpha} + \sin\frac{1}{n}\right)^{n}$ ($\alpha \in Q$) is equal to
(1) e^{α} (2) $-\alpha$ (3) $e^{1-\alpha}$ (4) $e^{1+\alpha}$

Q.104 Coordinate of the vertices B and C of a triangle ABC are (2, 0) and (8, 0) respectively. The vertex A is varying in such a way that $4\tan\frac{B}{2}\tan\frac{C}{2} = 1$. The locus of A is-
(1) $\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$ (2) $\frac{(x-5)^2}{16} + \frac{y^2}{25} = 1$
(3) $\frac{(x-5)^2}{25} + \frac{y^2}{9} = 1$ (4) $\frac{(x-5)^2}{9} + \frac{y^2}{25} = 1$

Q.105 Let $f(x) \lim_{n\to\infty} n^2\left(x^{\frac{1}{n}} - x^{\frac{1}{n+1}}\right)$; $x > 0$, then $\int xf(x)\,dx$ equals to
(1) $\frac{x^2}{2}\ell n\, x + c$ (2) $-\frac{x^2}{4}\ell n\ x + \frac{x^2}{2} + c$
(3) $\frac{x^2}{2}\ell n\, x + \frac{x^2}{4} + c$ (4) $\frac{x^2}{2}\ell n\, x - \frac{x^2}{4} + c$

Q.106 If vector $\hat{i} + 2\hat{j} + 2\hat{k}$ is rotated through an angle of 90° so as to cross the positive direction of y-axis then the vector in the new position is
(1) $-\frac{2}{\sqrt{5}}\hat{i} + \sqrt{5}\,\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$
(2) $\frac{2}{\sqrt{5}}\hat{i} - \sqrt{5}\,\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$
(3) $4\hat{i} - \hat{j} - \hat{k}$
(4) None of these

Q.107 If a, b, c are odd integers, then the equation $ax^2 + bx + c = 0$ can not have -
(1) imaginary roots (2) real roots
(3) irrational roots (4) rational roots

Q.108 If m and n are positive integers satisfying
$1 + \cos2\theta + \cos4\theta + \cos6\theta + \cos8\theta + \cos10\theta = \frac{\cos m\theta \sin n\theta}{\sin\theta}$, then (m + n) is equal to-
(1) 9 (2) 10 (3) 11 (4) 12

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Q.109 A triangle has base 10 cm long and the base angles of 50° and 70°. If the perimeter of the triangle is $x + y \cos z^\circ$, where $z \in (0, 90^\circ)$, then the value of $x + y + z$ equals –
(1) 60 (2) 55 (3) 50 (4) 40

Q.110 When $1! + 2! + 3! + 4! + \ldots\ldots + 2009!$ is divided by 35, then remainder will be –
(1) 31 (2) 32 (3) 33 (4) 36

BIOLOGY

Q.111 What would be the chromosome number of endosperm if it is developed from *Oenothera* type of embryo sac possessing chromosome number equal to 16 (haploid)?
(1) 32 (Diploid)
(2) 48 (Triploid)
(3) 16 (Haploid)
(4) None of these

Q.112 If the vasa differentia of a man are surgically cut or blocked :
(1) semen will be without sperms
(2) spermatogenesis will not take place
(3) testosterone will disappear from blood
(4) sperms in the semen become nonmotile

Q.113 A heterozygous round seeded (Rr) plant is crossed to recessive wrinkled (rr) seeded plant. The progeny would be :
(1) 20 rounded : 99 wrinkled
(2) 99 rounded : 301 wrinkled
(3) 301 rounded : 100 wrinkled
(4) 303 rounded : 301 wrinkled

Q.114 The length of DNA molecule greatly exceeds the dimensions of the nucleus in eukaryotic cells. How is this DNA accommodated ?
(1) DNase digestion
(2) Super-coiling in nucleosomes
(3) Deletion of nonessential genes
(4) Through elimination of repetitive DNA

Q.115 Down syndrome and Turner syndrome occur in human beings due to :
(1) nullisomic and monosomic conditions respectively
(2) monosomic and nullisomic conditions respectively
(3) trisomic and monosomic conditions respectively
(4) monosomic and trisomic conditions respectively

Q.116 DNA fingerprinting refers to :
(1) Molecular analysis of profiles of DNA samples
(2) Analysis of DNA samples using imprinting devices
(3) Techniques used for identification of fingerprints of individuals
(4) Techniques use for molecular analysis of different specimens of DNA

Q.117 Similarity in organisms with different genotypes indicates :
(1) microevolution (2) divergent evolution
(3) macroevolution (4) convergent evolution

Q.118 Agarose extracted from sea weeds finds use in :
(1) Tissue culture
(2) PCR
(3) Gel electrophoresis
(4) Spectrophotometry

Q.119 Which of the following is not used for disinfection of drinking water ?
(1) Ozone (2) Phenyl
(3) Chlorine (4) Chloramine

Q.120 Drinking of mineral water with very low levels of pesticides (about 0.02 ppm) for long periods may :
(1) cause cancer of the intestine
(2) produce immunity against mosquito
(3) cause leukaemia (blood cancer) in most people
(4) lead to accumulation of pesticide residues in body fat

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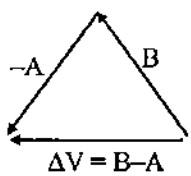
ANSWERS

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	4	4	4	3	4	2	4	2	4	2	1	2	2	2	3	1	3	4	1
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	2	2	2	4	4	3	1	1	3	3	2	2	3	1	1	3	1	2
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	1	2	2	1	2	3	4	4	2	4	3	2	4	1	4	3	1	2	2
Qus.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	4	4	1	2	3	1	4	1	1	4	1	3	3	3	2	2	2	4	1	3
Qus.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	4	3	2	3	2	3	1	3	2	4	1	2	2	2	4	3	3	1	4	1
Qus.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	3	2	3	1	4	4	4	3	4	3	1	1	4	2	3	1	4	3	2	4

SOLUTIONS

PART-I
One Mark Questions

PHYSICS

1. 

The direction of acceleration is the same as the direction of the changes in velocity, according to $a = \frac{v_f - v_i}{t}$. Because $\Delta v = v_f - v_i$, we can determine Δv graphically by adding v_f to the negative of v_i, or B + (–A). Placing the B vector "tip-to-tail" with the –A vector gives a direction for Δv (and therefore, a) to the left

2. The falling object, when released from rest, has an initial acceleration of 9.8 m/s^2 (if near the surface of the earth). As its velocity increases, it collides with air molecules at an increasing rate, thus reducing the rate at which it accelerates. (The acceleration is usually modeled as a function of v or v^2. depending on a number of factors.) The acceleration continues to decrease until the acceleration of the object is 0, at which point the velocity of the falling object remains constant. The only graph consistent with this analysis is d, where the acceleration curve can be seen to be approaching zero asymptotically.

3. This is a conservation of momentum problem, in which the total momentum of the glider at the beginning of the problem is equal to the sum of the momenta of the individual gliders at the end of the problem

$Mv = m_1v_1' + m_2v_2'$

$Mv = \frac{1}{4}Mv_1' + \frac{3}{4}Mv_2'$

$v = \frac{1}{4}v_1' + \frac{3}{4}v_2'$

$\Rightarrow v_2' = \frac{4}{3}(v - \frac{1}{4}v_1')$

$v_2' = \frac{4}{3}(2 - \frac{1}{4}5') = 1$ m/s.

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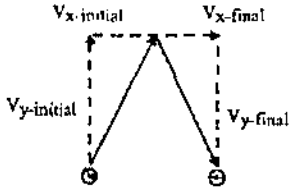
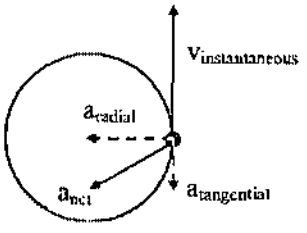
4. The billiard ball has velocity vectors in both the x and y directions, as shown here. The x-component of the velocity remains unchanged, but the y-component has changed from the positive to the negative direction, a change in velocity — and thus, momentum — in the negative y direction.



5. The overall momentum of the system at the beginning of the problem is 0, and with no external forces applied, this will continue to be the total momentum of the system. During the ball's motion to the right, the cart will have a motion to the left. However, once the cannonball is caught by the trap, they have the same velocity, which has to be zero if linear momentum is to be conserved

$m_{cart}v_{cart} + m_{cannonball}v_{cannonball} = m_{cart'}v_{cart'} + m_{cart}v_{cart}$
$(m_{cart} + m_{cannonball})v_{initial} = (m_{cart} + m_{cannonball})v_{final}$
$(m_{cart} + m_{cannonball})(0\ m/s) = (m_{cart} + m_{cannonball})v_{final}$
$v_{final} = 0\ m/s$

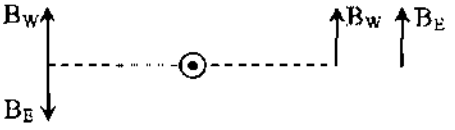
6. The instantaneous velocity of the object is tangent to its circular path, and we know that there's a radial (centripetal) aspect of the net acceleration that points towards the center of the circular path. Thus, we can conclude that the object is traveling in a circular path that is located to its left, as shown here.
We can also see that the net acceleration must include a tangential component of acceleration that is in the opposite direction of the instantaneous velocity, implying that the object is slowing down as it travels along this circular path.



7. This is a conservation of energy problem, and the total mechanical energy of the system remains constant :
$U_i + K_i = U_f + K_f$
$4.0\ J + 1.0\ J = -2.0\ J + K_f$
$K_f = 7.0\ J$

8. According to conservation of angular momentum, the angular momentum L of the star remains constant,
so when its moment of inertia I increases (due to the decreased radius), its angular velocity w goes up proportionally, according to :
$L_{initial} = L_{final}$
$I_i w_i = I_f w_f$
$w_f = \frac{I_i}{I_f} w_i$
The star's rotational kinetic energy, based on $K_{rotational} = \frac{1}{2} Iw^2$ also goes up. Although I has decreased, $K_{rotational}$ increases with the square of w, leading to a net increase in energy.

11. $B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q}{2TR} = \frac{\mu_0 2e}{2TR}$

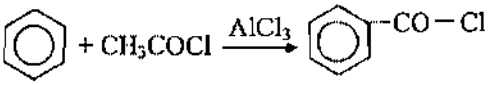
12. 

13. $\vec{F} = I\vec{\ell} \times \vec{B} \Rightarrow = I\ell(\hat{i}) \times B_0(\hat{i} + \hat{j} + \hat{k})$
$\vec{F} = B_0 I\ell(\hat{k} - \hat{j}) \Rightarrow \vec{F} = \sqrt{2}B_0 I\ell$

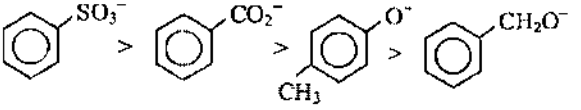
14. $\vec{F}_m = q\vec{v} \times \vec{B}$
$m\vec{a} = q\vec{v} \times \vec{B} \Rightarrow \vec{a} \perp \vec{v}$ so $\vec{a}.\vec{v} = 0$

16. $P = \frac{E_0 I_0}{2} \cos\frac{\pi}{3}$

CHEMISTRY

21. 

22. Stability of anion



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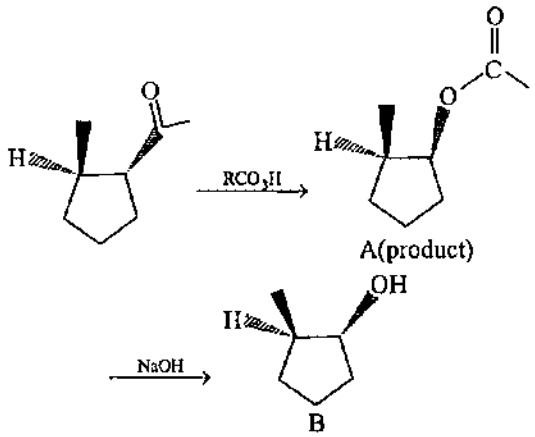
23. When attraction force is present then optically active isomer is more stable.

26.

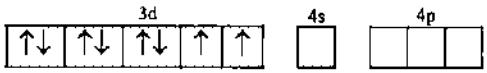


28. $NO^+ [BF_4]^-$
NO^+ has bond order = 3
NO has bond order = 2.5

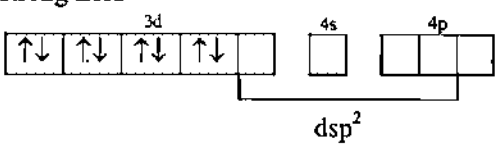
29. $BeCO_3$ has least thermal stability

30. Cis-$M(aa)_2b_2$ and $M(aa)_3$ show optical isomerism

31. $[NiX_4]^{-2}$
$Ni^{+2} = [Ar]\ 4s^0\ 3d^8$



magnetic moment is zero then ligand must be strong field



34. $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

35. $A = \frac{1}{8} \times 8 + \frac{1}{2} \times 4 = 1 + 2 = 3$

$B = \frac{1}{2} \times 4 + \frac{1}{4} \times 8 = 2 + 2 = 4$

A_3B_4

36. $\Delta H - T\Delta S = 0$
$4 \times 10^3 - T \times 10 = 0$
$T = 400$ K

37. $\Delta T_b = iK_b m$

$$0.16 = 3 \times 0.52 \times \frac{W_B}{134.5} \times \frac{1000}{100}$$

$W_B = 1.38$ g

38. More H_2O will form.

39. $K_{sp} = [Ag^+]\ [Cl^-]$
$CaCl_2 \rightarrow Ca^{+2} + 2Cl^-$
0.04 0.08
$4 \times 10^{-10} = K_{SP} = S \times (S + 0.08)$
$K_{sp} = 5 \times 10^{-9}$ M

40.

$$E_{Cu^{+2}/Cu} = E_{Cu^{+2}/Cu} - \frac{0.0591}{n} \log \frac{1}{[Cu^{+2}]}$$

$$= +0.34 - \frac{0.0591}{2} \log \frac{1}{0.01}$$

$$= 0.34 - \frac{0.0591}{2} \times 2 \quad = 0.2809 \text{ V}$$

MATHEMATICS

41.

$$\int \tan^{-1}\left(\frac{\tan x - 2\cot x}{3}\right) dx$$

$$= \int \tan^{-1}(\tan x)\, dx - \int \tan^{-1}(2\cot x)\, dx$$

$$= \int x\, dx - \int \frac{\pi}{2} - \cot^{-1}(2\cot x)\, dx$$

$$= \int x\, dx - \int \frac{\pi}{2} - \tan^{-1}\left(\frac{\tan x}{2}\right) dx$$

$$= \frac{x^2}{2} - \frac{\pi x}{2} + \alpha + c$$

42.

$$\frac{1}{2} \sum_{r=1}^{n} \frac{2r}{(r^2 + r + 1)(r^2 - r + 1)}$$

$$= \frac{1}{2} \sum_{r=1}^{n} \left(\frac{1}{(r^2 - r + 1)} - \frac{1}{(r^2 + r + 1)}\right)$$

$$= \frac{1}{2} \left(\frac{1}{(1^2 - 1 + 1)} - \frac{1}{(n^2 + n + 1)}\right)$$

$$= \frac{1}{2} \left(1 - \frac{1}{n^2 + n + 1}\right) = \frac{n^2 + n}{2(n^2 + n + 1)}$$

43. $(x + y + z)^9 = [x + (y + z)]^9$
$T_{r+1} = {}^9C_r x^{9-r} (y + z)^r$
Now the general term of $(y + z)^r = {}^rC_k y^{r-k} z^k$
Hence the general term of given expansion
$= {}^9C_r\ {}^rC_k x^{9-r}\ y^{r-k} z^k$
the coefficient of $x^4y^7z^3$ is 0 and coefficient of $x^2y^3z^4$ is ${}^9C_7\ {}^7C_3 = 36 \times 35 = 1260$
and number of terms in the expansion = ${}^{9+2}C_2$

$$= \frac{11 \times 10}{2} = 55$$

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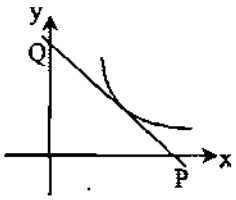
44. Symmetric about $x = 4$

$$2\left\{\left[\frac{x_1+x_2}{2}\right]+\left[\frac{x_3+x_4}{2}\right]+\left[\frac{x_5+x_6}{2}\right]+\left[\frac{x_7+x_8}{2}\right]\right\}$$

$= 2\{4+4+4+4\} = 16 \times 2 = 32$

45. $\because \sqrt{x}+\sqrt{y}=\sqrt{a}$



Slope of tangent at (x_1, y_1) is

$$\frac{1}{2}\frac{1}{\sqrt{x_1}}+\frac{1}{2}\frac{1}{\sqrt{y_1}}\left(\frac{dy}{dx}\right)=0 \Rightarrow \frac{dy}{dx}=\frac{-\sqrt{y_1}}{\sqrt{x_1}}$$

Equation of tangent is $y - y_1 = \frac{-\sqrt{y_1}}{\sqrt{x_1}}(x - x_1)$

$\because$ point P on x-axis is $(x_1 + \sqrt{x_1 y_1}, 0)$

Point Q on y-axis is $(0, y_1 + \sqrt{x_1 y_1})$

$\therefore OP + OQ = x_1 + y_1 + 2\sqrt{x_1 y_1} = (\sqrt{x_1} + \sqrt{y_1})^2 = a$

46. Given,

$$1 \geq |z-(4-3i)| \geq \begin{cases} |z|-|4-3i| \\ |4-3i|-|z| \end{cases}$$

$\Rightarrow |z| \leq 6$ and $|z| \geq 4$

$\Rightarrow 4 \leq |z| \leq 6 \Rightarrow \alpha = 4, \beta = 6$

Let $y = \frac{x^4+x^2+4}{x} = x^3 + x + \frac{4}{x}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$

Since $x \in (0, \infty)$, therefore

$x^3, x, \frac{1}{x}$ are positive.

Sum will be least when $x^3 = x = \frac{1}{x}$

$\Rightarrow x = 1$

$\therefore k = 6$

Hence, $k = \beta$

47. Given,

$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$ ...(1)

Replacing x by $\frac{1}{x}$, we get

$(1 + x + x^2)^n = a_0 x^{2n} + a_1 x^{2n-1} + a_2 x^{2n-2} + \ldots + a_{2n}$

$\Rightarrow a_0 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$

$= a_0 x^{2n} + a_1 x^{2n-1} + \ldots + a_{2n}$

On equating the coefficients of similar powers of x,

we get $a_0 = a_{2n}$, $a_1 = a_{2n-1}, \ldots, a_r = a_{2n-r}$

$\Rightarrow$ The value of given determinant is equal to 0 as 2nd and 3rd column are identical.

48. Clearly $a > 1$

$b^2 - 10b + 25 > 1$

$b^2 - 10b + 24 > 0$

$b \in (-\infty, 4) \cup (6, \infty)$

49. $\log_{1/4}\beta = -1 \Rightarrow \beta = 4$

$\log_\beta \alpha = -1 \Rightarrow \alpha = \frac{1}{\beta} = \frac{1}{4}$

$\gamma = \log_\alpha 8 = ; \log_{1/4} 8 = -\frac{3}{2}$

Now $\frac{1}{\alpha} + 1 = 4 + 1 = 5$

$\beta^2 + 4\gamma^2 = 16 + 4\left(-\frac{3}{2}\right)^2 = 25$

$\therefore \left(\frac{1}{\alpha}+1\right)^{\log_{\sqrt{5}}(\beta^2+4\gamma^2)} = 5^{\log_{\sqrt{5}} 25} = 625$

50. $|A^2BC^{-1}| = |A|^2 \cdot |B| \frac{1}{|C|} = \frac{12}{5}$

51. The total number of words is $6! = 720$. Let us write the letters of word ZENITH alphabetically, i.e. EHINTZ.

52. $\text{Arg}\left\{\frac{1}{(i^4)^{13} i} + (i^n + i.i^n - i^n - i.i^n + 4)i\right\}$

$\text{Arg}\{-i + 4i\} = \text{Arg}(3i) = \frac{\pi}{2}$

53. Lines are $(x - 2y)^2 + (x - 2y) - 6 = 0$

Let $x - 2y = t$

$t^2 + t - 6 = 0$

$(t + 3)(t - 2) = 0$

$\therefore t + 3 = 0$ or $t - 2 = 0$

$x - 2y + 3 = 0$ or $x - 2y - 2 = 0$

distance $= \sqrt{5}$

54. $\sin\left(\frac{1}{5}\cos^{-1} x\right) = 1 \Rightarrow \frac{1}{5}\cos^{-1} x = \frac{\pi}{2}$

$\cos^{-1} x = \frac{5\pi}{2}$, but $\cos^{-1} x \in [0, \pi]$, no solution.

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55. $x = \cot\alpha$ and $y = \cos\alpha$
$\Rightarrow \cot\alpha \cdot \cos\alpha = \sin\alpha$
$\tan\alpha = \pm 1 \Rightarrow \alpha = \pi/4$ or $3\pi/4$

56. Total number of bulbs = 200
Total number of defective bulbs
$= 1(7-6) + 2(6-5) + 3(5-4) + 4(4-3) + 5(3)$
$= 25.$
Answer is $\frac{25}{175} = \frac{1}{7}$.

57. Clearly $f(x + \pi) = f(x)$, $g(x + \pi) = g(x)$ and
$\phi\left(x + \frac{\pi}{2}\right) = \{(-1) f(x)\}\{(-1) g(x)\} = \phi(x)$.

58. $y = \frac{x^2}{8} + x\cos x + \cos 2x$
$= \frac{x^2}{8} + x\cos x + 2\cos^2 x - 1$
$= 2\left(\cos x + \frac{x}{4}\right)^2 - 1.$

59. $\text{Re}\left(\frac{z-2i+1-1}{z+1}\right) = 1$
$\Rightarrow \text{Re}\left(\frac{-1-2i}{z+1} + 1\right) = 1$
$\Rightarrow \text{Re}\left(\frac{-1-2i}{z+1}\right) = 0$
$\Rightarrow x + 1 + 2y = 0$

60. $am^2 + 2m + 1 = 0$
$m^2 + 2m + a = 0$
common root $m = 1$
other roots $= 1/a, a$
$m^2 - (a + 1/a)m + 1 = 0$
$\Rightarrow x^2 - (a + 1/a)xy + y^2 = 0$ (also $a < 1$)

BIOLOGY

61. Parthenogenesis is development of

62. Thalamus forms part of fruit where ovary is inferior.

63. LH hormone stimulates the ovulation.

64. In menstrual cycle after ovulation corpus luteum is develop in ovary which secrets the progesteron hormon so this phase is called as secretory phase.

65. Saheli chemically (centchroman or ormiloxifene) is a non steriodal female contraceptive.

66. 3 : 1 ratio is obtained on crossing heterozygous forms.

67. Test cross always shows only one type for each phenotype.

68. Crossing overy occurs between non sister chromatid of homologous chromosomes.

69. Down syndrome is caused due to trisomy 21st somatic chromosome.

70. B-DNA (common form) is right handed.
[B-DNA]

71. Members of a same species diversify them self & get settled in new environment, which causes divergent evolution.

72. Coccyx, Nictitating membrane (3rd eyelid) and muscles of ear pinna and vermiform appendix are vestigeal in human being.

73. DPT vaccine prevents against diptheria, pertussis & tetanus.

75. Green revolution in India was brought by M.S. Swami Nathan by increasing wheat production.

76. Dogs are the oldest pets with man due to their faithfulness, they were worshipped in Egypt.

77. Recombinant DNA means alignment of foreign DNA with host DNA.

78. Restriction endonuclease are highly specific and identify palindromic sites.

79. Inflow of domestic sewage causes algal bloom which decreases oxygen level in lake causing death of fishes.

80. E. coli are good indicator of water pollution.

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PART-II
Two Marks Questions

PHYSICS

81. Although the problem asks us to compare angular momenta of the two dumbbells, $L = I\omega$, and ω is identical for both dumbbells. Therefore, this is really a problem asking us to compare the moments of inertia I

$I_1 = \sum r^2 m$

$I_1 = \left(\frac{1}{2}d\right)^2 m + \left(\frac{1}{2}d\right)^2 m = \frac{d^2 m}{2}$

Now let's calculate I_2,

$I_2 = \sum r^2 m$

$I_2 = (d)^2 2m + (d)^2 2m$

$I_2 = 2d^2 2m = 4d^2 m$

$I_2 = 8\left(\frac{d^2 m}{2}\right) = 8I_1$

Because the moment of inertia I_2 is 8 times I_1, the angular momentum L_2 is $8L_1$ as well.

83. As intensity becomes one-fourth

$\therefore \cos\phi = -\frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3}, \frac{4\pi}{3}$.....

and $\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(\mu - 1)t$

84. $dU = \frac{dW}{2}$

by 1st law $\rightarrow dQ = 3dU = 3nC_v dT$

molar heat capacity $C = \frac{dQ}{ndT} = 3C_V$

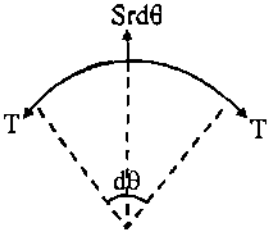
given $C = \frac{15R}{2}$

$\therefore C_V = \frac{5}{2}R$

$\therefore$ degree of freedom = 5

so gas is diatomic.

85.



Consider an element of wire forming angle θ at the centre

$2T\sin\frac{d\theta}{2} = 2 \times [S \times R \times d\theta]$

$\therefore T = 2SR$

$Y = \frac{Fl}{A\Delta l} \Rightarrow \frac{YA \times \Delta R}{R} = 2SR$

$\Delta R = \frac{2SR^2}{YA}$

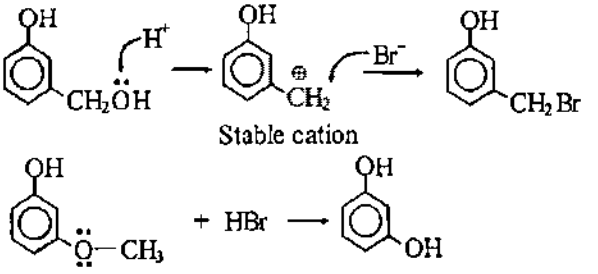
88. $e = \frac{LdI}{dt} \Rightarrow e \propto L$

Power $= e_1 I_1 = e_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{L_2}{L_1}$

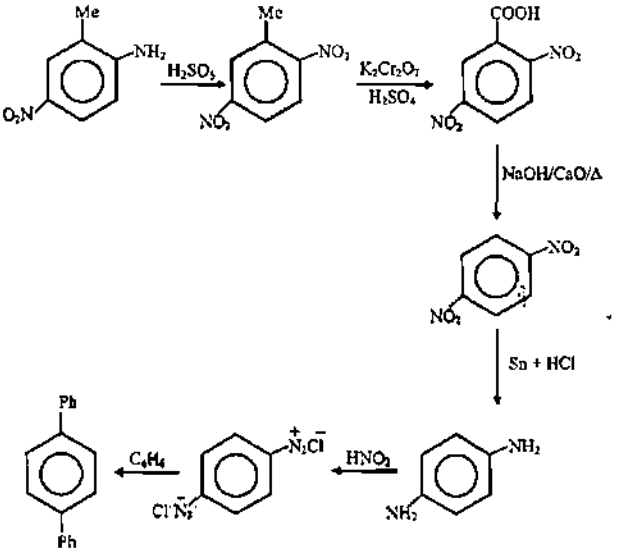
90 $e = Bv(2R)$ and Q is at high potential

CHEMISTRY

91.



92.



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94. moles of $As^+ = M \times V$
$= 1 \times 0.25 = 0.25$

$\text{moles} = \frac{1}{nF} Q$

$Q = 0.25 \times 1 \times F = \frac{1}{4} \times 96500$
$= 24125\ C$

96. $AgNO_3 \xrightarrow{\Delta} Ag + NO_2 + O_2$
$NO_2 \longrightarrow HNO_3 + HNO_2$
$Ag + HNO_3 \longrightarrow AgNO_3 + NO + H_2O$
$AgNO_3 + Na_2S_2O_{3(excess)} \rightarrow Na_3[Ag(S_2O_3)_2] + NaNO_3$

97. $MnO_2 + K_2CO_3 + air \xrightarrow{\Delta} K_2MnO_4$
$K_2MnO_4 + Cl_2 \longrightarrow KMnO_4$

98. $CH_3COCH_3 \rightarrow C_2H_4 + H_2 + CO$

0.4–x	x	x	x
0.4 – x	x	x	x

$0.4 + 2x = 0.5$
$x = 0.05$
$K = \frac{2.303}{10} \log \frac{0.4}{0.35} = 0.0133\ min^{-1}$

100. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
x
$CO + \frac{1}{2}O_2 \rightarrow CO_2$
$(10.5 - x)$
$2x + \frac{1}{2}(10.5 - x) = 9$
$\frac{3}{2}x + 5.25 = 9$
$x = 2.5$ mol.

MATHEMATICS

101. $z - 2 = re^{i\alpha}$
$\therefore (x - 2) + iy = r(\cos\alpha + i \sin\alpha)$
$r^2 = (x-2)^2 + y^2$ and $\tan\alpha = \frac{y}{x-2}$
The expression $\frac{z-2}{e^{i\theta}} + \frac{e^{i\theta}}{z-2} = re^{i(\alpha-\theta)} + \frac{1}{r}e^{-i(\alpha-\theta)}$
which is given as real
$\therefore r \sin(\alpha - \theta) - \frac{1}{r}\sin(\alpha - \theta) = 0$
$\Rightarrow r - \frac{1}{r} = 0$
$\Rightarrow r^2 = 1$
$\Rightarrow (x-2)^2 + y^2 = 1$

102. $f(x) = \pi - (\tan^{-1}x + \cot^{-1}x) + \sec^{-1}x$
$= \frac{\pi}{2} + \sec^{-1}x$
Range of $f(x) = \left[\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right]$

103. $\text{Limit} = e^{\lim_{n\to\infty} n\left(\left(\frac{n}{n+1}\right)^{\alpha} + \sin\frac{1}{n} - 1\right)}$
$= e^{\lim_{n\to\infty}\left(\frac{\sin(1/n)}{1/n} \cdot \frac{(1+(1/n))^{\alpha}-1}{1/n} \cdot \frac{1}{(1+(1/n))^{\alpha}}\right)}$
$= e^{1-\alpha}$

104. $4 \tan\frac{B}{2}\tan\frac{C}{2} = 1$
$\frac{s-a}{s} = \frac{1}{4}$
Solve it $b + c = 10$
Focus $F_1(2, 0)$ and $F_2(8, 0)$ and $2a = 10$
$\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$

105. $f(x) = \lim_{n\to\infty} \frac{x^{1/n} - x^{1/(n+1)}}{\frac{1}{n^2}}$
$= \lim_{n\to\infty} \frac{x^{1/n}\left(1 - x^{-1/n(n+1)}\right)}{\left[-\frac{1}{n(n+1)}\right]} \cdot \left[-\frac{n^2}{n(n+1)}\right]$
$= (-\log_e(x))(-1) = +\log_e x$
$\int x.f(x)\,dx = \int x.(\log_e x)\,dx$
$= \left[\log_e x.\frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2}\,dx\right.$
$= \log_e x.\frac{x^2}{2} - \frac{x^2}{4} + C$

106. Since $\hat{i} + 2\hat{j} + 2\hat{k}$ is rotated so as to cross y-axis, the vector in new position would be parallel to the plane $2x - z = 0$.

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107. Since a, b, c $\in$ Q, so both the roots should be rational or both irrational. If $\frac{p}{q}$ is a root of it, p, q $\in$ I, as a, b, c $\in$ odd numbers
$\Rightarrow$ $ap^2 + bpq + cq^2 = 0$ is never possible.

108. $1 + \cos2\theta + \cos4\theta + \cos6\theta + \cos10\theta$
$= Re[e^{i0} + e^{i2\theta} + e^{i4\theta} + ... + e^{i10\theta}]$
$= Re\frac{[e^{i12\theta} - 1]}{[e^{i2\theta} - 1]}$, $e^{i\theta} = (\cos\theta + i\sin\theta)$
$= \frac{\cos2\theta\cos12\theta - \cos12\theta - \cos2\theta + 1 + \sin2\theta\sin12\theta}{2(1 - \cos2\theta)}$
$= \frac{1 + \cos10\theta - 2\cos7\theta\cos5\theta}{4\sin^2\theta}$
$= \frac{\sin6\theta\cos5\theta}{\sin\theta}$
on comparing we get n = 6, m = 5
(m + n) = 11.

109. Perimeter = b + a + 10 = x + y cosz
Using sine rule
$\frac{10\times2}{\sqrt{3}} = \frac{a}{\sin50^\circ} = \frac{b}{\sin70^\circ}$
$a = \frac{20}{\sqrt{3}}\sin50^\circ$, $b = \frac{20}{\sqrt{3}}\sin70^\circ$
$a + b = \frac{20}{\sqrt{3}}(\sin50^\circ + \sin70^\circ)$
$= \frac{20}{\sqrt{3}} \times 2\cos10^\circ\sin60^\circ$
$= 2\cos10^\circ$
Perimeter $10 + a + b = 10 + 20\cos10^\circ = x + y\cos z$
On comparing we get
x + y + z = 10 + 20 + 10 = 40

110. 7! onwards are divisible by 35. Hence remainder of (1! + 2! + 3! + 4! + 5! + 6!), when divided by 35 is $\frac{873}{35} = 33$

BIOLOGY

111. Oenothera possess diploid endosperm

112. Vasa differentia transports sperms its absence will cause lack of sperms in semen.

113. Round and wrinkled seed would be obtained in 1 : 1 ratio.

114. DNA undergoes various level of packaging.

115. Down syndrome is due to trisomy of 21st chromosome & turners syndrome due to monosomy of sex chromosome (XO) condition.

117. Similarity in organism in spite of different genotype shows convergent evolution

118. These are basically obtained from brown and red algae and termed as phyco colloids

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KVPY (Stream SX) Model Test Papers
Practice Test -2

PART- I
One - Mark Questions

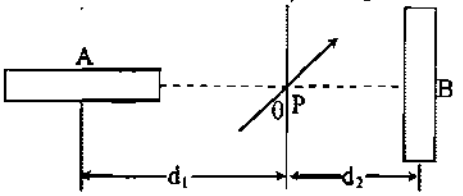
Questions 1 to 80 are Single correct answer type questions. Each correct answer carries 1 mark. 0.25 marks will be deducted for each wrong answer.

PHYSICS

Q.1 Two bar magnets of the same mass, same length and breadth but having magnetic moments M and 2M are joined together pole for pole and suspended by a string. The time period of assembly in a magnetic field of strength H is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time period of oscillation is :

(1) $\sqrt{3}$ sec (2) $3\sqrt{3}$ sec
(3) 3 sec (4) 6 sec

Q.2 Two magnets A and B are identical and these are arranged as shown. Their lengths are negligible in comparison to separation between them. A magnetic needle is placed between the magnets at point P and it gets deflected through an angle θ. The ratio of distances d_1 and d_2 will be

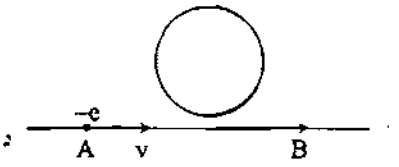


(1) $(2 \cot \theta)^{1/3}$ (2) $(2 \tan \theta)^{1/3}$
(3) $(2 \cot \theta)$ (4) $(2 \tan \theta)^{-1/3}$

Q.3 The tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produces deflections of 60° and 45° respectively. The ratio of the number of turns in the coil is :

(1) (4/3) (2) $(\sqrt{3}+1)/1$
(3) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ (4) $\sqrt{3}/1$

Q.4 An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the current induced if any in the loop ?

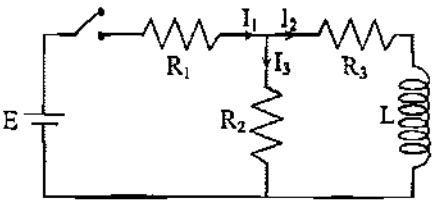


(1) No current will be induced
(2) The current will be clockwise
(3) The current will be anticlockwise
(4) The current will change direction as the electron passes by

Q.5 A solenoid has 2000 turns wound over a length of 0.3m. The area of its cross-section is 1.2×10^{-3} m^2. Around its central portion a coil of 300 turns is wound. If an initial current of 2 amp in the solenoid is reversed in 0.25 sec., the emf induced in the coil is equal to

(1) 6×10^{-4} V (2) 48 mV
(3) 6×10^{-2} V (4) 48 kV

Q.6 In the circuit shown in the following figure E = 10V, $R_1 = 2$ ohm, $R_2 = 3$ ohm and $R_3 = 6$ ohm and L = 5 henry. The current I_1 just after pressing the switch S is



(1) (10/4) amp (2) (10/5) amp
(3) (10/12) amp (4) (10/6) amp

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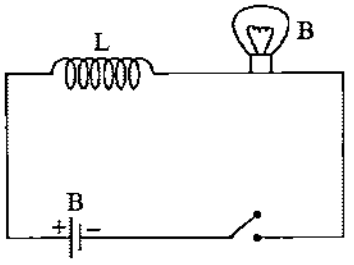
Q.7 In the Q.6, the current I_1 long after pressing the switch is

(1) (10/4) amp (2) (10/5) amp
(3) (10/12) amp (4) (10/6) amp

Q.8 When a ferromagnetic material is taken through a hysteresis cycle, the magnetic susceptibility cannot be :

(1) Zero (2) Infinity
(3) Negative (4) Constant

Q.9 In the circuit of figure the bulb will become suddenly bright if :

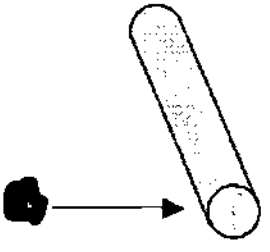


(1) Contact is made or broken
(2) Contact is made
(3) Contact is broken
(4) Would not become bright at all

Q.10 Which types of wire is suitable for flowing A.C. :

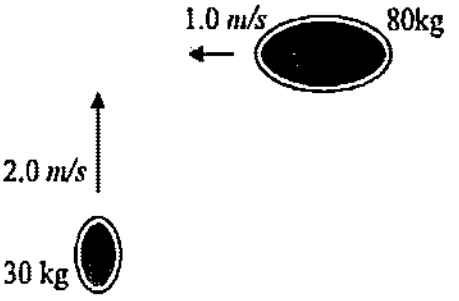
(1) Single strained wire
(2) Multi strained wire
(3) both
(4) None of these

Q.11 An asteroid traveling through space collides with one end of a long, cylindrical satellite as shown and sticks to the satellite. Which of the following is true of the isolated asteroid-satellite system in this collision?



(1) Kinetic energy K is conserved
(2) Total Energy E is conserved, but angular momentum L is not conserved
(3) Angular momentum L is conserved, but linear momentum P is not conserved
(4) Angular momentum L is conserved, and total energy E is conserved

Q.12 Two ice skaters, of mass 30 kg and 80 kg, are skating across the surface of a frozen lake on a collision course, with respective velocities of 2.0 m/s in a general north direction, and 1.0 m/s generally west, as shown above. After they collide, the pair of skaters move off in a direction north of west with a momentum of approximately 100 kg m/s. How much kinetic energy was lost in the collision ?



(1) 0 J (2) 110 J (3) 55 J (4) 70 J

Q.13 An ideal monoatomic gas is heated so that it expands at constant pressure. What percentage of the heat supplied to the gas is used to increase the internal energy of the gas ?

(1) 100% (2) 0%
(3) 60% (4) 50%

Q.14 When a tuning fork of frequency f is excited and held near one end of a straight pipe of length L open at both ends, the air column in the pipe vibrates in its fundamental mode and is in resonance with the tuning fork. The pipe is now kept vertical in a jar containing water so that half the length of the pipe is inside water. What should be the frequency of the tuning fork to be used to make the air column vibrate in its fundamental mode in resonance with the tuning fork now ?

(1) f (2) 2f
(3) f/2 (4) f/4

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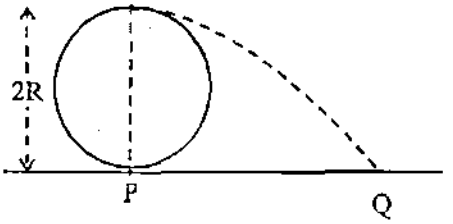
Q.15 A wheel of radius R is rolling along a horizontal surface with a speed u. A pebble trapped on the wheel gets separated from the highest point of the wheel arrives at position P (figure). The horizontal range PQ of the pebble is :



(1) $u\sqrt{R/g}$ (2) $2u\sqrt{R/g}$

(3) $4u\sqrt{R/g}$) (4) $u^2\sqrt{R/g}$

Q.16 A particle starts from the origin at time t = 0 with velocity $2\hat{j}$ and moves in the x-y plane with a constant acceleration of $2\hat{i} + 4\hat{j}$ where $\hat{i}$ and $\hat{j}$ are unit vectors along the x-direction and y direction respectively. What will be the x-coordinate of the particle when its y-coordinate becomes 12 m –

(1) 4 m (2) 6 m (3) 6.8 m (4) 8 m

Q.17 Choose the incorrect statement -

(1) A dimensionally correct equation may be correct

(2) A dimensionally correct equation may be incorrect

(3) A dimensionally incorrect equation may be correct

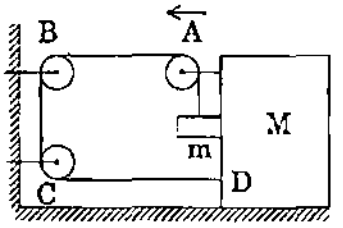
(4) A dimensionally incorrect equation may be incorrect

Q.18 Two solid bodies rotate about stationary mutually perpendicular intersecting axes with constant angular velocities ω_1 and ω_2. What is the magnitude of angular velocity of one with respect to the other ?

(1) $\omega_1 - \omega_2$ (2) $\sqrt{\omega_1^2 + \omega_2^2}$

(3) $\sqrt{\omega_1^2 - \omega_2^2}$ (4) $\omega_1 + \omega_2$

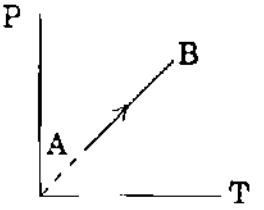
Q.19 In figure shown, find the magnitude of acceleration of m, given that string is inextensible and mass less and the acceleration of M is 2 m/s^2 towards left –



(1) $2\sqrt{3}$ m/s^2 (2) $3\sqrt{2}$ m/s^2

(3) $4\sqrt{2}$ m/s^2 (4) $2\sqrt{5}$ m/s^2

Q.20 During the process A-B for an ideal gas –



(1) Work done on the gas is zero

(2) Density of the gas is constant

(3) Slope of line AB from the T-axis is directly proportional to the number of moles of the gas

(4) All of the above

CHEMISTRY

Q.21 Among the oxides of nitrogen N_2O, NO and NO_2, molecules with unpaired electrons are :

(1) N_2O and NO

(2) NO and NO_2

(3) N_2O and NO_2

(4) NO_2 and its dimer

Q.22 Which of the following compounds have the same no. of lone pairs with their central atom ?

(I) XeF_5^- (II) BrF_3 (III) XeF_2 (IV) H_3S^+

(1) IV and I (2) I and III

(3) I and II (4) II and IV

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Q.23 Coordination compounds have great importance in biological systems. In this context which of the following statements is incorrect ?
(1) Carboxypeptidase-A is an enzyme and contains zinc
(2) Haemoglobin is the red pigment of blood and contains iron
(3) Cyanocobalamin is B^{12} and contains cobalt
(4) Chlorophylls are green pigments in plants and contains calcium

Q.24 An aqueous solution of titanium chloride, when subjected to magnetic measurement, measured zero magnetic moment. Assuming the octahedral complex in aqueous solution, the formulae of the complex is :
(1) $[Ti(H_2O)_6]Cl_2$
(2) $[Ti(H_2O)_6]Cl_4$
(3) $[TiCl_3(H_2O)_3]$
(4) $[TiCl_2(H_2O)_4]$

Q.25 Give the correct order of initials T or F for following statements. Use T if statement is true and F if it is false.
(i) Every mineral is an ore but every ore is not a mineral
(ii) Slag is product formed during extraction of metal by combinations of flux and impurities
(iii) Highly pure metals can be obtained by zone refining
(iv) Carnallite is an ore of magnesium and sodium
(1) TTTF (2) FTTF
(3) FTTT (4) TFTF

Q.26 Among the following statements, the incorrect statements is :
(1) calamine and cerrusite are carbonate ores
(2) rutile and cuprite are oxide ores
(3) zinc blends and pyrites are sulphide ores
(4) malachite and azurite are sulphate ores of Cu

Q.27 PbI_4 does not exist because :
(1) iodine is not a reactive
(2) Pb (IV) is oxidizing and I^- is strong reducing agent
(3) Pb (IV) is less stable than Pb (II)
(4) Pb^{4+} is not easily formed

Q.28 1 gm sample of H_2O_2 solution containing x % H_2O_2 by weight requires x ml of $KMnO_4$ solution for complete oxidation under acidic conditions. Normality of $KMnO_4$ solution is-
(1) 1.21 N (2) 0.122 N
(3) 0.588 N (4) 0.758 N

Q.29 Calculate the energy required to excite one litre of hydrogen gas at 1 atm and 298 K to the first excited state of atomic hydrogen. The energy for the dissociation of H–H bond is 436 kJ/mol.
(1) 112.5 kJ (2) 98.17 kJ
(3) 74.32 kJ (4) 55.21 kJ

Q.30 The excluded volume of a gas will be larger, if $\frac{T_C}{P_C}$ is-
(1) small (2) large
(3) equal to 1 (4) less than unity

Q.31 One mole of an ideal monoatomic gas at 27°C is subjected to a reversible isoentropic compression and until final temperature reached to 327°C. If the initial pressure was 1.0 atm then find the value of $\ln P_2$:
(Given : $\ln 2 = 0.7$)
(1) 1.75 atm (2) 0.176 atm
(3) 1.0395 atm (4) 2.0 atm

Q.32 The energy of a photon of wavelength $\lambda = 1$ meter is (Planck's constant = 6.625×10^{-34} Js, speed of light = 3×10^8 m/s)
(1) 1.988×10^{-25} J (2) 1.988×10^{-28} J
(3) 1.988×10^{-30} J (4) 1.988×10^{-27} J

Q.33 In metallic solids, the number of atoms for the face-centered and the body-centered cubic unit cell, are, respectively-
(1) 2, 4 (2) 2, 2 (3) 4, 2 (4) 4, 4

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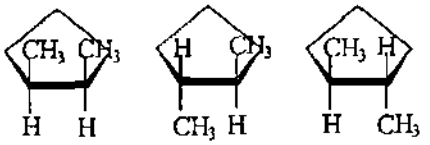
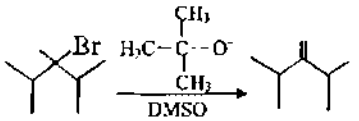
Q.34 Among the structures given, select the enantiomers

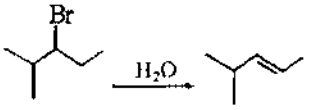


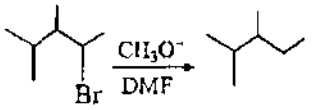
(I) (II) (III)

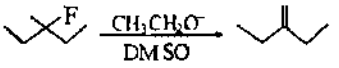
(1) I and II
(2) I and III
(3) II and III
(4) I, II and III

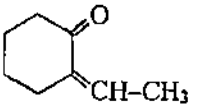
Q.35 Which of the following reactions does not represent the major product given ?

(1) 

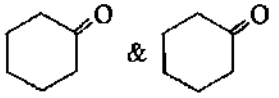
(2) 

(3) 

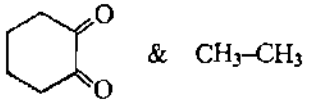
(4) 

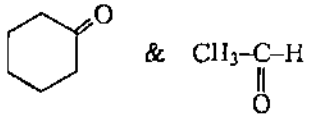
Q.36 If A + B $\xrightarrow{OH^{\ominus}}$ 

what are A & B

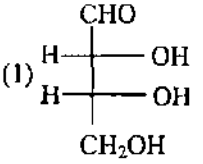
(1) 

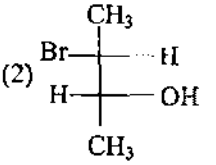
(2) $CH_3-\underset{\underset{O}{\|}}{C}-H$ & $CH_3-\underset{\underset{O}{\|}}{C}-H$

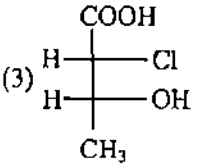
(3)  & CH_3-CH_3

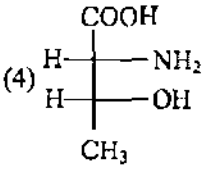
(4)  & $CH_3-\underset{\underset{O}{\|}}{C}-H$

Q.37 Which among the following is a *threo* isomer ?

(1) 

(2) 

(3) 

(4) 

Q.38 Selective oxidation of $R_1C \equiv C(CH_2)_nCH = CHR_2$ with CrO_3 in acetic acid gives-

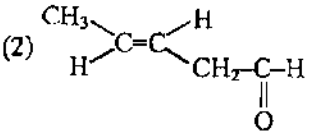
(1) $R_1C \equiv C(CH_2)_n\,CO_2H + R_2CO$
(2) $R_1CO_2H + HO_2C\,(CH_2)_nCH = CHR_2$
(3) $R_1CO-CO(CH_2)_nCH = CHR_2$
(4) $R_1C \equiv C(CH_2)_nCO - CHR_2$

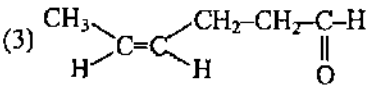
Q.39

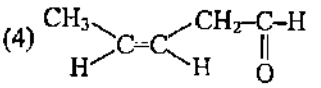
$$CH_3-CH\equiv C-CH_2-\underset{\underset{O}{\|}}{C}-H \xrightarrow{(i)\ Aq_2O\ /\ OH^{\ominus}} \xrightarrow{(ii)\ SOCl_2} \xrightarrow[S]{(iii)\ H_2-Pd-BaSO_4}$$

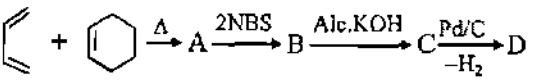
Final product is :

(1) $CH_3-CH=CH-CH_2-\underset{\underset{O}{\|}}{C}-H$

(2) 

(3) 

(4) 

Q.40 

The final product D in the above sequence of reactions is-

(1) Benzene (2) Anthracene
(3) Decalin (4) Naphthalene

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MATHEMATICS

Q.41 The value of $\int_{-6}^{6} \max(|2-|x||, 4-|x|, 3)dx$ is

(1) 40 (2) 50 (3) 60 (4) 30

Q.42 If $x = \frac{4\lambda}{1+\lambda^2}$ and $y = \frac{2-2\lambda^2}{1+\lambda^2}$ where λ is a real parameter and $x^2 - xy + y^2$ lies between [a, b] then (a + b) is

(1) 8 (2) 10 (3) 13 (4) 25

Q.43 Before a race, the chances of three runners A, B and C were estimated to be proportional to 5 : 3 : 2, but during the race, A meets with an accident which reduces his chance to 1/3. The respective chances of B and C now winning are

(1) $\frac{3}{5}, \frac{2}{5}$ (2) $\frac{2}{5}, \frac{4}{15}$ (3) $\frac{7}{15}, \frac{2}{15}$ (4) $\frac{3}{10}, \frac{2}{10}$

Q.44 Solution of the differential equation $\frac{dy}{dx} = \frac{y(x - y\ell ny)}{x(x\ell nx - y)}$ is

(1) $\frac{x\ell nx + y\ell ny}{xy} = C$ (2) $\frac{x\ell nx - y\ell ny}{xy} = C$

(3) $\frac{\ell nx}{x} + \frac{\ell ny}{y} \neq C$ (4) $\frac{\ell nx}{x} - \frac{\ell ny}{y} = C$

Q.45 If

$$f(n)\lim_{x\to 0}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\ldots\left(1+\sin\frac{x}{2^n}\right)\right)^{\frac{1}{x}},$$

then $\lim_{n\to\infty} f(n) =$

(1) 1 (2) e (3) 0 (4) ∞

Q.46 If parabola of latusrectum λ, touches a fixed equal parabola, the axes of the two curves being parallel, then the locus of the vertex of the moving curve is -

(1) a parabola of latusrectum 2λ
(2) a parabola of latusrectum λ
(3) an ellipse whose major axis is 2λ
(4) an ellipse whose minor axis is 2λ

Q.47 If S be the sum of coefficients in the expansion of $(px + qy - rz)^n$ (where p, q, r > 0), then the value of $\lim_{n\to\infty} \frac{S}{(S^{1/n}+1)^n}$, is :

(1) $\frac{pq}{r}$ (2) $e^{\frac{pq}{r}}$

(3) 0 (4) 1

Q.48 If A, B and C are exhaustive events satisfying $P((A\cup B)\cap \bar{C}) = \frac{1}{5}$, $P(B\cap C) - P(A\cap B\cap C) = \frac{1}{15}$ and $P(A \cap C) = \frac{1}{10}$, then $P(C \cap (A \cup B)')$ is equal to-

(1) $\frac{17}{30}$ (2) $\frac{18}{30}$ (3) $\frac{19}{30}$ (4) $\frac{20}{30}$

Q.49 If A(1 + i), B(3 + 4i) and C(z) are the vertices of a ΔABC in which $\angle BAC = \frac{\pi}{3}$ and AC = 2AB. Then z is-

(1) $3 + 4i + i\sqrt{3}(2 + 3i)$

(2) $(3 + 4i) + \frac{1}{\sqrt{3}} i(2 + 3i)$

(3) $(2 + 3i) + i\sqrt{3}(3 + 4i)$

(4) $(2 + 3i) + \frac{i}{\sqrt{3}}(3 + 4i)$

Q.50 Given that

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\log(n^2+r^2)-2\log n}{n} = \log 2 + \frac{\pi}{2} - 2,$$

then

$$\lim_{n\to\infty}\frac{1}{n^{2m}}\left[(n^2+1^2)^m(n^2+2^2)^m\ldots(2n^2)^m\right]^{1/n}$$

is equal to –

(1) $2^m e^{m(\pi/2-2)}$ (2) $2^m e^{m(2-\pi/2)}$

(3) $e^{m(\pi/2-2)}$ (4) $e^{2m(\pi/2-2)}$

Q.51 $\int \frac{(x^3-1)dx}{x^4\sqrt{6x^6-4x^3+2}}$ equals:

(1) $\frac{\sqrt{6x^6-4x^3+2}}{12x^3} + C$

(2) $\frac{\sqrt{6x^6-4x^3+2}}{12x^2} + C$

(3) $\frac{\sqrt{6x^6-4x^3+2}}{6x^3} + C$

(4) $\frac{1}{12}\ell n(6x^6-4x^3+2) + C$

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Q.52 From an external point P, pair of tangent lines are drawn to the parabola $y^2 = 4x$. If θ_1 and θ_2 be the inclinations of these tangents with the axis of x such that $\theta_1 + \theta_2 = \frac{\pi}{4}$, then the locus of P is

(1) $x - y + 1 = 0$ (2) $x + y - 1 = 0$
(3) $x - y - 1 = 0$ (4) $x + y + 1 = 0$

Q.53 The three planes $4y + 6z = 5$, $2x + 3y + 5z = 5$ and $6x + 5y + 9z = 10$
(1) meet in a point
(2) have a line in common
(3) form a triangular prism
(4) none of these

Q.54 Consider the family of lines
$(x + y - 1) + \lambda(2x + 3y - 5) = 0$ and
$(3x + 2y - 4) + \mu(x + 2y - 6) = 0$, the equation of a straight line that belongs to both the families is
(1) $x - 2y - 8 = 0$ (2) $x - 2y + 8 = 0$
(3) $2x + y - 8 = 0$ (4) $2x - y - 8 = 0$

Q.55 Let A, B, C be three events in a probability space. Suppose that P(A) = 0.5, P(B) = 0.3, P(C) = 0.2, $P(A \cap B) = 0.15$, $P(A \cap C) = 0.1$ and $P(B \cap C) = 0.06$, the greatest possible value of $P(A^c \cap B^c \cap C^c)$ is
[Note : A^c denotes compliment of event A]
(1) 0.31 (2) 0.25
(3) 0 (4) 0.26

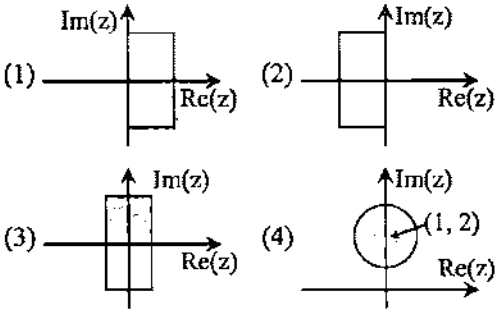
Q.56 If α and β are roots of the equation $ax^2 + bx + c = 0$ then roots of the equation $a(2x + 1)^2 - b(2x + 1)(3 - x) + c(3 - x)^2 = 0$ are-

(1) $\frac{2\alpha+1}{\alpha-3}, \frac{2\beta+1}{\beta-3}$ (2) $\frac{3\alpha+1}{\alpha-2}, \frac{3\beta+1}{\beta-2}$

(3) $\frac{2\alpha-1}{\alpha-2}, \frac{2\beta+1}{\beta-2}$ (4) None of these

Q.57 The number of integral value(s) of 'α' for which expression $\log(\log_{1/3}(\log_7(\sin x + \alpha)))$ is meaningful $x \in R$, is
(1) 1 (2) 2 (3) 3 (4) 5

Q.58 If the shaded portion represents the set of complex numbers then which of the following set of complex numbers satisfy the inequality $\tan^{-1}(\log_3|2z - 1|) > \tan^{-1}(\log_3|2z + 1|)$



Q.59 If $0 \le \arg(z) \le \frac{\pi}{4}$, then the least value of $\sqrt{2}|2z - 4i|$ is
(1) 6 (2) 1 (3) 4 (4) 2

Q.60 (a, b) and (c, d) are solutions of simultaneous equation $5(\log_y x + \log_x y) = 26$, $xy = 64$ then $a + b + c + d =$
(1) 51 (2) 17 (3) 68 (4) 34

BIOLOGY

Q.61 Apogamy is :
(1) reproduction of virus
(2) development of bacteria
(3) failure of fusion of gametes
(4) loss of function of reproduction

Q.62 In maize, aleuron layer develops in outermost region of :
(1) Pericarp (2) Epicarp
(3) Testa (4) Endosperm

Q.63 Major part of semen is secreted by :
(1) Seminal vesicle
(2) Prostate gland
(3) Cowper's gland
(4) Bartholin's gland

Q.64 Which one of the following is not a phase of the menstrual cycle ?
(1) Lutcal phase (2) Estrous phase
(3) Follicular phase (4) Menstrual phase

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Q.65 Copper-T :
(1) acts as a barrier
(2) prevents cleavage
(3) suppresses fertilization
(4) prevents implantation of blastocysts

Q.66 When a tall pea plant (TT) is crossed with a dwarf plant (tt), what will be the F_2 generation?
(1) All tall plants
(2) All dwarf plants
(3) Both tall and dwarf plants in 3 : 1 ratio
(4) Both tall and dwarf plants in 1 : 1 ratio

Q.67 Test cross involves :
(1) crossing the F_1 hybrid with a double recessive genotype
(2) crossing between two genotypes with dominant trait
(3) crossing between two genotypes with recessive trait
(4) crossing between two F_1 hybrids

Q.68 Alleles of different genes found on same chromosome may be separated by :
(1) epistasis
(2) pleiotropy
(3) crossing over
(4) continuous variation

Q.69 Trisomy of chromosome number 21 in man causes :
(1) Thalassemia (2) Down syndrome
(3) Turner syndrome (4) Sickle-cell anaemia

Q.70 Width of DNA molecule is :
(1) 15 Å (2) 34 Å
(3) 25 Å (4) 20 Å

Q.71 Which is incorrect?
(1) Wings of insects and bats are analogous
(2) Wings of bats and birds are homologous
(3) Wings of insects and birds are analogous
(4) Wings of insects and birds are homologous

Q.72 Which of the following is not a vestigial organ in man?
(1) Nail
(2) Tail vertebra
(3) Wisdom tooth
(4) Vermiform appendix

Q.73 Diphtheria is caused by :
(1) Bordetella (2) Clostridium
(3) Treponema (4) Corynebacteria

Q.74 The most important long acting antibody representing about 80% of the antibody that is able to pass across the placenta is :
(1) IgD (2) IgG
(3) IgM (4) IgA

Q.75 Central Rice Research Institute is at :
(1) New Delhi (2) Kolkata
(3) Cuttack (4) Mumbai

Q.76 The closest pets of human beings are :
(1) cat and dog (2) dog and sheep
(3) cat and cow (4) cattle and buffaloes

Q.77 Introduction of foreign genes for improving genotype is :
(1) tissue culture (2) vernalization
(3) biotechnology (4) genetic engineering

Q.78 Which of the following pairs is correctly matched?
(1) Central dogma - Codon
(2) Okazaki fragments - Splicing
(3) RNA polymerase - RNA primer
(4) Restriction enzymes - Genetic engineering

Q.79 *Escherichia coli* is used as an indicator organism to determine pollution of water with :
(1) heavy metals
(2) faecal matter
(3) industrial effluents
(4) pollen of aquatic plants

Q.80 Common indicator organism of water pollution is :
(1) Escherichia coli
(2) Entamoeba histolytica
(3) Lemna pancicostata
(4) Eicchornia crassipes

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PART- II

Questions given below are single correct type Questions. Each correct answer carries +2. 0.5 marks will be deducted for each wrong answers.

PHYSICS

Q.81 Which statement is correct–

(A) Electric field of nonconducting charged sheet = $\sigma/2\epsilon_0$

(B) Electric field of conducting charged sheet = σ/ϵ_0

(C) Electric field inside charged conducting sphere is zero

(1) A, B, C (2) B, C, D

(3) A, B, C, D (4) A, B, D

Q.82 Match the facts given in Column-I with the system given in column-II

	Column-I		Column-II
(A)	Electric field strength is zero in the volume	(P)	A non conducting solid sphere charged uniformly
(B)	Electric field strength is non-zero in the volume but zero at the centre	(Q)	A conducting spherical shell charged uniformly
(C)	Electric field strength is maximum on the surface	(R)	A non conducting hollow sphere charged uniformly
(D)	Electric field strength outside the system varies inversely as the square of the distance from the centre	(S)	A conducting solid sphere charged uniformly

(1) A→Q,R; B→P; C→Q,R,S; D→P,Q,R,S

(2) A→Q,R,S; B→P; C→P,Q,R,S; D→P,Q,R,S

(3) A→Q,R,S; B→P,Q; C→P,Q,R,S; D→P,Q,R

(4) A→Q,S; B→P; C→P,Q,R,S; D→Q,R,S

Q.83 Match Column-I with Column-II

	Column-I		Column-II
(A)	Force on an electron in an atom	(P)	Gravitational force
(B)	Force between a proton and a neutron inside nucleus	(Q)	Strong force
(C)	Force between a proton and proton inside nucleus	(R)	Coulomb force
(D)	Conservative force	(S)	Electric force

(1) A→P,R,S; B→P,Q; C→P,Q,R,S; D→P,R,S

(2) A→P,R,S; B→P; C→P,Q,R; D→P,R,S

(3) A→R,S; B→P,Q; C→P,Q,R,S; D→R,S

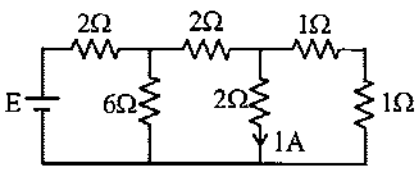
(4) A→P,R; B→P,Q; C→ Q,R,S; D→P,R,S

Q.84 Which transistor is best from following :

(1) NPN (2) PNP

(3) Both (4) None of these

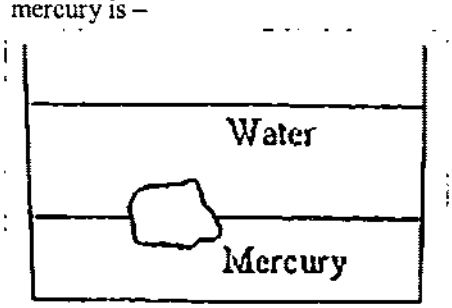
Q.85 What is value of E in given circuit :



(1) 6V (2) 12V

(3) 3V (4) 2V

Q.86 A piece of granite floats at the interface of mercury and water contained in a beaker (Fig.). If the densities of granite, water and mercury are ρ, ρ_1 and ρ_2 respectively, the ratio of the volume of granite in water to the volume in mercury is –



(1) $(\rho_2 - \rho)/(\rho - \rho_1)$ (2) $(\rho_2 + \rho)/(\rho + \rho_1)$

(3) $\rho_1\rho_2/\rho$ (4) ρ_1/ρ_2

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Q.87 A simple pendulum is constructed by attaching a mass m to a thin rod of length λ. The pendulum is pulled back to some angle $\theta > 30°$ from the vertical and released. Which of the following techniques could be used to change the frequency f of this pendulum ?

I. Changing the mass m on the end of the pendulum

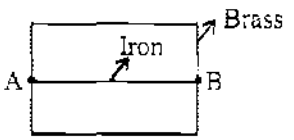
II. Changing the length λ of the pendulum

III. Changing the angle θ from which the pendulum is released

(1) I only (2) I and II only

(3) II only (4) II and III only

Q.88 The diagram shows an iron wire AB mounted in a brass frame and attached to the frame at both ends A and B. The length of AB at 0°C is 300 cm and the diameter of the wire is 0.6 mm. What extra tension will be set up in the stretched wire when the temperature of the system is raised to 40°C in powers of 10^6 dyne? Given $\alpha_{brass} = 8 \times 10^{-6}$ /°C, $\alpha_{iron} = 12 \times 10^{-6}$/°C and $Y_{iron} = 21 \times 10^{11}$ dyne/cm^2



(1) 0.95 (2) 0.165

(3) 0.142 (4) 0.125

Q.89 Two identical trucks each of mass M (excluding sacks of rice) move on national highways with speeds v_1 and v_2 towards each other. When they meet each other, a sack of rice of mass m is thrown from one truck to the other and an identical sack of rice is thrown from the second to the first. Calculate their velocities v_1' and v_2' after the exchange of sacks, given m = 50 kg, M = 200 kg,

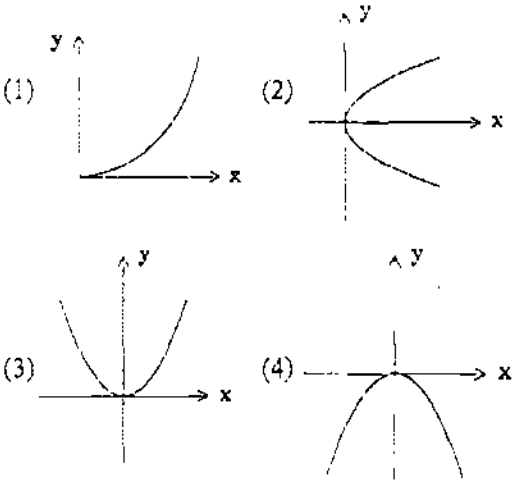
$|\vec{v}_1| = 50$ m/s and $|\vec{v}_2| = 200$ m/s.

Friction of the road may be neglected –

(1) 0, – 150 m/s (2) 150 m/s, – 50 m/s

(3) 100 m/s, 150 m/s (4) 100 m/s, 50 m/s

Q.90 A particle moves in the x-y plane with velocity $\vec{v} = a\hat{i} - bx\hat{j}$, where a and b are constants. Initially, the particle was at the origin. The trajectory of the particle is



CHEMISTRY

Q.91 Predict product (s) in the following reaction,

$P_4 - OH^- \longrightarrow$ Product (s)

(1) PH_3 (2) PO_4^{2-}

(3) $H_2PO_2^-$ (4) Both (1) & (3)

Q.92 Which of the following ion will show paramagnetism ?

(1) V^{5+} (2) V^{4+} (3) O^{2-} (4) VO_3^-

Q.93 The product of I^- with MnO^-_4 in alkaline medium is :

(1) I_2 (2) IO^-_3 (3) IO^- (4) IO_4^-

Q.94 For a complex reaction A $\xrightarrow{k}$ products

$E_{a_1} = 180$ kJ/mol ; $E_{a_2} = 80$ kJmol;

$E_{a_3} = 50$ kJ/mol

Overall rate constant k is related to individual rate constant by the equation $k = \left(\frac{k_1 . k_2}{k_3}\right)^{2/3}$.

Activation energy (kJ/mol) for the overall reaction is-

(1) 100 (2) 43.44 (3) 150 (4) 140

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Q.95 If the equilibrium constant for the reaction $H^+(aq) + OH^-(aq) \rightleftharpoons H_2O(l)$ is 10^{13} at certain temperature then what is the E° for the reaction, $2H_2O(l) + 2e^- \rightleftharpoons H_2(g) + 2OH^-(aq)$.

Given : $\frac{2.303RT}{F} = 0.066$

(1) 1.230 V (2) –0.858 V
(3) –0.80 V (4) –0.8274 V

Q.96 Diborane is formed the elements as shown in equation (1)

$2B(s) + 3H_2(g) \rightarrow B_2H_6(g)$ ....(1)

Given that

$H_2O(l) \rightarrow H_2O(g)$ $\Delta H_1° = 44$ kJ

$2B(s) + 3/2O_2(g) \rightarrow B_2O_3(s)$ $\Delta H_2° = -1273$ kJ

$B_2H_6(g) + 3O_2(g) \rightarrow B_2O_3(s) + 3H_2O(g)$ $\Delta H_3° = -2035$ kJ

$H_2(g) + 1/2\ O_2(g) \rightarrow H_2O(l)$ $\Delta H_4° = -286$ kJ

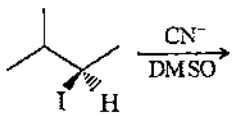
the $\Delta H°$ for the reaction (1) is-

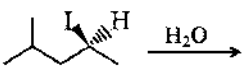
(1) 36 kJ (2) 509 kJ
(3) 520 kJ (4) –3550 kJ

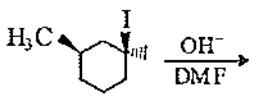
Q.97 A solution containing 8.0 g of nicotine in 92 g of water freezes 0.925 degrees below the normal freezing point of water. If the molal freezing point depression constant $K_2 = 1.85°C\ mol^{-1}$ then the molar mass of nicotine is-

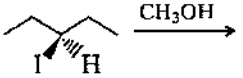
(1) 16 (2) 80
(3) 320 (4) 160

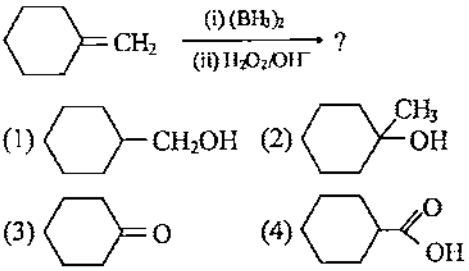
Q.98 Which reaction results in the formation of a pair of enantiomers ?

(1) 

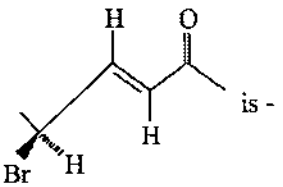
(2) 

(3) 

(4) 

Q.99 

Q.100 The IUPAC name (including the stereochemical notation for the chiral carbon) of



is -

(1) (S)-2-Bromo-(Z)-hex-3-en-4-one
(2) (R)-2-Bromo-(E)-hex-3-en-4-one
(3) (R)-5-Bromo-(Z)-hex-3-en-2-one
(4) (S)-5-Bromo-(E)-hex-3-en-4-one

MATHEMATICS

Q.101 Let f(x) be a twice differentiable function for all real values of x and satisfies f(1) = 1, f(2) = 4, f(3) = 9. Then which of the following is definitely true?

(1) $f''(x) = 2\ \ x \in (1, 3)$
(2) $f''(x) = f'(x) = 5$ for some $x \in (2, 3)$
(3) $f''(x) = 3\ \ x \in (2, 3)$
(4) $f''(x) = 2$ for some $x \in (1, 3)$

Q.102 If S_1, S_2 and S_3 denote the sums upto 10 terms of three sequences in A.P., whose first terms are unity and common difference are in H.P., then the value of $\frac{2S_1S_3 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3}$ equals to-

(1) 1 (2) 5 (3) 10 (4) 20

Q.103 If $|f(x)| + \sqrt{1 + \cos 2\pi x} = \tan^2\left(\frac{\pi x}{9}\right) f(x)$;

then f(3) is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $-\frac{1}{\sqrt{2}}$ (4) $-\frac{1}{2\sqrt{2}}$

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Q.104 If g(x) is a differential real valued function satisfying $g''(x) - 3g'(x) > 3\ x \geq 0$ and $g'(0) = -1$, then $g(x) + x$ for $x > 0$ is
(1) an increasing function
(2) a decreasing function
(3) a constant function
(4) Data insufficient

Q.105 The tangent at a point whose eccentric angle 60° on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ meet the auxiliary circle at L and M. If LM subtends a right angle at the centre, then eccentricity of the ellipse is-

(1) $\frac{1}{\sqrt{7}}$ (2) $\frac{2}{\sqrt{7}}$ (3) $\frac{3}{\sqrt{7}}$ (4) $\frac{1}{2}$

Q.106 The direction cosines of two lines are related by $\lambda + m + n = 0$ and $a\lambda^2 + bm^2 + cn^2 = 0$. The lines are parallel if -
(1) $a + b + c = 0$ (2) $a^{-1} + b^{-1} + c^{-1} = 0$
(3) $a = b = c$ (4) None of these

Q.107 The maximum value of $\sec^{-1}\left(\frac{7-5(x^2+3)}{2(x^2+2)}\right)$ is

(1) $\frac{5\pi}{6}$ (2) $\frac{5\pi}{12}$ (3) $\frac{7\pi}{12}$ (4) $\frac{2\pi}{3}$

Q.108 If $\alpha = \lim_{n\to\infty}\left(\frac{1}{n^3+1}+\frac{4}{n^3+1}+.....+\frac{n^2}{n^3+1}\right)$ and $\beta = \lim_{x\to 0}\frac{\sin 2x}{\sin 8x}$ then the quadratic equation whose roots are α, β is
(1) $12x^2 - 7x + 1 = 0$ (2) $x^2 + 19x - 120 = 0$
(3) $x^2 - 17x + 60 = 0$ (4) $x^2 - 7x + 12 = 0$

Q.109 Let $n_1 = \sin 7 + \cos 7$, $n_2 = \sqrt{\sin 7} + \sqrt{\cos 7}$, $n_3 = \sqrt{1+\sin 14}$, $n_4 = 1$, then-
(1) $n_2 > n_3 > n_4 > n_1$ (2) $n_3 > n_4 > n_2 > n_1$
(3) $n_3 = n_1 > n_4 > n_2$ (4) $n_2 > n_1 = n_3 > n_4$

Q.110 The interval in which function $f(x) = \sin^{-1}(x^2 - 3x + 3)$ is increasing, is-
(1) [1, 2] (2) $\left[\frac{3}{2}, \infty\right)$
(3) $\left[\frac{3}{2}, 2\right]$ (4) None of these

BIOLOGY

Q.111 In embryo sac of a typical angiosperm, there are :
(1) Egg cell, synergids and antipodals
(2) Egg cell, synergids and secondary nucleus
(3) Egg cell, synergids, central cell and polar nuclei
(4) Egg cell, synergids, polar nuclei and antipodals

Q.112 Ovulation in the human female normally takes place during the menstrual cycle :
(1) at the mid secretory phase
(2) at the end of the proliferative phase
(3) just before the end of the secretory phase
(4) at the beginning of the proliferative phase

Q.113 A pure tall and a pure dwarf plant were crossed to produce offsprings. Offsprings were self crossed, then find out the ratio between true breeding tall to true breeding dwarf?
(1) 1 : 1 (2) 3 : 1
(3) 2 : 1 (4) 1 : 2

Q.114 If there are 16 chromosomes in the roots of onion, then the number of chromosomes in spore mother cell will be :
(1) 2 (2) 8
(3) 4 (4) 16

Q.115 The chromosomal pattern of Turner syndrome usually have :
(1) 2A + XXY
(2) One X
(3) 2A + XXX
(4) Two X only

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Q.116 Which of the following is the best way to determine paternity ?
(1) Gene counting
(2) Protein analysis
(3) DNA fingerprinting
(4) Chromosome counting

Q.117 Which of the following is the correct order of evolutionary history of man?
(1) Peking man, Homo sapiens, Neanderthal, Cro-Magnon
(2) Peking man, Neanderthal, Homo sapiens, Cro-Magnon
(3) Heidelberg man, Peking man, Neanderthal, Cro-Magnon
(4) Peking man, Neanderthal, Homo sapiens, Heidelberg man

Q.118 Plasmids are :
(1) cDNA
(2) viral RNA
(3) mitochondrial DNA
(4) circular extra chromosomal DNA in bacteria

Q.119 Fluoride pollution mainly affects :
(1) teeth (2) heart
(3) brain (4) kidney

Q.120 "Minamata disease" is produced due to the pollution of water with :
(1) Crude oil
(2) Silver nitrate
(3) Mercuric chloride
(4) Methyl isocyanate

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ANSWERS

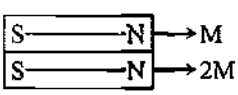
Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	4	4	2	2	1	4	3	2	4	3	3	1	3	1	3	2	4	4
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	3	4	2	2	4	2	3	2	2	1	1	3	3	2	4	2	1	4	4
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	1	2	1	2	1	3	3	1	1	3	3	2	2	1	2	3	2	3	3
Qus.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	4	1	2	3	3	1	3	2	4	4	1	4	2	3	1	4	4	2	1
Qus.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	3	2	1	1	2	1	4	1	1	3	4	2	2	4	2	1	4	2	1	4
Qus.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	4	3	1	1	2	2	4	1	4	3	4	2	3	4	2	3	3	4	1	3

SOLUTIONS

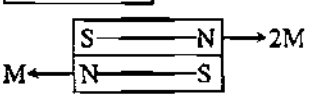
PART-I
One Mark Questions

PHYSICS

1.



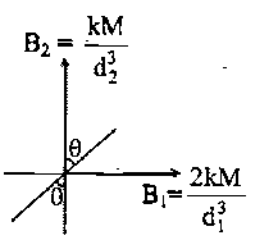
$$T_1 \propto \frac{1}{\sqrt{M+2M}}$$



$$T_2 \propto \frac{1}{\sqrt{2M-M}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{M}{3M}} \Rightarrow T_2 = \sqrt{3}\,T_1$$

2.



$$B_1 = B_2 \tan\theta$$

$$\Rightarrow \frac{2kM}{d_1^3} = \frac{kM}{d_2^3}\tan\theta \quad \frac{2}{\tan\theta}$$

$$= \left(\frac{d_1}{d_2}\right)^3 \Rightarrow \frac{d_1}{d_2} = (2\cot\theta)^{1/3}$$

3. For tangent galvanometer

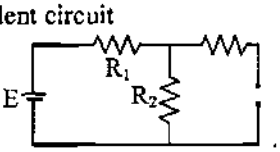
$$\frac{\mu_0 NI}{2R} = B_H \tan\theta \Rightarrow \frac{N_1}{N_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

4. First e^- come near then goes away.

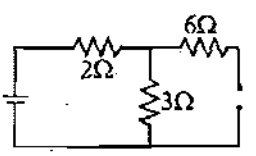
5.

$$e = \frac{L[I-(-I)]}{t} = \frac{2LI}{t} = 2 \times \frac{\mu_0 N^2 A}{\ell} \times \frac{I}{t}$$

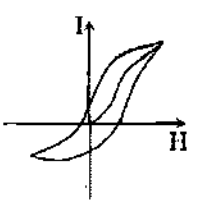
6. equivalent circuit



7.



8.



Slope of I–H curve for ferro may be positive, negative and zero so never be constant

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9. When contact is broken $\boxed{e+E}$ net emf

11. For isolated systems, all three conservation laws are always in effect : total energy is conserved (although kinetic energy K is not conserved in this perfectly inelastic collision), linear momentum is conserved, and angular momentum is conserved.

12. The kinetic energy lost in the collision can be found by subtracting the skater's final K from their initial K :

$$K_{initial} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$K_{final} = \frac{1}{2} (m_1 + m_2) v_f^2$$

$$\Delta K = K_{final} - K_{initial}$$

$$\Delta K = \left(\frac{1}{2}(m_1 + m_2)v_f^2\right) - \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right)$$

$$\Delta K = \left(\frac{1}{2}(30+80)\left(\frac{100\text{kg}\bullet\text{m/s}}{110\text{kg}}\right)^2\right) - \left(\frac{1}{2}(30\text{kg})(2\text{m/s})^2 + \frac{1}{2}(80\text{kg})(1\text{m/s})^2\right)$$

$$\Delta K = 45\text{ J} - (60+40) = -55\text{J}$$

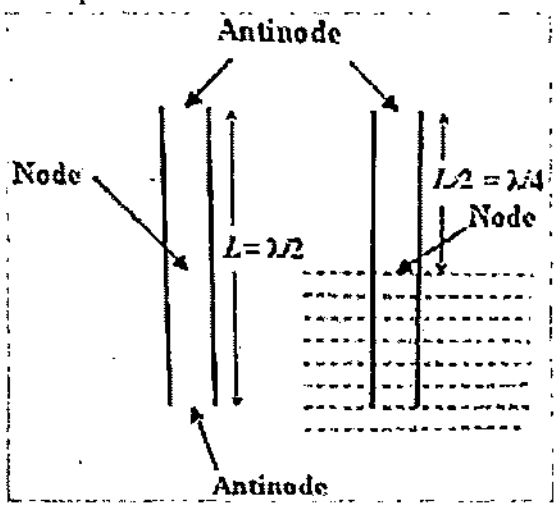
13. Since a monoatomic gas molecule has 3 degrees of freedom, the molar heat capacity (also called molar specific heat) of the monatomic gas at constant volume (C_V) is (3/2) R where R is universal gas constant. Its molar heat capacity at constant pressure (C_p) is (5/2) R. This follows from Meyer's relation, $C_p = C_v + R$.

This means that when heat energy equal to (5/2) R joule is supplied to one mole of a monatomic gas to make it expand at constant pressure, (3/2) R.

This means that when heat energy equal to (5/2) R joule is supplied to one mole of a monatomic gas to make it expand at constant pressure, (3/2) R joule is used to increase the temperature by 1 K and the remaining R joule is used for doing work against the forces which oppose the expansion. 3/2 is 60% of 5/2.

14. In the fundamental mode the air column in the open pipe (pipe open at both ends) vibrates with consecutive antinodes at its ends so that the length L of the pipe is equal to $\lambda/2$ where λ is the wave length of sound in air. Therefore, $\lambda = 2L$.

The air column in the closed pipe (pipe closed at one end) on the other hand vibrates in its fundamental mode, with a node at the closed end (at the water surface inside the pipe) and the neighbouring antinode at the open end so that $L/2 = \lambda/4$. Again we obtain $\lambda = 2L$. The frequencies in the two cases are same.



15. Considering the vertical motion of the projectile (pebble), the time taken to reach the ground after leaving the wheel is given by

$2R = 0 + (1/2)gt^2$, from which $t = 2\sqrt{(R/g)}$

[We have used the equation of the one dimensional motion, $x = x_0 + v_0 t + (1/2)\,at^2$]

The horizontal range of the pebble is PQ = horizontal velocity × time of flight

The centre of mass of the wheel is moving with speed u. The pebble at the topmost point of the wheel is moving horizontally with speed u with respect to the wheel so that the horizontal velocity of the pebble with respect to the ground is u + u = 2u.

Therefore, horizontal range

$PQ = 2u \times 2\sqrt{(R/g)} = 4u\sqrt{(R/g)}$

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16. The position vector r_1 of the particle at the instant t is given by

$r_1 = v_0t + 1/2at^2$ where v_0 is the initial velocity and a is the constant acceleration

We have $v_0 = 2\hat{j}$ and $a = 2\hat{i} + 4\hat{j}$

Therefore, $r_1 = 2\hat{j}t + (1/2)(2\hat{i} + 4\hat{j})t^2$

$= t^2\hat{i} + (2t + 2t^2)\hat{j}$

The above equation shows that the x-coordinate of the particle at time t is t^2 and the y-coordinate is $(2t + 2t^2)$

The time t at which the y-coordinate becomes 12 metre is given by

$2t + 2t^2 = 12$

Or, $2t^2 + 2t - 12 = 0$

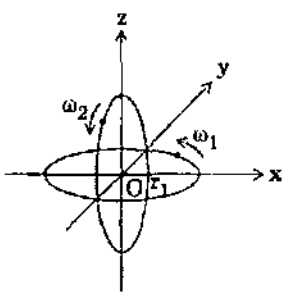
This gives $t = [-2 \pm \sqrt{(4 + 96)}]/4 = 2$ seconds, ignoring the negative time.

Since the x-coordinate of the particle is t^2, its value when the y-coordinate becomes 12 m (at time 2 seconds) is 4m.

17. With the help of dimensional analysis, we cannot arrive at constants and algebraic, exponential and trigonometric functions. Though dimensionally equation is satisfied, but it may not be correct.

18. Let us consider any two particles of the body. Let a particle of the first body rotate in the x-y plane in a circle of radius r_1.

Similarly, let a particle of body 2 move in the y-z plane in a circle of radius $r_2$as shown in the figure.



Then $\vec{\omega_1} = \omega_1\hat{k}$ and $\vec{\omega_2} = \omega_2\hat{i}$

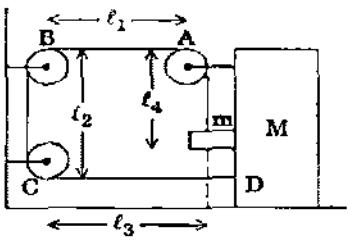
$\vec{\omega_{21}} = \vec{\omega_2} - \vec{\omega_1} = \omega_2\hat{i} - \omega_1\hat{k}$

$\therefore |\vec{\omega_{21}}| = \sqrt{\omega_1^2 + \omega_2^2}$

19. Let X be the leftward displacement of M and x and y be the leftward and downward displacement of m.

Let $AB = \ell_1$; $BC = r_2$: $CD = \ell_3$ and $Am = \ell_4$

When M moves towards left, say by x, then



$AB = (\ell_1 - x)$

$BC = \ell_2$

$CD = \ell_3 - x$

$Am = \ell_4 + y$

$\therefore \ell_1 - x + \ell_2 + \ell_3 - x + \ell_4 + y = \ell_1 + \ell_2 + \ell_3 + \ell_4$

$\therefore 2x = y$

$2\hat{x} = \hat{y}$

Further $x = x'$

$\therefore \hat{x} = \hat{X}$

Given $\hat{x} = 2 m/s^2$

$\therefore \hat{x} = 2 m/s^2$ and $\hat{y} = 4 m/s^2$

Acceleration of m is $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$ m/s^2.

20. P-T graph is a straight line passing through the origin.

Therefore V = constant

$\therefore$ work done on the gas is zero.

Further, density of the gas, $\rho = \frac{m}{V}$

$\therefore \rho \propto \frac{1}{V}$

Since volume of the gas is constant, density of the gas is constant.

$PV = nRT$

$\Rightarrow P = \left(\frac{nR}{V}\right)T$

i.e., slope of P-T line is proportional to n.

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CHEMISTRY

21. $:\dot{N}=\ddot{O}:$ and

22. XeF_5^- and BrF_3 have 2 lone pairs each.

24. $[Ti(H_2O)_6]Cl_4$
In Ti^{+4} all electrons are paired.

26. Both are carbonate ores

27. $PbI_4 \xrightarrow{\Delta} PbI_2 + I_2$
Pb^{+4} is oxidsing and I^- is reducing agent.

28. meq. of $H_2O_2 = \frac{x}{100\times17}\times1000 = \frac{10x}{17}$

$\frac{10x}{17} = xN_2 \Rightarrow N_2 = \frac{10}{17} = 0.588$

29. $n = \frac{1\times1}{0.082\times298} = 0.041$

$E = 0.041\,(\underset{\text{BE/mole}}{\underline{436}} + \underset{\text{excitation energy for 2mole H atoms}}{\underline{1968}}) = 98.56\ KJ$

30. $\frac{T_C}{P_C} = \frac{\frac{8a}{27Rb}}{\frac{a}{27b^2}} = \frac{8b}{R}$ or $\frac{T_C}{P_C} \propto b$

31. $T_1 = 300K,\ T_2 = 600K,\ P_1 = 1\ atm,\ P_2 = ?$

$\left(\frac{P_1}{P_2}\right)^{\gamma-1} = \frac{T_1}{T_2} \Rightarrow (\gamma-1)\log\left(\frac{P_1}{P_2}\right) = \log\left(\frac{T_1}{T_2}\right)$

$(1.66-1)[\ln P_1 - \ln P_2] = -\ln 2$

$\ln P_2 = \frac{-\ln 2}{-0.66} = 1.06$

32. $E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{1}$

33. $FCC \rightarrow 6\times\frac{1}{2} + \frac{1}{8}\times4$

$BCC \rightarrow 1 + \frac{1}{8}\times8$

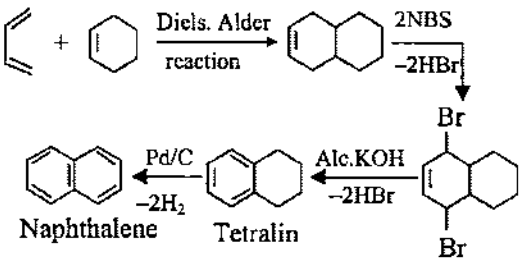
34. Mirror images.

35. Elimination product is non major product

37. In compound (2), the two identical atoms (H,H) lie on the opposite sites of the Fisher projection formula and hence it represents the threo-isomer.

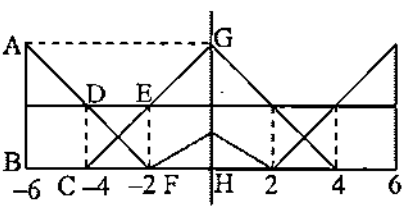
38. A double bond is more reactive than a triple bond and hence it is more easily oxidized i.e, (1) is correct.

40.



MATHEMATICS

41.



$\int_{-6}^{6} \max(|2-|x||, 4-|x|, 3)dx$

$= 2(\triangle ABCD + \square DEFC + \triangle EFGF)$

$= 2\left[\frac{1}{2}(3+4)\times2 + 2\times3 + \frac{1}{2}(3+4)\times2\right]$

$= 2[14+6] = 40.$

42. Let $\lambda = \tan\theta$

$\therefore x = 2\sin2\theta \Rightarrow y = 2\cos2\theta$

$\therefore x^2 + y^2 - xy = 4 - 2\sin4\theta$

$x^2 + y^2 - xy \in [2, 6]$

$\therefore a + b = 8$

43. $P(B) = \frac{2}{3}.\frac{3}{5} = \frac{2}{5}$

$\Rightarrow P(C) = \frac{2}{3}.\frac{2}{5} = \frac{4}{15}$

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44. Given, $\frac{dy}{dx} = \frac{y(x - y\ell ny)}{x(x\ell n\, x - y)}$

$\Rightarrow x^2 \ell n\, x\, dy - xy\, dy = xy\, dx - y^2 \ell n\, y\, dx$

$\Rightarrow \frac{\ell nx}{y^2} dy - \frac{1}{xy} dy = \frac{1}{xy} dx - \frac{\ell n\, y}{x^2} dx$

(on dividing by x^2y^2)

$\Rightarrow \frac{1}{xy} dx - \frac{\ell n\, x}{y^2} dy + \frac{1}{xy} dy - \frac{\ell n\, y}{x^2} dx = 0$

$\Rightarrow d\left(\frac{\ell n\, x}{y}\right) + d\left(\frac{\ell n\, y}{x}\right) = C$

On integrating both sides, we get

$x\, \ell n\, x + y\, \ell n\, y = C\, xy$

45. $f(n) = (1)^{\infty}$ apply the formula

$f(n) = \lim_{x\to 0} e^{\frac{1}{x}\left(\left(1+\sin\frac{x}{2}\right)\left(1+\sin\frac{x}{2^2}\right)\ldots\left(1+\sin\frac{x}{2^n}\right)-1\right)}$

$= \lim_{x\to 0} e^{\frac{\left(1+\left(\sin\frac{x}{2}+\sin\frac{x}{2^2}+\ldots+\sin\frac{x}{2^n}\right)+\left(\sin\frac{x}{2}.\sin\frac{x}{2^2}+\ldots\right)-1\right)}{x}}$

$= \lim_{x\to 0} e^{\left(\frac{\sin\frac{x}{2}}{x}+\frac{\sin\left(\frac{x}{2^2}\right)}{x}+\ldots+\frac{\sin\frac{x}{2^n}}{x}\right)} = e^{\left(\frac{1}{2}+\frac{1}{2^2}+\ldots\frac{1}{2^n}\right)}$

sum of infinite G.P. $= e^{\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)}} = e.$

46. Let (h, k) be the coordinates of the vertex of the moving parabola and its equation be

$(y-k)^2 = -4a(x-h)$ ... (i)

Let the equation of fixed parabola be

$y^2 = 4ax$ ... (ii)

From Eqs. (i) and (ii)

$(y-k)^2 = -4a\left(\frac{y^2}{4a} - h\right)$

$\Rightarrow 2y^2 - 2ky + k^2 - 4ah = 0$

Since, two parabolas touch each other, therefore

$D = 0$

$\Rightarrow 4k^2 - 8(k^2 - 4ah) = 0 \Rightarrow k^2 = 8ah$

$\Rightarrow$ Locus of the vertex of the moving parabola is

$y^2 = 8ax$

whose latusrectum $= 8a = 2(4a) = 2\ell$

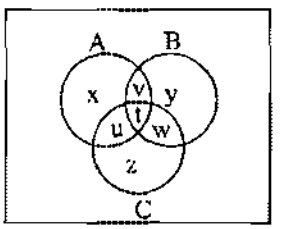
47. $S = (p + q - r)^n$ {Putting $x = y = z = 1$}

$\lim_{n\to\infty} \frac{S}{(S^{1/n}+1)^n} = \lim_{n\to\infty} \frac{(p+q-r)^n}{\{(p+q-r)+1\}^n}$

$= \lim_{n\to\infty} \left(\frac{p+q-r}{(p+q-r+1)}\right)^n$

$= 0 \quad \left\{\text{as}\left(\frac{p+q-r}{p+q-r+1}\right) < 1\right\}$

48.



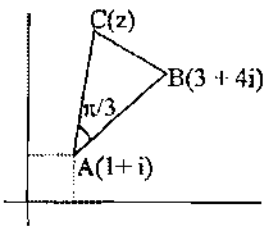
$\frac{1}{5} = x + v + y$

$\frac{1}{15} = w \Rightarrow \frac{1}{10} = u + t$

$x + y + z + u + v + w + t = 1$

$z = ?$

49.



$\frac{z-(1+i)}{AC} = \frac{(2+3i)}{AB} e^{i\pi/3}$

$\Rightarrow z - 1 - i = 2(2 + 3i)\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$\Rightarrow z - 1 - i = \frac{4 - 6\sqrt{3} + i(6 + 4\sqrt{3})}{2}$

$\Rightarrow z = 3 + 4i + i\sqrt{3}(2 + 3i)$

50. Given $\frac{\sum \log\left(\frac{n^2+r^2}{n^2}\right)}{n} = \log 2 + \frac{\pi}{2} - 2$

Let

$A = \lim_{n\to\infty} \frac{1}{n^{2n}} \left[(n^2+1^2)^n (n^2+2^2)^n \ldots (2n^2)^n\right]^{1/n}$

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$$\Rightarrow A = \frac{1}{n}\lim_{n\to\infty}\log\left[\left(1+\frac{1}{x^2}\right)^m + \log\left(1+\frac{2^2}{x^2}\right)^m + \ldots\log\left(1+\frac{n^2}{x^2}\right)^m\right]$$

$$= \frac{1}{n}\sum_{r=1}^{n}\lim_{n\to\infty}\log\left(1+\frac{r^2}{n^2}\right)^m = \frac{m}{n}\lim_{n\to\infty}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{x^2}\right)$$

$$\log A = m\left(\log 2 + \frac{\pi}{2} - 2\right) \Rightarrow A = 2^m e^{m\left(\frac{\pi}{2}-2\right)}$$

51. $\int \frac{(1-x^{-3})x^{-4}}{\sqrt{6-4x^{-3}+2x^{-6}}}$

Put $x^{-3} = t \Rightarrow -3x^{-4}\,dx = dt$

$$= -\frac{1}{3}\int \frac{(1-t)}{\sqrt{6-4t+2t^2}}\,dt$$

Again put $6-4t+2t^2 = z$

$$(-4+4t)dt = dz$$

$$(1-t)dt = -\frac{1}{4}dz$$

$$= \frac{1}{12}\int \frac{1}{\sqrt{z}}dz = \frac{1}{12}\cdot\frac{(z)^{1/2}}{1/2} + C$$

$$= \frac{1}{6}\sqrt{2t^2-4t+6} + C = \frac{1}{6}\sqrt{\frac{2}{x^6}-\frac{4}{x^3}+6} + C$$

$$= \frac{1}{6x^3}\sqrt{6x^6-4x^3+2} + C$$

52. Since, $y = mx + \frac{1}{m}$ or $m^2h - mk + 1 = 0$,

we have $m_1 + m_2 = \frac{k}{h}$, and $m_1m_2 = \frac{1}{h}$

Given, $\theta_1 + \theta_2 = \frac{\pi}{4}$

$\therefore \tan(\theta_1+\theta_2) = \tan\frac{\pi}{4}$

$$\Rightarrow \frac{m_1+m_2}{1-m_1m_2} = 1 \Rightarrow \frac{k}{h} = 1 - \frac{1}{h}$$

$\Rightarrow y = x - 1 \Rightarrow x - y - 1 = 0$

53. Solution of these three equations is infinite.
So planes contain straight line.

54. I[st] family lines passes through (–2, 3) and II[nd] (–1, 7/2)
Required line $x - 2y + 8 = 0$

55. $P(A^C \cap B^C \cap C^C)$
$= 1 - P(A) - P(B) - P(C) + P(AB) + P(BC) + P(CA) - P(ABC)$

$P(A^C \cap B^C \cap C^C)_{max}$
$= 1 - P(A) - P(B) - P(C) + P(AB) + P(BC) + P(CA)$
$= 0.31$

56. $\frac{a(2x+1)^2}{(x-3)^2} + \frac{b(2x+1)}{(x-3)} + c = 0$

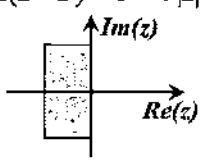
$$\Rightarrow \frac{2x+1}{x-3} = \alpha \text{ or } \frac{2x+1}{x-3} = \beta$$

$\Rightarrow 2x + 1 = \alpha x - 3\alpha \Rightarrow x(\alpha - 2) = 1 + 3\alpha$

$$\Rightarrow x = \frac{1+3\alpha}{\alpha-2}, \frac{1+3\beta}{\beta-2}$$

57. Let $y = \log(\log_{1/3}(\log_7(\sin x + \alpha)))$, $\forall x \in R$ will exist
if $\log_{1/3}(\log_7(\sin x + \alpha)) > 0$
but $\log_7(\sin x + \alpha) > 0$ ...(1)
Take antilog $\log_7(\sin x + \alpha) < 1$ ...(2)
From (1) and (2)
$0 < \log_7(\sin x + \alpha) < 1$
$1 < \sin x + \alpha < 7$
$1 - \sin x < \alpha < 7 - \sin x$
$\therefore$ max. value of $1 - \sin x = 2$
min. value of $7 - \sin x = 6$
$\therefore 2 < \alpha < 6$ & $\alpha \in I$
$\therefore \alpha = 3, 4, 5$

58. $\log_3|2z-1| > \log_3|2z+1|$
$\Rightarrow (2z-1)(2\bar{z}-1) > (2z+1)(2\bar{z}+1)$
$\Rightarrow 4|z|^2 - 2(z+\bar{z}) + 1 > 4|z|^2 + 2(z+\bar{z}) + 1$



$\Rightarrow 4.2\,\mathrm{Re}(z) < 0 \Rightarrow \mathrm{Re}(z) < 0$

59. Since, $0 \le \arg(z) \le \frac{\pi}{4}$, represent the region of complex plane lying in the first quadrant and bounded by x-axis and the line $y = x$.

$|2z - 4i| = 2|z - 2i|$

Least value of $|z-2i|$ is length of perpendicular from (0, 2) to $y = x$, which is $\sqrt{2}$.

So, the least value of $\sqrt{2}|2z-4i|$ is 4.

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60. $5(\log_y x + \log_x y) = 26,\ xy = 64$

$\Rightarrow 5\left(\log_y x + \frac{1}{\log_y x}\right) = 26$

Put $\log_y x = t$

$\therefore 5(t^2 + 1) = 26t \Rightarrow 5t^2 - 26t + 5 = 0$

$\Rightarrow t = 5, \frac{1}{5}$

$\therefore \log_y x = 5, \frac{1}{5}$

So $x = y^5, y^{1/5}$

But $xy = 64 \Rightarrow y^5 . y = 64$ or $y^{1/5} . y = 64$

$\therefore y = 2$ or 32

$\therefore$ Solution set will be (32, 2) & (2, 32)

$\therefore a + b + c + d = 32 + 2 + 2 + 32 = 68$

BIOLOGY

61. Apogamy leads to formation of haploid sporophyte.

62. Alcurone and endosperm are usually triploid.

63. Maximum part of semen is seminal fluid which is secreted by seminal vesicle.

64. Estrous phase is the part of estrous cycle which is found in non-primate mammales.

65. Copper-T releases the copper ion which inhibites the motility of sperms.

66. Mendelian monohybrid cross gives 3 : 1 ratio.

67. It is used to identify homozygous & heterozygous F_1 progeny.

68. Crossing over leads to recombination of alleles by reparation between homologous chromosome.

69. It is known as mangoloid idiocity which results from trisomy due to chromosomal non disjunction during gamete formation.

71. Both have different origin & similar function, so they are analogous organs.

72. Nails are useful body parts which provides strength to digits of helps in holding object.

73. Diptheria is caused by corynebacterium diptheriae, which produces diptheria toxin that after protein function in the host by inactivating elongation factor (EF-2) required for transcription. It results is pharyngitis.

74. IgG is found in blood plasma & provides protection against antigen & provide protection to foetus.

77. Foreign genes are introduced to provide resistance or eliminate disease.

78. Restriction enzymes are key tools in genetic engineering.

79. E. coli commonly found in faecal material pollutes the drinking water in rivers, ponds. It's presence shows contamination.

80. E. coli number is decreased in polluted water.

PART-II

Two Marks Questions

PHYSICS

86. The weight of a floating body is equal to the weight of the displaced fluid. If V and v represent the total volume of the piece of granite and volume of granite in water respectively, we have

$V\rho g = v\rho_1 g + (V - v)\rho_2 g$

Or, $v(\rho_1 - \rho_2) = V(\rho - \rho_2)$

Therefore, $v/V = (\rho - \rho_2)/(\rho_1 - \rho_2)$

The ratio required in the question is $v/(V - v)$ and is given by

$v/(V - v) = (\rho - \rho_2)/[(\rho_1 - \rho_2) - (\rho - \rho_2)]$

Or,

$v/(V - v) = (\rho - \rho_2)/(\rho_1 - \rho) = (\rho_2 - \rho)/(\rho - \rho_1)$

87. For small angles of θ (typically less than 15°), the frequency of oscillation for a simple pendulum is approximately.

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$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

For increasingly large value of θ, however, the acceleration no longer varies linearly with displacement. Thus, for larger angles, the frequency f will be a affected by the angle of release ∅, as well as by the length of the pendulum.

88. We have $L = L_0(1 + \alpha\theta)$

For brass $L_B - L_0^B = L_0^B \alpha_B.\theta$

For iron $L_{iron} - L_0^{iron} = L_0^{iron} \alpha_{iron}.\theta$

Effective increase in length of the iron wire

$$\Delta\ell = \left(L_0^{iron}\alpha_{iron} - L_0^B\alpha_B\right)\times\theta$$

Extra tension = Y × Strain × A

$$= Y \times \pi r^2 \times \frac{\Delta\ell}{L_0^{iron}} \quad [L_0^B = L_0^{iron}]$$

$$= Y \times \pi r^2 \times \left(\alpha_{iron} - L_0^B.\frac{\alpha_B}{L_0^{iron}}\right)\times\theta$$

$$= 21 \times 10^{11} \times 3.14 \times (0.3 \times 10^{-4})^2 \times (12 \times 10^{-6} - 8 \times 10^{-6}) \times 40$$

$$= 95 \times 10^4 \text{ dynes} \simeq 0.945 \times 10^6 \text{ dynes}$$

89. Let the first truck move to the right and the second to the left. Let us take the rightward direction as positive. In the horizontal direction, friction being absent, no external force is acting on the system. Hence momentum is conserved.

The momentum carried away by a sack thrown from the first is mv_1. the momentum brought in by the second sack to the first is $m(-v_2)$.

∴ By the law of conservation of momentum of (truck + sacks of rice)

$(M + m)\,v_1 - mv_1 + m(-v_2) = (M + m)\,v_1'$

where v_1' is the new velocity of truck 1.

$$\therefore Mv_1 - mv_2 = (M + m)\,v_1'$$

$$\therefore v_1' = \frac{M}{M+m}v_1 - \frac{m}{M+m}v_2$$

$$= \frac{200}{250}.50 - \frac{50}{250}.200 = 0$$

Similarly considering conservation of momentum of the second (truck ÷ sacks of rice), we get for the second truck.

$$v_2' = -\left[\frac{M}{M+m}v_2 - \frac{m}{M+m}.v_1\right]$$

$$= -\left[\frac{200}{250}\times 200 - \frac{50}{250}\times 50\right]$$

$$= -150 \text{ m/s}.$$

90. $\vec{v} = a\hat{i} + bx\hat{j}$

$v_x = a$ and $v_y = bx$

$$\frac{dx}{dt} = a$$

$$\therefore x = at + C$$

Since $x = 0$ at $t = 0$, $C = 0$

$$\therefore x = at$$

$$\frac{dy}{dx} = bx = abt \qquad \text{.....(1)}$$

$$\therefore y = \frac{abt^2}{2} + C$$

$$\therefore y = \frac{abt^2}{2}$$

as $y = 0$ at $t = 0$ .....(2)

From equation (1) and (2), $y = \frac{abt^2}{2} = \frac{b}{2a}x^2$

Hence the trajectory is a parabola symmetrical about the y-axis.

CHEMISTRY

91. $P_4 + OH^- \longrightarrow PH_3 + H_2PO_2^-$

93. $MnO_4^- + OH^- \longrightarrow MnO_4^{-2} + IO_3^- + H_2O$

94. $E = \frac{2}{3}(180 + 80 - 50) = 140$ kJ/mole

95. $2H_2O(l) \rightleftharpoons 2H^+(aq.) + 2OH^-(aq.)$ $\quad \Delta G_1^\circ$

$2H^+(aq.) + 2e \rightleftharpoons H_2(g)$ $\quad \Delta G_2^\circ$

$2H_2O(l) + 2e \rightleftharpoons H_2(g) + 2OH^-(aq.)$

$\Delta G = \Delta G_1^\circ + \Delta G_2^\circ$

$\Rightarrow -2FE^\circ = -RT\ln(10^{-26}) - 2 \times F \times O$

$$E^\circ = \frac{0.066}{2}\log(10^{-26}) = -0.858$$

96. $2B(s) + \frac{3}{2}O_2(g) \rightarrow B_2O_5(s)$

$\Delta H_2^\circ = -1273$ kJ

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$$3H_2(g) + \frac{3}{2}O_2(g) \rightarrow 3H_2O(l)$$

$$\Delta H_4^\circ \times 3 = -286 \times 3$$

$$B_2O_3(s) + 3H_2O(g) \rightarrow B_2H_6(g) + 3O_2(g)$$

$$\Delta H_3^\circ = 2035 \text{ kJ}$$

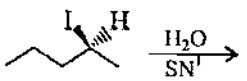
$$3H_2O(g) \rightarrow 3H_2O(g)$$

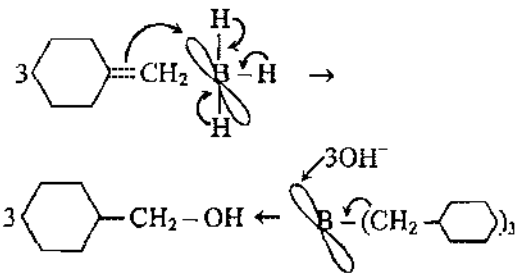
$$\Delta H_1^\circ = 44 \times 3$$

$$3B(s) + 3H_2(g) \rightarrow B_2H_6(g) \quad \Delta H = 36$$

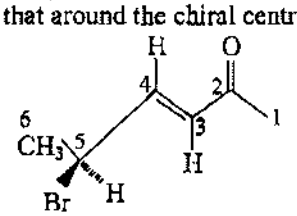
97. $\frac{\frac{8}{x}}{\frac{92}{1000}} \times 1.85 = 0.925$

$x = 173.9$

98.  d, ℓ racemic mixture

99. 

100. The configuration about the double bond is (E) while that around the chiral centre, i.e., C_5 is S.



Hence the correct name is
(S)-5-bromo-(E)-hex-3-en-2-one.

MATHEMATICS

101. Let $g(x) = f(x) - x^2$. We have
$g(1) = 0, g(2) = 0, g(3) = 0$
$[\because f(1) = 1, f(2) = 4, f(3) = 9]$
From Rolle's theorem on $g(x)$, $g'(x) = 0$ for at least $x \in (1, 2)$.
Similarly, $g(x) = 0$ for at least one $x \in (2, 3)$.
$g''(x) = 0 \Rightarrow f''(x) = 2$ for some $x \in (1, 3)$

102. Let the common difference of the three A.P's be d_1, d_2 and d_3

Then $S_1 = \frac{10}{2}[2.1 + (10-1)d_1] \Rightarrow d_1 = \frac{S_1 - 10}{5.9}$

Similarly, we have $d_2 = \frac{S_2 - 10}{5.9}$ and $d_3 = \frac{S_3 - 10}{5.9}$

Since d_1, d_2 and d_3 are given to be in H.P., therefore

$$\frac{1}{d_2} - \frac{1}{d_1} = \frac{1}{d_3} - \frac{1}{d_2}$$

$$\Rightarrow \frac{1}{S_2 - 10} - \frac{1}{S_1 - 10} = \frac{1}{S_3 - 10} - \frac{1}{S_2 - 10}$$

$$\Rightarrow \frac{S_1 - S_2}{(S_2 - 10)(S_1 - 10)} = \frac{S_2 - S_3}{(S_2 - 10)(S_3 - 10)}$$

$$\Rightarrow \frac{S_1 - S_2}{S_1 - 10} = \frac{S_2 - S_3}{S_3 - 10}$$

$$\Rightarrow \frac{2S_1S_3 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3} = 10$$

103. $|f(x)| + \sqrt{2\cos^2 \pi x} = \left(\tan^2 \frac{\pi x}{9}\right) f(x)$

At $x = 3$, $|f(3)| + \sqrt{2} = 3\, f(3)$

If $f(3) \geq 0 \Rightarrow f(3) = \frac{1}{\sqrt{2}}$

& if $f(3) < 0$; $f(3) = \frac{\sqrt{2}}{4}$ not possible

104. $\frac{d}{dx}(g'(x)e^{-3x}) > 3.e^{-3x}$

$\Rightarrow \frac{d}{dx}(g'(x)e^{-3x} + e^{-3x}) > 0$

$\Rightarrow e^{-3x}(1 + g'(x))$ is an increasing function.

Now, $e^{-3x}(1 + g'(x)) > (g'(0) + 1) \Rightarrow x > 0$

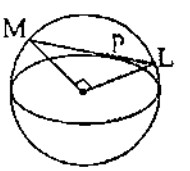
$\Rightarrow g'(x) + 1 > 0$

$\Rightarrow g(x) + x$ is an increasing function.

105. Point P will be $(a\cos 60°, a\sin 60°)$

Tangent will be $\frac{x\cos 60°}{a} + \frac{y\sin 60°}{b} = 1$

$$\frac{x}{2a} + \frac{y\sqrt{3}}{2b} = 1 \qquad \ldots(1)$$



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Homogenise $x^2 + y^2 = a^2$ with the help of eqn. (1)

We get $x^2 + y^2 = a^2\left[\frac{x}{2a} + \frac{y\sqrt{3}}{2b}\right]^2$

$$x^2 + y^2 = a^2\left[\frac{x^2}{4a^2} + \frac{3y^2}{4b^2} + \frac{xy\sqrt{3}}{2ab}\right]$$

$$\frac{3x^2}{4} + y^2\left(1 - \frac{3a^2}{4b^2}\right) - \frac{a\sqrt{3}xy}{2b} = 0$$

For 90° at centre

$$\frac{3}{4} + 1 - \frac{3a^2}{4b^2} = 0$$

$$\frac{3}{4}\cdot\frac{a^2}{a^2(1-e^2)} = \frac{7}{4}$$

$$1 - e^2 = \frac{3}{7}$$

$$\Rightarrow\ e^2 = \frac{4}{7} \Rightarrow e = \frac{2}{\sqrt{7}}$$

106. For $n = -(\ell + m)$, the second relation gives
$a\ell^2 + bm^2 + c(\ell + m)^2 = 0$
or $(a + c)\,\ell^2 + 2c\ell m + (b + c)\,m^2 = 0$.
For parallel lines, the two roots must be equal
$\Rightarrow 4c^2 - 4(b + c)(a + c) = 0$
$\Rightarrow ab + bc + ca = 0$

107. $$\left(\sec^{-1}\left(\frac{-8 - 5x^2}{2x^2 + 4}\right)\right)_{\max} = \sec^{-1}(-2) = \frac{2\pi}{3}$$

108. $$\alpha = \lim_{n\to\infty}\frac{1 + 4 + a + \ldots + n^2}{n^3 + 1}$$
$$= \lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6(n^3+1)}$$
$$\beta = \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{3x}{\sin 2x}\right)\times\left[\frac{2x}{8x}\right]$$
$$\beta = \frac{1}{4}$$
$$\alpha\beta = \frac{1}{12}$$
$$\alpha + \beta = \frac{7}{12}$$
$$12x^2 - 7x + 1 = 0$$

109. $\because \sqrt{1 + \sin 14} > 1$
$$\sqrt{1 + \sin 14} = \sqrt{(\sin 7 + \cos 7)^2}$$
$$= \sin 7 + \cos 7 \ \&\ \sqrt{\sin 7} > \sin 7$$
$$\sqrt{\cos 7} > \cos 7$$
$\therefore n_2 > n_1 = n_3 > n_4$

110. $-1 \le x^2 - 3x + 3 \le 1$
$\Rightarrow x^2 - 3x + 4 \ge 0$ and $x^2 - 3x + 2 \le 0$
$\Rightarrow x \in [1, 2]$
$$f'(x) = \frac{2x - 3}{\sqrt{1 - (x^2 - 3x + 3)^2}} \ge 0 \Rightarrow x \ge \frac{3}{2}$$

BIOLOGY

111. A typical embryo sac is 7 called and 8 nuclei structure with egg, synergids, polar nuclei and antipodal.

112. At the end of proliferative phase level of LH is increased which stimulates the ovulation.

115. Turners syndrome is due to monosomy (XO). results is sterile female.

116. In DNA finger printing technique. DNA is isolated & segmented images are developed. It is unique for each individual.

117. There are prehistonic man, whose fossils have been traced at different places they are named after the place they lived in & shows some evolutionary evidence over previous one.

119. Fluroide pollution is mainly affects the tooth which is known as flurosis.

120. Minamata disease caused by pollution of murcury in marine water, Murcury gets in food chain with small animals & fishes, which are consumed by man.

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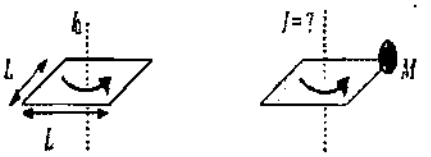
KVPY (Stream SX) Model Test Papers
Practice Test -3

PART- I
One - Mark Questions

Questions 1 to 80 are Single correct answer type questions. Each correct answer carries 1 mark. 0.25 marks will be deducted for each wrong answer.

PHYSICS

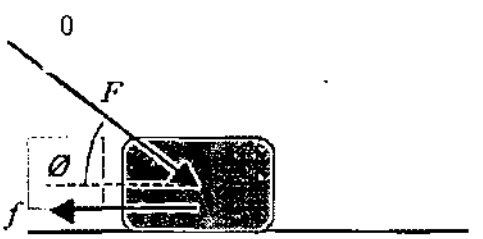
Q.1 A square of mass M and sides of length L has a moment of inertia I_0 when rotated about an axis perpendicular to its surface and passing through its center, as shown. Now a lump of clay, also of mass M is attached to one corner of the square as shown. What is the new moment of inertia of the masses about the same axis of rotation?



(1) $I_0 + \frac{ML^2}{4}$ (2) $I_0 + \frac{ML^2}{2}$

(3) $I_0 + \frac{\sqrt{2}ML^2}{2}$ (4) $I_0 + 2ML^2$

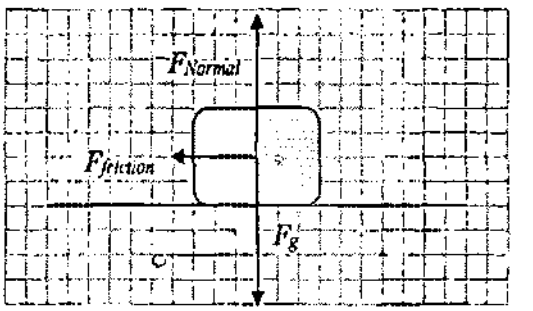
Q.2 A block of mass m is pushed across a rough surface by an applied force F, directed at an angle θrelative to the horizontal as shown. The block experiences a friction force f in the opposite direction. What is the coefficient of friction between the block and the surface?



(1) $\frac{mg}{F\sin\phi}$ (2) $\frac{f}{F\sin\phi + mg}$

(3) $\frac{f}{mg}$ (4) $\frac{mg}{f}$

Q.3 The free-body diagram shows all forces acting on a box supported by a stationary horizontal surface, where the length of each force vector is proportional to its magnitude. Which statement below is correct ?



(1) The box must be moving to the left, due to the force of friction acting in that direction
(2) The box must be accelerating to the right, as indicated by the force of friction in the opposite direction
(3) The box must be moving to the right, as indicated by the force of friction in the opposite direction
(4) None of these statements is correct

Q.4 A mass m in three-dimensional space is subjected to three forces : F_1, F_2 and F_3.F_1 and F_2 have the same magnitude, with F_1 in the positive-x direction, and F_2 in the positive-y direction. If the mass has an acceleration of 0, which of the following statements is false?
(1) The magnitude of F_3 is the same as that of F_1
(2) The object is in equilibrium and could be stationary
(3) F_3 lies in the x-y plane
(4) The object is in equilibrium and could be moving

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Q.5 A cube of mass 30 g wettable by water floats on the surface of water. Each face of the cube is 4 cm long. Surface tension of water = 70 dynes/cm. The distance of the lower face of the cube from the surface of water is ($g = 980\ cm\ s^{-2}$) :

(1) 1.9 cm (2) 1.93 cm
(3) 1.95 cm (4) 1.98 cm

Q.6 An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. The distance of the near surface of the slab from the mirror is 1 cm. The final image from the mirror will be formed at :

(1) 4.56 cm (2) 5.65 cm
(3) 6.56 cm (4) 7.67 cm

Q.7 A simple pendulum has a period T on the earth. What is the period T' of this same pendulum on the moon, where the acceleration due to gravity is 1/6 that of the earth?

(1) $\frac{T}{\sqrt{6}}$ (2) $\frac{T}{6}$ (3) $\sqrt{6}T$ (4) $36T$

Q.8 The only force acting on a block is along x-axis is given by $F = -\left(\frac{4}{x^2+2}\right)N$. When the block moves from $x = -2$ m to $x = 4$ m, the change in kinetic energy of block is –

(1) Positive
(2) Negative
(3) Zero
(4) May be positive or negative

Q.9 An aeroplane is flying in a horizontal circle at a speed of 540 km/h. Banked for this turn, the wings of the plane are tilted at an angle 45° from the horizontal. Assume that a lift force acting perpendicular to the wings holds the aircraft in the sky. The radius of the circle in which the plane is flying is (Take $g = 10\ m/s^2$)

(1) 1000 m (2) 2250 m
(3) 500 m (4) 4500 m

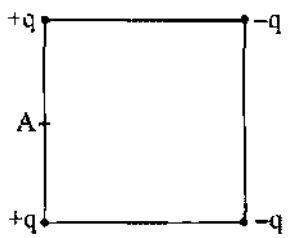
Q.10 A piano tuner stretches a steel piano wire with a tension of 800 N. The steel wire is 0.4 m long and has a mass 2 g. The number of the highest harmonic that could be heard by a person who is capable of hearing frequencies up to 10^4 Hz is :

(1) 100 (2) 150 (3) 50 (4) 20

Q.11 The electric intensity due to a dipole of length 10 cm and having a charge of 500 μC, at a point on the axis 20 cm from one of the charges in air is :

(1) 9.28×10^7 N/C (2) 20.5×10^7 N/C
(3) 6.25×10^7 N/C (4) 13.1×10^{11} N/C

Q.12 Four electric charges +q, +q, –q and –q are placed at the corners of a square of side 2L (See fig.). The electric potential at point A, midway between the two charges +q and +q, is



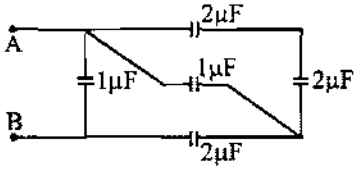
(1) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}(1+\sqrt{5})$

(2) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$

(3) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$

(4) zero

Q.13 The effective capacity between A and B in figure will be :



(1) 0.5 μF (2) 1.5 μF
(3) 2 μF (4) 2.5 μF

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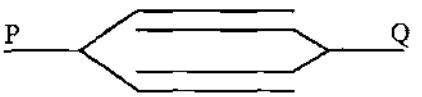
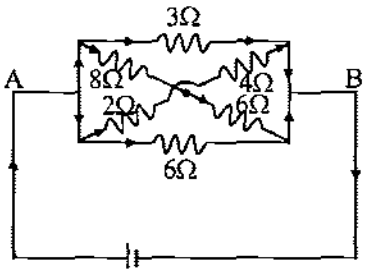
Q.14 Figure shows four plates each of area A and separated from another by a distance d. What is the capacitance between P and Q ?



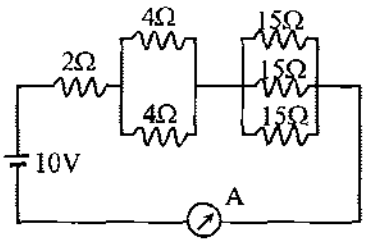
(1) $\frac{\varepsilon_0 A}{d}$ (2) $2\frac{\varepsilon_0 A}{d}$ (3) $3\frac{\varepsilon_0 A}{d}$ (4) $4\frac{\varepsilon_0 A}{d}$

Q.15 In the adjoining figure, the emf of the cell is 1.8 V and the internal resistance is (2/3) Ω. The current in 3 Ω resistance is :



(1) 0.4 A (2) 0.8 A
(3) 0.2 A (4) 0.1 A

Q.16 The current through the circuit shown in figure is 1 A. If each of the 4Ωresistor is replaced by 2Ωresistor, the current in circuit will become nearly :



(1) 1.11 A (2) 1.25 A
(3) 1.34 A (4) 1.68 A

Q.17 A particle of mass m, charge Q and kinetic energy T enters a transverse uniform magnetic field of induction $\vec{B}$. After 3 seconds the kinetic energy of the particle will be :

(1) T (2) 4T (3) 3T (4) 2T

Q.18 Let $\vec{E}$ and $\vec{B}$ denote the electric and magnetic fields in a certain region of space. A proton moving with a velocity $\vec{v}$ along a straight line enters the region and is found to pass through it undeflected. Indicate which of the following statements are consistent with the observations?

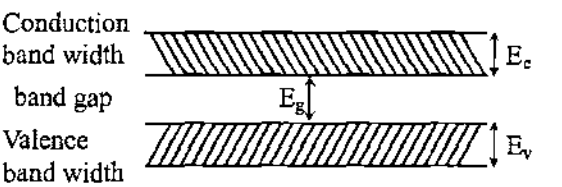
(i) $\vec{E} = 0$ (ii) $\vec{E} \neq 0$ (iii) $\vec{E} \neq 0$ (iv) $\vec{E}$, $\vec{v}$

(1) (iii, iv) (2) (i, ii, iv)
(3) (i, ii, iii) (4) (ii, iv)

Q.19 If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities?

(1) $\frac{5}{8}$ (2) $\frac{4}{5}$ (3) $\frac{5}{4}$ (4) $\frac{4}{7}$

Q.20 If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(1) All E_c, E_g, E_v increase
(2) E_c and E_v increase, but E_g decreases
(3) E_c and E_v decrease, but E_g increases
(4) All E_c, E_g, E_v decrease

CHEMISTRY

Q.21 Select correct statement about hydrolysis of BCl_3 and NCl_3 :

(1) NCl_3 is hydrolysed and gives HOCl but BCl_3 is not hydrolysed.
(2) Both NCl_3 and BCl_3 on hydrolysis gives HCl.
(3) NCl_3 on hydrolysis gives HOCl but BCl_3 gives HCl.
(4) Both NCl_3 and BCl_3 on hydrolysis gives HOCl.

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Q.22 The geometry with respect to the central atom of the following molecules are :
$N(SiH_3)_3$; Me_3N ; $(SiH_3)_3P$
(1) Pyramidal, Pyramidal, Planar
(2) Planar, Pyramidal, Planar
(3) Pyramidal, Pyramidal, Pyramidal
(4) Pyramidal, Planar, Pyramidal

Q.23 A complex has a composition corresponding to the formula $CoBr_2Cl.\ 4NH_3$. What is the structural formula if conductance measurements show two ions per formula unit? Silver nitrate solution gives an immediate precipitate of AgCl but no AgBr.
(1) $[CoBrCl(NH_3)_4]\ Br$
(2) $[CoCl(NH_3)_4]\ Br_2$
(3) $[CoBr_2Cl\ (NH_3)_4]$
(4) $[CoBr_2(NH_3)_4]Cl$

Q.24 In $[Pt(NH_3)_2Cl_2]$, Pt-Cl bond length is 2Å and Cl–Cl distance is 2.88 Å then the compound is
(1) tetrahedral (2) square pyramidal
(3) cis-square planar (4) trans-square planar

Q.25 When ZnS and PbS minerals are present together, then NaCN is added to separate them in the froth floatation process as a depressant, because :
(1) $Pb(CN)_2$ is precipitated while no effect on ZnS
(2) ZnS forms soluble complex $Na_2[Zn(CN)_4]$
(3) PbS forms soluble complex $Na_2[Pb(CN)_4]$
(4) They cannot be separated by adding NaCN

Q.26 The electrolysis of pure alumina is not feasible because :
(1) it is bad conductor of electricity and its fusion temperature is high
(2) it is volatile in nature
(3) it is decomposed when fused
(4) it is amphoteric

Q.27 Which of the following is correct statement ?
(1) F_2 has higher dissociation energy than Cl_2
(2) F has higher electron affinity than Cl
(3) HF is stronger acid than HCl
(4) Boiling point increase down the group in halogens

Q.28 The concentration of a substance undergoing a chemical reaction becomes one-half of its original value after time t regardless of the initial concentration. The reaction is an example of a-
(1) zero order reaction
(2) first order reaction
(3) second order reaction
(4) third order reaction

Q.29 In the nuclear reaction ${}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + X$. X is-
(1) ${}^{0}_{-1}e$ (2) ${}^{0}_{1}e$
(3) H (4) ${}^{2}_{1}H$

Q.30 Zinc sulphate contains 22.65% of zinc and 43.9 % of water of crystallization. If the law of constant proportions is true, then the weight of zinc required to produce 20 g of the crystal will be-
(1) 45.3 g (2) 4.53 g
(3) 0.453 g (4) 453 g

Q.31 In what ratio by mass carbon monoxide and nitrogen should be mixed so that partial pressure exerted by each gas is same ?
(1) 1 : 1 (2) 1 : 2
(3) 2 : 1 (4) 3 : 4

Q.32 Water and chlorobenzene are immiscible liquids. Their mixture boils at 90° C under a reduced pressure of 9.58×10^4 Pa. The vapour pressure of pure water at 90°C is 7.03×10^4 Pa and molecular weight of chlorobenzene is 112.5. On what precent basis, chlorobenzene in the distillate is equal to-
(1) 50 (2) 60
(3) 70 (4) 80

Q.33 ΔC_p for a reaction is given by 2.0 + 0.2 T cal/deg. Its enthalpy of reaction at 10 K is –14.2 kcal. Its enthalpy of reaction at 100 K in kcal will be
(1) –13.03 (2) –15.37
(3) 16.02 (4) 07.08

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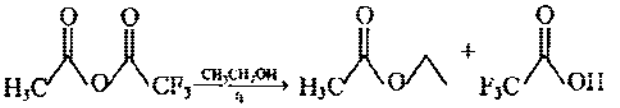
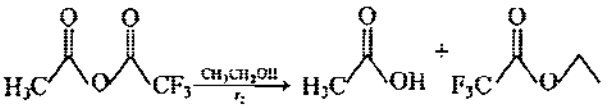
Q.34 Consider the following reactions :

Reaction I :



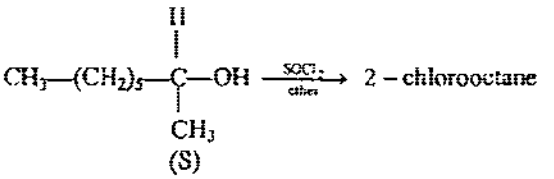
Reaction II :



Which of the following is a correct comparison of the rate of reactions I and II ?

(1) $r_I = r_{II}$ (2) $r_I > r_{II}$ (3) $r_I < r_{II}$ (4) None

Q.35 Consider the given reaction



Which is the following statement is true for the above reaction?

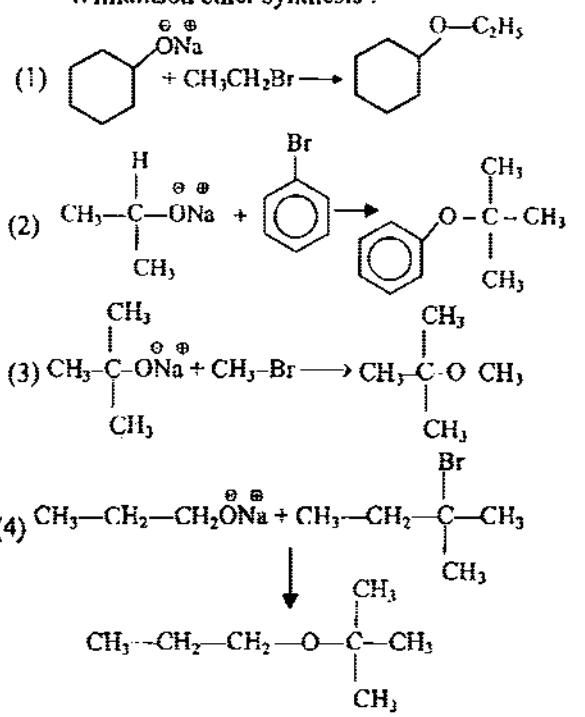
(1) Configuration of the product is S

(2) Configuration of the product is R

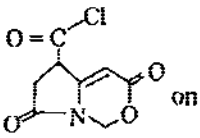
(3) Product is 1 : 1 racemic mixture

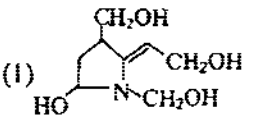
(4) Product is 2 : 1 racemate

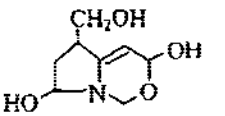
Q.36 Consider the following reactions involving Williamson ether synthesis :

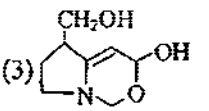


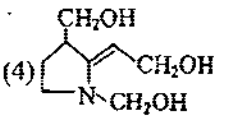
Which of the above reactions constitute good synthesis?

Q.37 Compound  on reduction with $LiAlH_4$ produces –

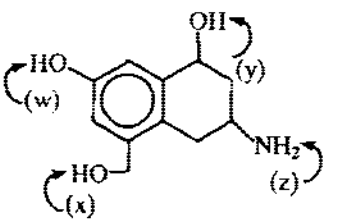
(1) 

(2) 

(3) 

(4) 

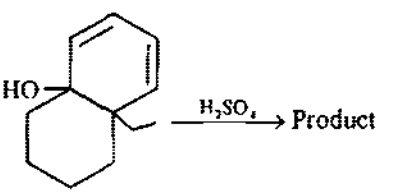
Q.38 Decreasing order of acidic strength of different groups is –

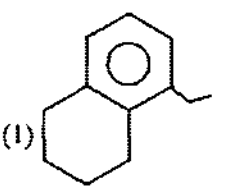


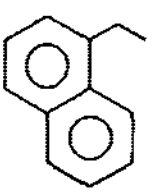
(1) w > x > y > z (2) w > z > x > y

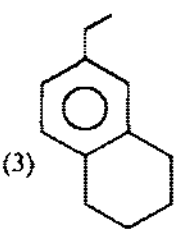
(3) z > w > x > y (4) z > x > w > y

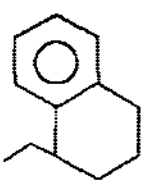
Q.39 Identify the major product :



(1) 

(2) 

(3) 

(4) 

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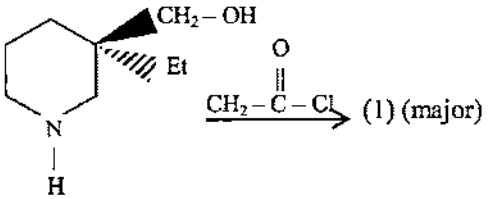
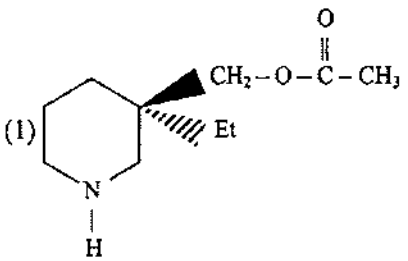
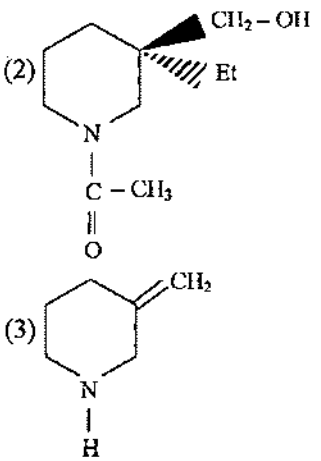
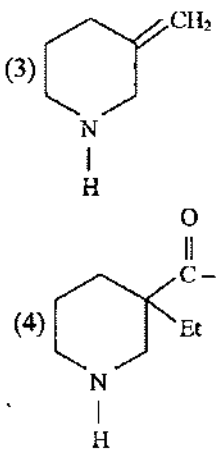
Q.40









MATHEMATICS

Q.41 What is the function whose graph is the reflection about the line $x + y = 0$ of the inverse function $f^{-1}(x)$ of a function $f(x)$:

(1) $-f(x)$ (2) $-f(-x)$
(3) $-f^{-1}(x)$ (4) $-f^{-1}(-x)$

Q.42 If the chords of contact of tangents from two points $(-4, 2)$ and $(2, 1)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are at right angle, then the eccentricity of the hyperbola is -

(1) $\frac{\sqrt{7}}{2}$ (2) $\sqrt{\frac{5}{3}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\sqrt{2}$

Q.43 If $f(x) = 2e^x - c \ln x$ monotonically increases for every $x \in (0, \infty)$, then the true set of values of c is -

(1) $\left(-\infty, \frac{-1}{e}\right)$ (2) $\left(-\infty, \frac{-2}{e}\right)$
(3) $[0, \infty)$ (4) $(-\infty, 0]$

Q.44 Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis in P_1 and P_2. If S is the focus of the parabola, then $\frac{1}{(SP_1)} + \frac{1}{(SP_2)}$ is equal to -

(1) $\frac{4}{a}$ (2) $\frac{2}{a}$ (3) $\frac{1}{a}$ (4) $\frac{1}{4a}$

Q.45 Range of $y = \cos\left(\pi \sin\left(\frac{\pi}{2}\cos(\pi \sin x)\right)\right)$, where $x \in R$, is -

(1) $[-\pi, \pi]$ (2) $[-1, 1]$
(3) $[0, 1]$ (4) $[-1, 0]$

Q.46 A circle C_1 is drawn having any point P on x-axis as its centre and passing through the centre of circle C: $x^2 + y^2 = 1$. A common tangent to C_1 and C touches circles at Q and R respectively. Then Q (x, y) also satisfies -

(1) $x^2 = 1$ (2) $x^2 + y^2 = 1$
(3) $y^2 - 1 = 0$ (4) None of these

Q.47 The value of $\lim_{x\to\infty}\left\{\left(x+\frac{1}{x}\right)e^{\frac{1}{x}}-x\right\}$ is equal to (where {.} denotes fractional part of function) :

(1) 0 (2) 1
(3) 1/2 (4) does not exist

Q.48 Six people, all of different weights, are trying to build a human pyramid, that is they get into the formation

A
B C
D E F

We say that some one not in the bottom row is "supported by" each of the two closest people beneath her or him. The number of possible different pyramids, if nobody can be supported by anybody of lower weight, are-

(1) 24 (2) 18 (3) 16 (4) 14

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Q.49 Let $f(x) = \begin{cases} (x+2)^3, & -3 < x \le -1 \\ x^{\frac{2}{3}}, & -1 < x < 2 \end{cases}$ and

$g(x) = \int_{-3}^{x} f(t)dt, \ -3 < x < 2.$

Then the number of extremum points of g'(x) is
(1) 0 (2) 1 (3) 2 (4) 3

Q.50 If f(x) is continuous and increasing function such that domain of $g(x) = \sqrt{f(x) - x}$ be R and $h(x) = \frac{1}{1-x}$, then the domain of $\phi(x) = \sqrt{f(f(f(x))) - h(h(h(x)))}$ is -
(1) R (2) {0, 1}
(3) R – {0, 1} (4) R – (0, 1)

Q.51 The probability that the length of a randomly chosen chord of a circle lies between 2/3 and 5/6 of its diameter is -
(1) 1/4 (2) 5/12 (3) 1/16 (4) 5/16

Q.52 The sum of all the roots of the equation $\sin\left(\pi \log_3\left(\frac{1}{\pi}\right)\right) = 0$ in $(0, 2\pi)$, is -
(1) 3/2 (2) 4 (3) 9/2 (4) 13/3

Q.53 Solution set of the equation $3^{2x^2} - 2.3^{x^2+x+6} + 3^{2(x+6)} = 0$ is -
(1) {–3, 2} (2) {6, –1}
(3) {–2, 3} (4) {1, –6}

Q.54 If $a, b, c, d \in R$ such that $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$, then the equation $ax^3 + bx^2 + cx + d = 0$ has -
(1) at least one root in (–1, 0)
(2) at least one root in (0, 1)
(3) no root in (–1, 1)
(4) no root in (0, 2)

Q.55 If matrix $A = [a_{ij}]_{3\times3}$, matrix $B = [b_{ij}]_{3\times3}$, where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0$, then $A^4 . B^3$ is -
(1) skew-symmetric matrix
(2) singular matrix
(3) symmetric matrix
(4) zero matrix

Q.56 Suppose that 'a' and 'b' are two roots of the equation $x^2 + 3x + 5 = 0$. If $t = \frac{a+2}{b+2}$ and $s = \frac{b+2}{a+2}$ are the two roots of $x^2 - mx + 1 = 0$, then the value of 'm' is -
(1) –5/2 (2) –5/3 (3) 5/2 (4) 5/3

Q.57 If $f(x) = ae^{2x} + be^x + cx$ satisfies the conditions $f(0) = -1, f'(\log 2) = 31$ & $\int_0^{\ell n 4} (f(x) - cx)\,dx = \frac{39}{2}$, then -
(1) $a = 4$ (2) $b = -6$
(3) $c = 2$ (4) $a = 3$

Q.58 If a, b, c are first three terms of a GP, the harmonic mean of a and b is 12, and arithmetic mean of b and c is 3, then -
(1) no term of this GP is square of an integer
(2) arithmetic mean of a, b, c is 7
(3) $b = \pm 6$
(4) common ratio of this GP is 2

Q.59 Let α and β be the solutions of the quadratic equation $x^2 - 1154x + 1 = 0$, then the value of $\sqrt[4]{\alpha} + \sqrt[4]{\beta}$ is equal to -
(1) 4 (2) 5 (3) 6 (4) 8

Q.60 Let $u = (\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$ has
(1) no solution for x
(2) exactly one solution for x
(3) exactly two distinct solutions for x
(4) exactly three distinct solutions for x

BIOLOGY

Q.61 Sexual reproduction leads to :
(1) euploidy
(2) polyploidy
(3) aneuploidy
(4) genetic recombination

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Q.62 Formation of an embryo from synergid or any other haploid cell refers to :
(1) Parthenocarpy
(2) Parthenogenesis
(3) Apogamy
(4) Both (2) and (3) correct

Q.63 The function of vas deferens is to :
(1) store the sperms
(2) mature the sperms
(3) conduct the sperms
(4) none of these

Q.64 Labium majora of a female mammal is homologous to :
(1) scrotal sac (2) prostate gland
(3) epididymis (4) seminal vesicle

Q.65 Contraceptive oral pills used by females help in birth control by :
(1) killing of ova
(2) killing of sperms
(3) preventing ovulation
(4) forming barrier between sperms and ova

Q.66 Which of the following is the monohybrid ratio?
(1) 3 : 1 (2) 9 : 7
(3) 1 : 2 (4) 9 : 3 : 3 : 1

Q.67 Mendel's law of independent assortment can be demonstrated by :
(1) test cross (2) dihybrid cross
(3) back cross (4) monohybrid cross

Q.68 When closely placed genes on the same chromosome are inherited together the phenomenon is known as :
(1) linkage (2) crossing over
(3) gene interaction (4) multiple allelism

Q.69 In man, chromosomes are :
(1) metacentric (2) acrocentric
(3) submetacentric (4) all of these

Q.70 Double helical model of DNA molecule was proposed by :
(1) Erwin Chargaff
(2) Watson and Crick
(3) Heinz Fraenkel Conrat
(4) Singer and Nicolson

Q.71 The organs which look different but have the same basic structure and origin are known as :
(1) homologous (2) vestigial
(3) heterologous (4) analogous

Q.72 Which one of the following is not a vestigial structure in *Homo sapiens*?
(1) Epiglottis (2) Third molar
(3) Plica semilunaris (4) Pyramidalis muscle

Q.73 Cholera, leprosy and diphtheria are :
(1) viral diseases (2) fungal diseases
(3) bacterial diseases (4) functional diseases

Q.74 Most abundant immunoglobulin is :
(1) IgG (2) IgD
(3) IgA (4) IgE

Q.75 Inbreeding is possible between two members of a :
(1) species (2) genus
(3) family (4) none of these

Q.76 The most important livestocks of India are :
(1) cattle and cat (2) cattle and elephant
(3) cattle and dog (4) cattle and buffalo

Q.77 The technique which involves addition or deletion of genes is :
(1) gene therapy
(2) gene splicing
(3) genetic engineering
(4) artificial synthesis

Q.78 Manipulation of DNA in genetic engineering became possible due to the discovery of :
(1) primase
(2) DNA ligase
(3) transcriptase
(4) restriction endonuclease

Q.79 Water pollution on the surface of pond can be minimised by :
(1) Lichen (2) Hydrilla
(3) Phytoplankton (4) Bacteria and Algae

Q.80 Which is a degradable pollutant?
(1) DDT (2) Mercury salts
(3) Domestic wastes (4) Aluminium oil

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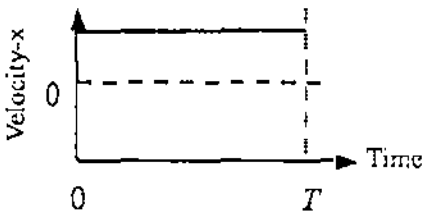
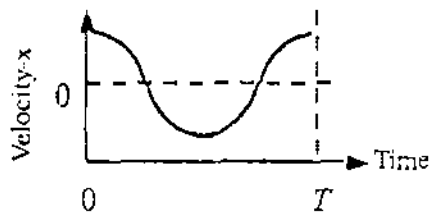
PART- II

Questions given below are single correct type Questions. Each correct answer carries +2. 0.5 marks will be deducted for each wrong answers.

PHYSICS

Q.81 A particle moves constantly in a circle centered at the origin with a period T. If its position at time t = 0 seconds is (A, 0) meters, which graph represents v_x, the x-component of the particle's velocity, as a function of time?

(1)


(2)


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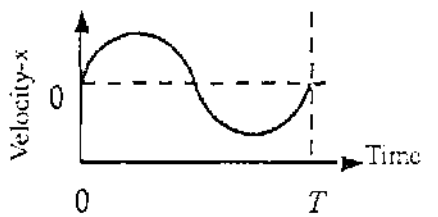
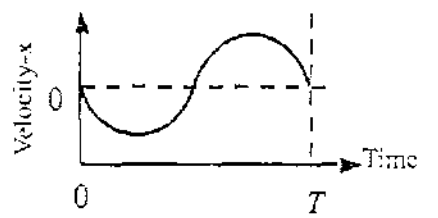
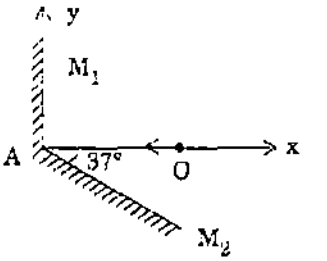


(4)


Q.82 Two plane mirrors are placed as shown in figure. A point object O is approaching the intersection point A of mirrors with a speed of 100 cm/s. The velocity of image of the object formed by M_2 with respect to velocity of image of object formed by M_1 is :



(1) $-128\hat{i}+96\hat{j}$ cm/s (2) $-28\hat{i}-48\hat{j}$ cm/s

(3) $128\hat{i}-48\hat{j}$ cm/s (4) $100\hat{i}+48\hat{j}$ cm/s

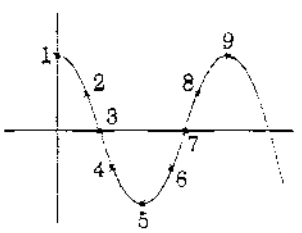
Q.83 A nut is screwed onto a bolt with 12 turns per cm and diameter more than 1 cm. The bolt is lying in a horizontal position. The nut spins at 216 rpm. Time taken by the nut to cover 3 cm along the bolt is :

(1) 10 s (2) 12 s (3) 14 s (4) 16 s

Q.84 A motor bike accelerates from rest at a constant rate of 3 ms^{-2} for some time and then moves with uniform velocity for the same duration. Then it retards at a constant rate of 6 ms^{-2} and comes to rest. The bike was in motion for a period of 5 seconds. The total distance travelled by the bike is :

(1) 21 m (2) 24 m (3) 27 m (4) 30 m

Q.85 Figure shows a sinusoidal wave of period T travelling to the right along a string at time t = 0. Which of the following statement is incorrect?



(1) The point 3 on the string is moving upward with maximum speed

(2) The point 5 on the string has the greatest upward acceleration

(3) The point 9 on the string has the greatest downward acceleration

(4) The point 2 on the string has a downward velocity and upward acceleration

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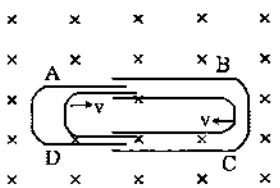
Q.86 What is the minimum number of geostationary satellites required for uninterrupted global coverage?
(1) 3 (2) 5 (3) 7 (4) 9

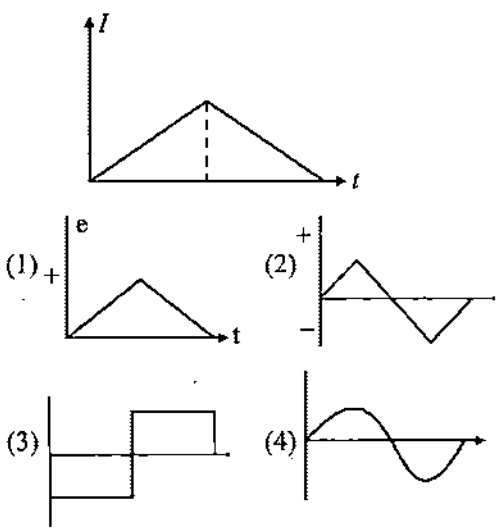
Q.87 For sky wave propagation of 10 MHz signal, what should be the minimum electron density in ionosphere ?
(1) 1.2×10^{12} m^{-3} (2) 10^{6} m^{-3}
(3) 10^{14} m^{-3} (4) 10^{22} m^{-3}

Q.88 One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be :

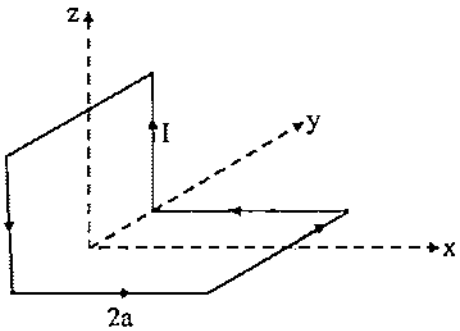


(1) Blv (2) $-Blv$ (3) zero (4) $2Blv$

Q.89 The current I in an inductance coil varies with time t according to the graph shown in figure. Which one of the following plots shpws the variation of voltage in the coil with time?



Q.90 A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop of length 2a. The magnetic field due to this loop at the point P(a, 0, a) points in the direction :



(1) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (2) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$
(3) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (4) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

CHEMISTRY

Q.91 SbF_5 reacts with XeF_4 to form an adduct. The shapes of cation and anion in the adduct are respectively :
(1) square planar, trigonal bipyramidal
(2) T-shaped, octahedral
(3) square pyramidal, octahedral
(4) square planar, octahedral

Q.92 Which of the following statements are correct about Zn, Cd and Hg ?
(I) they exhibit high enthalpies of atomization as the d-subshell is full
(II) Zn and Cd do not show variable oxidation states while Hg shows + I and + II
(III) Compound of Zn, Cd and Hg are paramagnetic in nature
(IV) Zn, Cd and Hg are called soft metals
(1) I, II, III (2) I, III
(3) II, IV (4) IV only

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Q.93 Silicate having one monovalent corner oxygen atom in each tetrahedron unit is :
(1) sheet silicate
(2) cyclic silicate
(3) single chain silicate
(4) double chain silicate

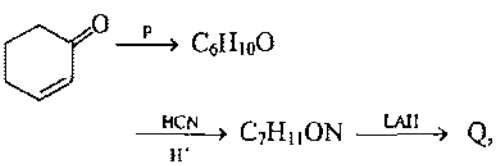
Q.94 The solubility product of $Mg(OH)_2$ is 1.0×10^{-12}. Concentrated aqueous NaOH solution is added to a 0.01 M aqueous solution of $MgCl_2$. The pH at which precipitation occur is-
(1) 7.2 (2) 7.8 (3) 8.0 (4) 9.0

Q.95 A metal with an atomic radius of 141.4 pm crystallizes in the face centred cubic structure. The volume of the unit cell in pm^3 is-
(1) 2.74×10^7 (2) 2.19×10^7
(3) 6.40×10^7 (4) 9.20×10^7

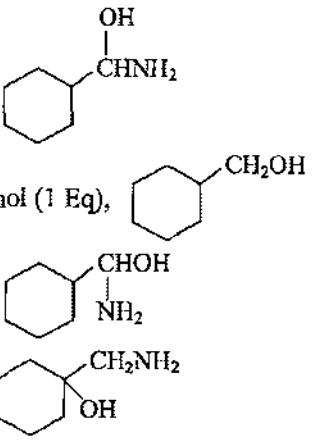
Q.96 A saturated solution of H_2S in 0.1 M HCl at 25°C contains a S^{2-} ion concentration of 10^{-23} mol L^{-1}. The solubility products of some sulphides are : CuS = 10^{-44}, FeS = 10^{-14}, MnS = 10^{-15}, CdS = 10^{-25}. If 0.01 M solutions of these salts in 0.1 M HCl are saturated with H_2S, which of these will be precipitated ?
(1) All
(2) All except MnS
(3) All except MnS and FeS
(4) Only CuS

Q.97 Given standard electrode potentials
$Fe^{2+} + 2e^- \rightarrow Fe$; $E^\circ = -0.44$ V
$Fe^{3+} + 3e^- \rightarrow Fe$; $E^\circ = -0.036$ V
The standard electrode potential (E°) for $Fe^{3+} + e \rightarrow Fe^{2+}$ is-
(1) – 0.476 V (2) – 0.404 V
(3) + 0.404 V (4) + 0.772 V

Q.98



P and Q are -

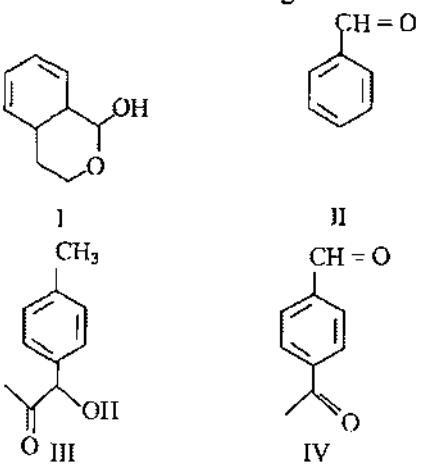


(1) $NaBH_4$ (1 Eq),
(2) $Ni–H_2$, in ethanol (1 Eq),
(3) $LiAlH_4$ (1 Eq),
(4) $Pd–H_2$ (1 Eq),

Q.99 Analyse the given information :

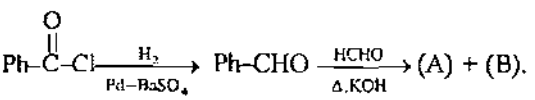
	2,4-dinitro Phenyl hydrazine	Fehling's solution test	Iodoform test	Tollen's test
(A)	turns red	–ve	+ve	+ve
(B)	turns red	+ve	+ve	+ve
(C)	turns red	–ve	– ve	+ve

and consider the following molecules :



I II III IV

(1) A-IV; B-III; C-I (2) A-IV; B-III; C-II
(3) A-III; B-IV; C-II (4) A-III; B-IV; C-I

Q.100 

Product (A) & (B) are -
(1) $Ph–CO_2H$, Ph–OH
(2) $Ph–CO_2^-$, HCO_2^-
(3) $Ph–CH_2OH$, $H–CO_2^-$
(4) $Ph–CO_2^-$, CH_3OH

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MATHEMATICS

Q.101 The numbers 1, 2, 3,....., n are arrange in random order. The probability that the digits 1, 2, 3,...., k (k < n) appear as neighbours in that order is-

(1) 1/n! (2) k!/n!

(3) (n – k)!/ n! (4) (n – k + 1)!/n!

Q.102 If f(x) satisfies $x + |f(x)| = 2f(x)$ then $f^{-1}(x)$ satisfies

(1) $3x + |f^{-1}(x)| = 2f^{-1}(x)$

(2) $x + |f^{-1}(x)| = 2f^{-1}(x)$

(3) $f^{-1}(x) - |x| = 2x$

(4) $3x - |f^{-1}(x)| = 2f^{-1}(x)$ where $f(x) \geq 0$

Q.103 A function y = f(x) satisfies the condition $f'(x) \sin x + f(x) \cos x = 1$, f(x) being bounded when x > 0. if $I = \int_0^{\pi/2} f(x)\,dx$, then

(1) $\frac{\pi}{2} < I < \frac{\pi^2}{4}$ (2) $\frac{\pi}{4} < I < \frac{\pi^2}{2}$

(3) $1 < I < \frac{\pi}{2}$ (4) $0 < I < 1$

Q.104 If *A*, *B* & *C* are matrices of order 2 such that $|A| = \frac{1}{8}$, $|B| = 5$, $|C| = 3$ then $|(5AC)(2B)^{-1}|$ is equal to:

(1) $\frac{15}{16}$ (2) $\frac{3}{4}$ (3) $\frac{5}{8}$ (4) $\frac{15}{32}$

Q.105 A lamp of negligible height is placed on the ground λ_1 m away from a wall. A man λ_2 m tall is walking at a speed of $\frac{\ell_1}{10}$ m/s from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is:

(1) $-\frac{5\ell_2}{2}$ m/s (2) $-\frac{2\ell_2}{5}$ m/s

(3) $-\frac{\ell_2}{2}$ m/s (4) $-\frac{\ell_2}{5}$ m/s

Q.106 $\int \sqrt{\frac{(2012)^{2x}}{1-(2012)^{2x}}}\,(2012)^{\sin^{-1}(2012)^x}\,dx =$

(1) $(\log_{2012} e)\,(2012)^{\sin^{-1}(2012)^x} + C$

(2) $(\log_{2012} e)^2\,(2012)^{x+\sin^{-1}(2012)^x} + C$

(3) $(\log_{2012} e)^2\,(2012)^{\sin^{-1}(2012)^x} + C$

(4) $\frac{(2012)^{\sin^{-1}(2012)^x}}{(\log_{2012} e)^2} + C$

Q.107 If the slope of chord PQ of $f(x) = x^3 - 2x^3 + 10$ is 9, then relation between the AM(A) and GM(G) of abscissae of points P and Q is-

(1) $9G^2 - (7A^2 - G^2)(G^6 + 2) = 0$

(2) $6G^6 - (7A^2 - G^2)(G^6 + 2) = 0$

(3) $9G^6 - (4A^2 - G^2)(G^6 + 2) = 0$

(4) $6G^6 - (4A^2 - G^2)(G^6 + 2) = 0$

Q.108 Let $f(x) = \int_0^x \cos\left(\frac{t^2 + 2t + 1}{5}\right) dt$, $0 < x < 2$.

Then $f(x)$

(1) increases monotonically

(2) decreases monotonically

(3) has one point of local maximum

(4) has one point of local minimum

Q.109 The value of the sum

$$\frac{1}{3^2+1} + \frac{1}{4^2+2} + \frac{1}{5^2+3} + \frac{1}{6^2+4} + \ldots \infty,$$

is equal to -

(1) $\frac{13}{36}$ (2) $\frac{12}{36}$

(3) $\frac{15}{36}$ (4) $\frac{18}{36}$

Q.110 If $f(x) = [x] + [x + 1/3] + [x + 2/3]$, then ([.] denotes the greatest integer function)

(1) $f(x)$ is continuous at $x = 1, 10, 15$

(2) $f(x)$ is continuous at $x = n/3$, where n is any integer

(3) $\int_0^{2/3} f(x)dx = 1/3$

(4) $\lim_{x \to 2/3} f(x) = 2$

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BIOLOGY

Q.111 Allium type bisporic type female gametophyte is always :
(1) 8-nucleate and 7-celled
(2) 16-nucleate and 13-called
(3) 8-nucleate and 8-celled
(4) 4-nucleate and 4-celled

Q.112 Withdrawal of which of the following hormones is the immediate cause of menstruation?
(1) FSH (2) FSH-RH
(3) Estrogen (4) Progesterone

Q.113 In human being, brown eye (B) is dominant over blue eye (b). A brown-eyed couple has a blue-eyed child. What is the possible genotypes of the couple?
(1) Bb × bb (2) Bb × bB
(3) BB × bb (4) BB × bB

Q.114 Radioactive thymine when added to the medium surrounding living mammalian cell gets incorporated into a newly synthesized DNA. Which of the following type of chromatin is expected to be radioactive if cells are exposed to radioactive thymidine as soon as they enter the S-phase?
(1) Euchromatin
(2) Heterochromatin
(3) Both euchromatin and heterochromatin
(4) Neither euchromatin or heterochromatin

Q.115 Syndrome in human in which an individual somatic cell contain only one sex chromosome XO is called :
(1) Patau syndrome
(2) Down syndrome
(3) Turner syndrome
(4) Klinefelter syndrome

Q.116 In DNA fingerprinting which of the following is true?
(1) House keeping or luxury genes are uses as probes
(2) Specific metabolic genes are used as probes
(3) VNTR is used as probes
(4) All of the above

Q.117 Among the human ancestors the brain size was more than 1000 cc in :
(1) Ramapithecus
(2) Homo habilis
(3) Homo erectus
(4) Homo neanderthalensis

Q.118 What are true of plasmids ?
(1) They are found in viruses
(2) They are main parts of chromosomes
(3) They are widely used in gene transfer
(4) They contain gene for vital activities

Q.119 Which of the following toxic materials was present in Minamata bay of Japan?
(1) Hg (2) Mg
(3) Cd (4) Pb

Q.120 "itai-itai" ("ouch-ouch") disease is caused by the poisoning of :
(1) Lead (2) Mercury
(3) Chromium (4) Cadmium

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ANSWERS

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	3	1	3	4	3	2	2	4	3	3	3	2	1	1	1	3	3	3
Qus.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	2	4	3	2	1	4	2	1	2	1	3	1	3	1	3	4	1	1	2
Qus.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	3	4	3	2	1	1	3	3	3	1	3	3	2	2	2	2	1	3	2
Qus.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	4	4	3	1	3	1	2	1	4	2	1	1	3	1	1	4	1	4	4	3
Qus.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	4	1	1	1	4	1	1	4	3	4	2	3	1	4	3	3	4	4	1	3
Qus.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	4	4	1	4	2	3	3	3	1	3	1	4	2	1	3	3	4	3	1	4

SOLUTIONS

PART-I
One Mark Questions

PHYSICS

1. The moment of inertia for the system can be calculated by adding the two individual moments of inertia as following :

$I_{total} = I_0 + I_{clay}$

$I_{clay} = MR^2$

$R = \sqrt{2}\frac{L}{2}$

$I_{clay} = M\left(\sqrt{2}\frac{L}{2}\right)^2 = \frac{ML^2}{2}$

$I_{total} = I_0 + \frac{ML^2}{2}$

2. The key to finding the coefficient of friction μ is in calculating the correct normal force acting on the block

$\Sigma F_y = ma$

$-F_{applied-y} - F_g + F_{normal} = 0$

$F_{normal} = F\sin\phi + mg$

$\mu = \frac{F_{friction}}{F_{normal}} = \frac{f}{F\sin\phi + mg}$

3. The diagram suggests that the box is currently moving to the right, and in the process of slowing down due to a force of friction that is causing it to accelerate in the opposite direction (slowing down the box).

4. By definition, the object is in equilibrium, either static (unmoving) or dynamic (moving with a constant velocity).

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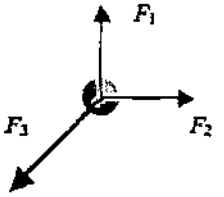
If the object has acceleration a = 0, the net force acting on the mass must be 0 as well :



$F_{net} = ma$

$F_{net} = m(0) = 0$

with forces F_1 and F_2 in the x-y plane, the force that will counteract them must lie in the x-y plane as well, as shown. The magnitude of that force F_3 is equal to the vector sum of F_1 and F_2 and can be calculated as follows :

$$\sum F_x = 0 = \vec{F}_2 - \vec{F}_{3-x}$$

$$\sum F_y = 0 = \vec{F}_1 - \vec{F}_{3-y}$$

$$|F_3| = \sqrt{F_{3-x}^2 + F_{3-y}^2}$$

$$= \sqrt{F_1^2 + F_2^2}$$

$$= F_1\sqrt{2}$$

both graphically and analytically, we can see that the magnitude of F_3 is not the same as that of F_1.

5. Mass of the cube = 30 g

length of each face of the cube = 4 cm

let y be the distance of the lower face of the cube from the surface of water.

When the cube floats freely in water

$a^2y\rho g = mg + 4a\sigma$

$$\therefore y = \frac{mg + 4a\sigma}{a^2\rho g}$$

$$= \frac{30\times 980 + 4\times 4\times 70}{4\times 4\times 1\times 980}$$

$$\frac{29400 + 1120}{15680}$$

$$\frac{30520}{15680} = 1.95 \text{ cm}$$

6. $u = 21$ cm ; $f = \frac{R}{2} = \frac{10}{2} = 5$ cm

on introducing the glass slab, the object as well as the image will be shifted from the mirror through a distance

$$d = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1 \text{ cm},$$

so that apparent distance of the object = 20 cm (i.e.) u = 20 cm

By the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$v = \frac{20}{3} \text{ cm} = 6.67 \text{ cm}$$

$\therefore$ distance of the final image from the mirror = 6.67 + 1 = 7.67 cm.

7. The length of the pendulum hasn't changed and the only other factor that determines the period of a pendulum is the acceleration due to gravity

$$T' = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T' = 2\pi\sqrt{\frac{\ell}{\frac{1}{6}g}}$$

$$T' = \sqrt{6}\left(2\pi\sqrt{\frac{\ell}{g}}\right)$$

$$T' = \sqrt{6}\,T$$

8. As $W = \Delta K$

Force is along negative x-axis and displacement is along + x-axis

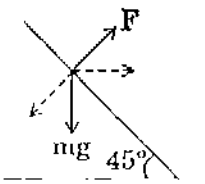
$\therefore$ W = negative

Hence

ΔK = negative

9. Lift force $F = \frac{mg}{\cos 45°}$

The centripetal force is provided by the component of weight



$$F\cos 45° = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{mg} = \frac{v^2}{g} = \frac{\left(540\times\frac{5}{18}\right)^2}{10} = 2250 \text{ m}$$

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10. $n = \frac{p}{2\ell}\sqrt{\frac{T}{m/\ell}} = \frac{p}{2}\sqrt{\frac{T}{m\ell}}$

$10^4 = \frac{p}{2}\sqrt{\frac{800}{2\times10^{-3}\times0.4}}$

$\Rightarrow p = 20$

The highest harmonic that could be heard $= 20$

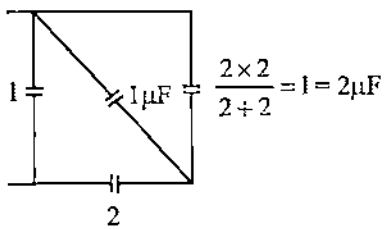
11. $E_{axis} = \frac{2kpr}{(r^2 - a^2)^{3/2}}$

$r = 20 + 5$

$a = 5$ cm

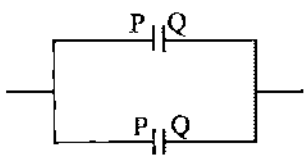
12. $2\left(\frac{Kq}{L} - \frac{Kq}{\sqrt{5}L}\right)$

13.



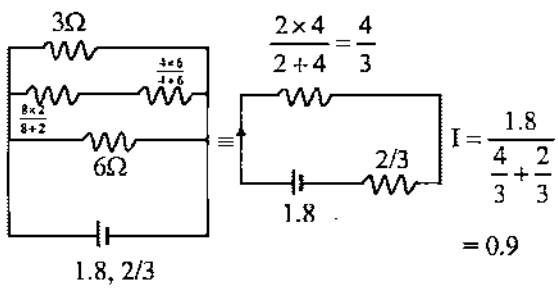
$\frac{2\times2}{2+2} = 1 = 2\mu F$

14.



$C_{net} = 2C = \frac{2\epsilon_0 A}{d}$

15.



$\frac{2\times4}{2+4} = \frac{4}{3}$

$I = \frac{1.8}{\frac{4}{3}+\frac{2}{3}} = 0.9$

0.9 is divided in ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{6}$

17. K.E. is constant in magnetic field

18. $F_L = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{V}\times B)$

19. $I = ncAv_d$

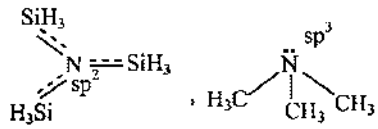
$\frac{I_1}{I_2} = \frac{n_1}{n_2}\times\frac{v_{d_1}}{v_{d_2}} \Rightarrow \frac{v_{d_1}}{v_{d_2}} = \frac{I_1}{I_2}\times\frac{n_2}{n_1}$

CHEMISTRY

21. $NCl_3 + 3H_2O \rightarrow NH_3 + 3HOCl$

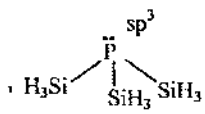
$BCl_3 + 3H_2O \rightarrow 3HCl + H_3BO_3$

22.



pπ - dπ back bonding

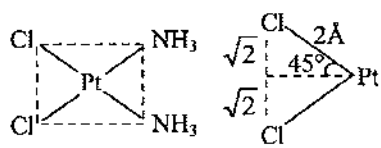
No back bonding



No back bonding

23. $[CoBr_2(NH_3)_4]Cl$

24.



28. First order

$t_{1/2} = \frac{0.693}{k}$

29. $Z = 90 - 91 = -1$

$A = 234 - 234$

$= 0$

30. To prepare 20 g of the crystals,

zinc required $= \frac{22.65}{100}\times 20$

$= 4.53$ g

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31. $CO = w_1 g = \frac{w_1}{28}$ mol,

$N_2 = w_2 g = \frac{w_2}{28}$ mol,

$P_{CO} = \frac{w_1/28}{w_1/28 + w_2/28}$

$= \frac{w_1}{w_1 + w_2}$

Similarly $p_{N_2} = \frac{w_2}{w_1 + w_2}$

As $p = P_{N_2}$,

Hence $w_1 = w_2$ i.e. $w_1 : w_2$

1 : 1

32. In steam distillation

$$\frac{\text{weight of liquid distilled}}{\text{weight of water distilled}} = \frac{P^\circ_l \times M_l}{P^\circ_w \times M_w}$$

$$= \frac{(9.58\times10^{-4} - 7.03\times10^{4})\times112.5}{7.03\times10^{4}\times18}$$

$$= \frac{2.27}{1}$$

i.e. in a total weight of 3.37 g, weight of chlorobenzene = 2.27 g

$\therefore$ % of chlorobenzene $= \frac{2.27}{3.37} \times 100$

$= 67.4$

$= 70\ \%$

33. By Kirchoff's equation,

$$\frac{\Delta H_2 - \Delta H_1}{T_2 - T_1} = \Delta C_P$$

or $\Delta H_2 - \Delta H_1 = \int_{T_1}^{T_2} \Delta C_p dT$

$$\int_{10K}^{100K} (2.0 + 0.2T) dT$$

$$= \left[2.0T + 0.2\frac{T^2}{2}\right]_{10K}^{100K}$$

$$= \left[2.0T + 0.1T^2\right]_{10K}^{100K}$$

$= [2.0 \times 100 + 0.1 \times (100)^2] - [2.0 + 0.1 \times 10^2]$

$= 1170$ cal

$= 1.17$ kcal

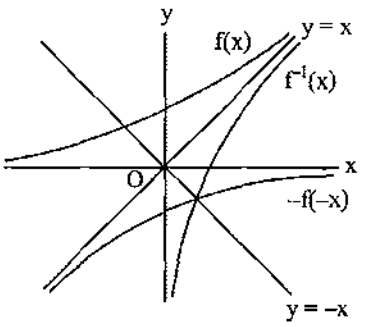
$\therefore \Delta H_2 = \Delta H_1 + 1.17$ kcal

$= -14.2 + 1.17$

$= -13.03$ kcal

MATHEMATICS

41.



42. Chord of contact w.r.t. (–4, 2) is $-\frac{4x}{a^2} - \frac{2y}{b^2} = 1$

Slope $m_1 = \frac{-2b^2}{a^2}$ ...(1)

Chord of contact w.r.t. (2, 1) is $\frac{2x}{a^2} - \frac{y}{b^2} = 1$

Slope $m_2 = \frac{2b^2}{a^2}$

$\therefore m_1 m_2 = -1$

$\frac{4b^4}{a^4} = 1$

$\Rightarrow 2b^2 = a^2$

$\Rightarrow 2a^2(e^2 - 1) = a^2$

$\Rightarrow e = \sqrt{\frac{3}{2}}$

43. $f(x) = 2e^x - c\ln x$

$f'(x) = 2e^x - \frac{c}{x} \geq 0$

$\Rightarrow \frac{2xe^x - c}{x} \geq 0$

$\Rightarrow c \leq 2xe^x$

$\Rightarrow c \leq 0$

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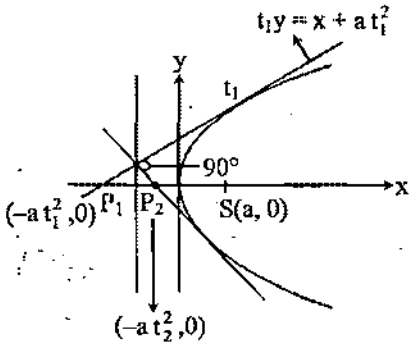
44. $SP_1 = a(1 + t_1^2)$; $SP_2 = a(1 + t_2^2)$

$t_1t_2 = -1$



$$\frac{1}{SP_1} = \frac{1}{a(1+t^2)}; \frac{1}{SP_2} = \frac{t^2}{a(1+t^2)}$$

$$\therefore \frac{1}{SP_1} + \frac{1}{SP_2} = \frac{1}{a}$$

45. $-\pi \le \pi \sin x \le \pi$

$\Rightarrow -1 \le \cos(\pi \sin x) \le 1$

$$\Rightarrow -1 \le \sin\left(\frac{\pi}{2}(\cos(\pi \sin x))\right) \le 1$$

$$\Rightarrow -\pi \le \pi \sin\left(\frac{\pi}{2}\cos(\pi \sin x)\right) \le \pi$$

$\Rightarrow -1 \le y \le 1$

46. Clearly tangent $xx_1 + yy_1 - \alpha(x + x_1) = 0$ at $Q(x_1, y_1)$ on the circle

$(x - \alpha)^2 + y^2 - \alpha^2 = 0$ is also tangent to $x^2 + y^2 - 1 = 0$

$$\Rightarrow \frac{|\alpha x_1|}{\sqrt{(x_1 - \alpha)^2 + y_1^2}} = 1$$

$\Rightarrow |x_1| = 1$

47. $$x = \frac{1}{t} \lim_{t \to 0} \left\{\left(\frac{1}{t} + t\right)e^t - \frac{1}{t}\right\}$$

$$\lim_{t \to 0} \left\{\frac{e^t + t^2e^t - 1}{t}\right\}$$

$$\lim_{t \to 0} \frac{e^t + 2te^t + t^2e^t}{1} = 1 \text{ (using L' Hospital rule)}$$

$\therefore \{1\} = 0$

48. Without any loss of generality assume the wts to be 1, 2, 3, 4, 5, 6.

It is obvious that 1 should be at the top of pyramid.

If 2, 3 make second row then

1

2 3 --------2! $\Rightarrow$ 12 ways

4 5 6 ---------3!

if 2, 4 make second row

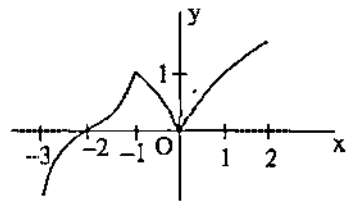
1

2 4 --------2 $\Rightarrow$ 4 ways

3 5 6 ---------2

Total number of ways = 16

49. $g'(x) = f(x)$



so two points of extrema at $x = -1$ and $x = 0$

50. $$h(x) = \frac{1}{1 - x}, x \ne 1$$

$$h(h(x)) = \frac{x - 1}{x}, x \ne 0, 1$$

$\Rightarrow h(h(h(x))) = x, x \ne 0, 1$

$g(x) \ge 0 \ \forall\ x \in R \Rightarrow f(x) \ge x$

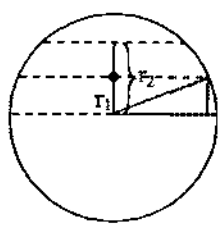
$\Rightarrow f(f(x)) \ge f(x) \ge x$ (as f(x) is an increasing function)

$\Rightarrow f(f(f(x))) \ge f(f(x)) \ge f(x) \ge x$

$\Rightarrow f(f(f(x))) - x \ge 0$ for $x \in R - \{0, 1\}$

51. $$r_1 = \sqrt{r^2 - \frac{25}{36}r^2} \Rightarrow r_1 = \frac{\sqrt{11}}{6}r$$

$$r_2 = \sqrt{r^2 - \frac{4}{9}r^2} \Rightarrow r_2 = \frac{\sqrt{5}}{3}r$$



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$$\text{Prob.} = \frac{\pi\left(\frac{5}{9}r^2 - \frac{11}{36}r^2\right)}{\pi r^2}$$

$$= \frac{20-11}{36} = \frac{1}{4}$$

52. $\pi \log_3\left(\frac{1}{x}\right) = k\pi,\ k \in I$

$$\log_3\left(\frac{1}{x}\right) = k \Rightarrow x = 3^{-k}$$

Possible values of k are $-1, 0, 1, 2, 3, \ldots\ldots$

$$S = (3+1) + \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\infty\right)$$

$$= 4 + \frac{(1/3)}{1-(1/3)}$$

$$= 4 + \frac{1}{2} = \frac{9}{2}$$

53. $(3^{x^2} - 3^{(x+6)})^2 = 0$

$\Rightarrow x^2 = x + 6$

$\Rightarrow (x^2 - x - 6) = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3$ or $x = -2$

54. $\frac{a+2c}{b+3d} + \frac{4}{3} = 0$

$\Leftrightarrow 3a + 4b + 6c + 12d = 0$

$\Leftrightarrow \frac{1}{4}a + \frac{b}{3} + \frac{c}{2} + d = 0$

Consider

$$f(x) = \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx$$

Then, $f(0) = 0 = f(1)$

$\therefore$ $f(x)$ satisfies the conditions of Rolle's theorem in $[0, 1]$.

Hence, $f'(x) = 0$ has at least one solution in $(0, 1)$.

55. Since, matrix A is skew-symmetric.

$\therefore |A| = 0$

$\therefore |A^4 . B^3| = 0$

56. $a + b = -3;\ ab = 5;\ s + t = m$

$$s = \frac{b+2}{a+2};\ t = \frac{a+2}{b+2};$$

$$s + t = \frac{(b+2)^2 + (a+2)^2}{(a+2)(b+2)}$$

$$= \frac{(a^2+b^2) + 4(a+b) + 8}{ab + 2(a+b) + 4}$$

$$m = \frac{(a^2+b^2) + 4(a+b) + 8}{ab + 2(a+b) + 4}$$

$$s + t = -\frac{5}{3} = m$$

57. $f(x) = ae^{2x} + be^x + cx$

Since, $f(0) = a + b$

i.e., $a + b = -1$ ....(i)

$f'(x) = 2ae^{2x} + be^x + c$

$\therefore f'(\log 2) = 2ae^{2\log 2} + be^{\log 2} + c$

$= 8a + 2b + c$

$= 8a + 2b + c = 31$ ....(ii)

$$\int_0^{\ell n 4} (ae^{2x} + e^x + cx - cx)dx$$

$$= \int_0^{\ell n 4} (ae^{2x} + be^x)dx$$

$$= \left(\frac{ae^{2x}}{2} + be^x\right)_0^{\ell n 4}$$

$$= \frac{ae^{2\ell n 4}}{2} + be^{\ell n 4} - \frac{a}{2} - b$$

$$= 8a + 4b - \frac{a}{2} - b = \frac{15a}{2} + 3b$$

$$= \frac{39}{2}$$

i.e., $15a + 6b = 39$ ....(iii)

from equation (i), (ii) & (iii)

$9a = 45$

$\therefore a = 5,\ b = -6$ and $c = 3$

58. a, b, c are in GP

$$\frac{2ab}{a+b} = 12,\ \frac{b+c}{2} = 3$$

$$\frac{2a.ar}{a+ar} = 12,\ ar + ar^2 = 6$$

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$ar = 6(1 + r)$, $ar(1 + r) = 6$

i.e., $6(1 + r)^2 = 6$ i.e., $(1 + r)^2 = 1$

$\therefore 1 + r = \pm 1$

i.e., $r = 0$ or $r = -2$

$\therefore r = -2$

if $r = -2$, then $a = 3$

$\therefore$ The GP is 3, – 6, 12, –24, 48.

Clearly, no term of the series is square of an integral.

AM of a, b, c is $\frac{3-6+12}{3} = 3$.

59. $\alpha + \beta = 1154$ and $\alpha\beta = 1$

$(\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha\beta}$

$= 1154 + 2 = 1156 = (34)^2$

$\sqrt{\alpha} + \sqrt{\beta} = 34$

Again,

$(\alpha^{1/4} + \beta^{1/4})^2 = \sqrt{\alpha} + \sqrt{\beta} + 2(\alpha\beta)^{1/4}$

$= 34 + 2 = 36$

$\alpha^{1/4} + \beta^{1/4} = 6$

60. $x^{(\log_2 x)^2 - 6\log_2 x + 12} = 256$

$(t^2 - 6t + 12)t = 8$,

where $t = \log_2 x$

$\Rightarrow (t-2)^3 = 0$

$\Rightarrow t = 2$

$\Rightarrow x = 4$

BIOLOGY

61. Meiosis involves crossing over which forms basis of recombinations.

62. Embryo formed from synergid or haploid cell doesn't involve fusion of gametes and so formed sporophyte is haploid.

63. Vas deferens transfer sperms from testis to the sits of storage i.e. seminal vesicle.

65. Contraceptive pills prevents ovulation, because they are synthetic derivative of estrogen & progesterone.

67. Independent assortment of character can be shown only upon crossing 2 character i.e. dihybrid cross.

68. Linkage prevents crossing over and so separation of genes.

71. Homologous organs are the organ with above feature.

72. Epiglottis covers glottis during eating food, preventing its entry into respiratory tract, it is not vestigial.

73. There are caused by vibrio cholerae, mycobacterium laprae & corynebacterium diptheriae respectively.

74. IgG is the most abundent immunoglobulin in plasma.

76. Cattles & buffalo are most important sources of diary products.

77. Gene therapy is used to replace the defective gene by a normal gene.

78. Restriction endonuclease is key tool for genetic engineering.

PART-II

Two Marks Questions

PHYSICS

81. Based on the description of the particle's position at time t = 0,

we know that the equation that describes the particle's x-coordinate as a function of time is

$$x = A\cos(\omega t)$$

To determine the x-component of the velocity, we take the time derivative of this function :

$$v_x = \frac{dx}{dt} = \frac{d}{dt} A\cos(\omega t) = -\omega A \sin(\omega t)$$

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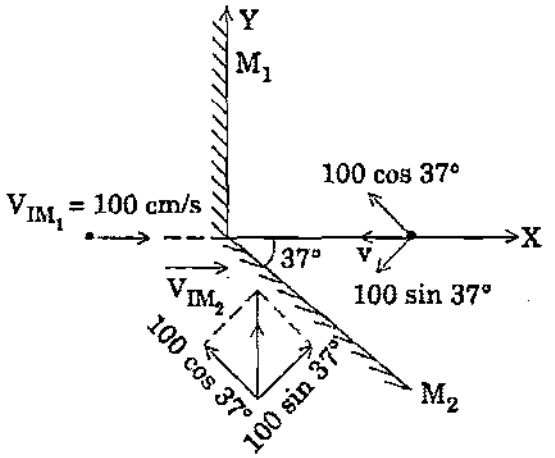
This is essentially a negative sine function, which matches the graph in answer 4. The problem may be solved conceptually, too, by considering the x-component of the rotating particle as a mass on a horizontally-stretched spring. The mass is released from a position +A at time t = 0, at which point it begins to accelerate in the x-direction, beginning a simple harmonic oscillation. This motion is consistent with answer.

82. The components of various velocities are as shown in figure



$\overline{V}_{IM_2}$ is given by the vector sum of components of velocity of image with respect to M_2 along the normal and perpendicular to the normal

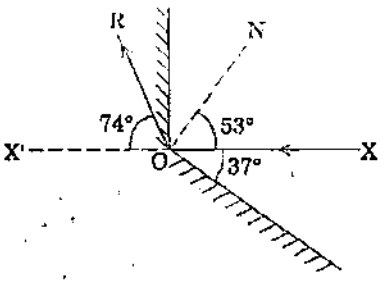
$$\overline{V}_{IM_2} = [100\sin^2 37°\,\hat{i} + 100\sin 37°\cos 37°\,\hat{j}] + [-100\cos^2 37°\,\hat{i} + 100\sin 37°\cos 37°\,\hat{j}]$$

$$= [-28\hat{i} + 96\hat{j}] \text{ cm/s}$$

$$\therefore \overline{V}_{IM_2,IM_1} = \overline{V}_{IM_2} - \overline{V}_{IM_1}$$

$$= (-128\hat{i} + 96\hat{j}) \text{ cm/s}$$

Aliter : For the reflection at mirror M_2 the incident ray OX is reflected along OR (ON – normal)



$\angle X'OR = 74° = 2\theta$

Then $\overline{V}_{IM_2} = -v\cos 2\theta\,\hat{i} + v\sin 2\theta\,\hat{j}$

83. Pitch of the screw = $\frac{1}{12}$ cm

number of revolutions made to cover 3 cm = $\frac{3}{1/12} = 36$

spin frequency of the nut = 216 rpm

$$= \frac{216}{60} \text{ Hz}$$

angular speed $\omega = 2\pi v = 2\pi \times \frac{216}{60}$

$$= 7.2\,\pi \text{ rad/s}$$

$$\omega = \frac{\theta}{t}$$

$$\Rightarrow t = \frac{\theta}{\omega} = \frac{2\pi n}{\omega} = \frac{2\pi \times 36}{7.2\pi} = 10 \text{ s}$$

84. (A) In $v = u + at$ or $v = u - at$, if $|a|$ is doubled, the time t is halved.

Hence, if t is the time for acceleration, then $\frac{t}{2}$ is the time taken for retarded motion/

$\therefore$ total time duration for travel is

$$t + t + \frac{t}{2} = \frac{5t}{2}$$

$$\frac{5t}{2} = 5s \text{ (given)}$$

$\therefore t = 2s$

(i) Distance covered in accelerated motion

$$S_1 = ut + \frac{1}{2}at^2$$

$$= 0 \times t + \frac{1}{2} \times 3 \times 2^2 = 6 \text{ m}$$

(ii) Distance covered in uniform motion

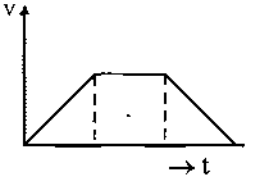
$$v = u + at$$

$$= 0 + 3 \times 2$$

$$= 6 \text{ ms}^{-1}$$

$\therefore S_2 = vt = 6 \times 2 = 12$ m

(iii) Distance travelled in retarded motion



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$$S_3 = ut - \frac{1}{2}at^2$$

$$= 6 \times 1 - \frac{1}{2} \times 6 \times 1^2 = 3m$$

$\therefore$ total distance traversed

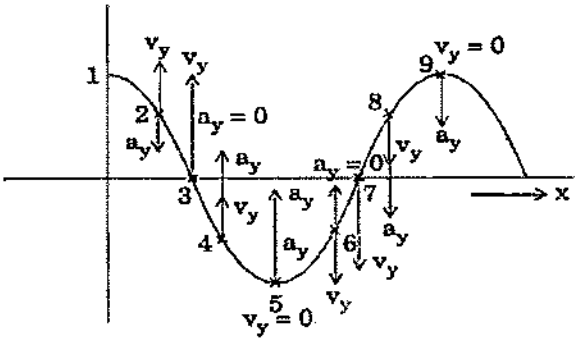
$S = S_1 + S_2 + S_3 = 6 + 12 + 3 = 21$ m

(N.B) this problem can also be solved by drawing v-t graph

85. Displacement $y(x, t) = A \cos(kx - \omega t)$

At $t = 0$, point 1 on the string has maximum displacement.

Velocity $v_y(x, t) = \omega A \sin(kx - \omega t)$



At $t = 0$, v_y is maximum at $kx = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi\lambda}{2 \times 2\pi} = \frac{\lambda}{4}$ i.e., point 3 on the string.

Hence (1) is correct.

Acceleration $a_y(x, t) = \omega^2 A \cos(kx - \omega t)$

At $t = 0$, a_y is maximum downward acceleration

if $\cos kx = 1 \Rightarrow kx = 2\pi \Rightarrow x = \lambda$

i.e., point 9 on the string has maximum downward acceleration.

Hence (3) is correct.

At $t = 0$, $kx = -\pi$, $x = \frac{\lambda}{2}$ i.e., the string has maximum upward acceleration.

Hence (2) is correct

Velocity v_y is positive and acceleration a_y is negative when $0 < kx < \frac{\pi}{2}$

v_y is negative and a_y is positive $\pi < kx < \frac{3\pi}{2}$

v_y and a_y are positive when $\frac{\pi}{2} < kx < \pi$

v_y and a_y are negative when $\frac{3\pi}{2} < kx < \pi$

(4) is incorrect.

86.

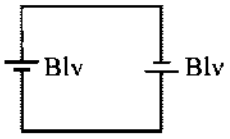
one satellite cover 120° angle for 360° 3 satellite

87. $f_c = 9\sqrt{N_0}$

$\Rightarrow \frac{f_c}{9} = \sqrt{N_0}$

$N = \frac{f_c^2}{81}$

88. Equivalent cell



$e = Blv + Blv = 2Blv$

89. $e = L\frac{dI}{dt}$

$e \Rightarrow$ Rate of change of current

90. Magnetic field is perpendicular to plane of coil.

CHEMISTRY

91. $SbF_5 + XeF_4 \rightarrow XeF_3^+ [SbF_6]^-$

94. $(0.01) \times [OH^-]^2 = 1 \times 10^{-12}$

$[OH^-] = 10^{-10}$

$[OH^-] = 10^{-5}$

$pOH = 5$

$pH = 9$

95. $a = \frac{4 \times 141.4}{\sqrt{2}} = 400$ pm

$V = (400)^3 = 6.4 \times 10^7$ pm^3

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96. $[S^{2-}] = 10^{-23}$ mol L^{-1}, $[M^{2+}] = 10^{-2}$ M.
Ionic product $[M^{2+}][S^{2-}] = 10^{-25}$. Ionic product is greater than K_{sp} of CuS and CdS.

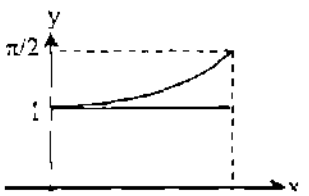
97. $Fe^{2+} + 2e^- \rightarrow Fe$,
$\Delta G^\circ_1 = -nFE^\circ$
$= -2F(-0.44) = 0.88\ F$ ...(i)
$Fe^{3} + 3e^- \rightarrow Fe$
$\Delta G^\circ_2 = nFE^\circ$
$= -3F(0.036) = 0.108\ F$ ...(ii)
Eqn.(i) – Eqn.(ii) gives
$Fe^{+3} + e^- \rightarrow Fe^{+2}$,
$\Delta G = -nFE^\circ = -0.772\ F$
or $E^\circ = \frac{0.772}{1} = 0.772$ V

MATHEMATICS

101. The number of ways of arranging n numbers is n! In each order obtained, we must now arrange the digits 1, 2,...., k as group and the n – k remaining digits. This can be done in (n – k + 1)! ways. Therefore, the probability for the required event is (n – k + 1)!/ n!

102. Let y = f(x)
given equation $x + |y| = 2y$
case (1) $y \geq 0$
$x = y \Rightarrow f^{-1}(x) = x\ \forall\ x \geq 0$
case (2) $y < 0$
$x = 3y$
$f^{-1}(x) = 3x, \quad x < 0$
which satisfies in option (4).

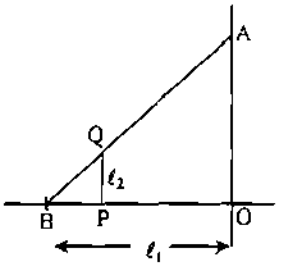
103.



Since, $\sin x \frac{dy}{dx} + y \cos x = 1$
$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$
$IF = e^{\int \cot x\, dx} = e^{\ln(\sin x)} = \sin x$
$y \sin x = \int \operatorname{cosec} x \cdot \sin x\, dx$
$y \sin x = x + C$
If $x = 0$, y is finite
$\therefore C = 0$
$\therefore y = \frac{x}{\sin x}$
Now, $I < \frac{\pi^2}{4}$ and $I > \frac{\pi}{2}$
Hence, $\frac{\pi}{2} < I < \frac{\pi^2}{4}$

104. $|(5AC)(2B)^{-1}|$
$= |5AC|\,|2B^{-1}|$
$= 25\,|A|\,|C| \times \frac{1}{2^2}\,\frac{1}{|B|}$
$= 25 \times \frac{1}{8} \times 3 \times \frac{1}{4} \times \frac{1}{5}$
$= \frac{15}{32}$

105.



Let BP = x. From similar Δ's property, we get
$\frac{AO}{\ell_1} = \frac{\ell_2}{x}$
$\Rightarrow AO = \frac{\ell_1 \ell_2}{x}$
$\Rightarrow \frac{d(AO)}{dt} = \frac{-\ell_1 \ell_2}{x^2}\frac{dx}{dt}$, when
$x = \frac{\ell_1}{2}$, $\frac{d(AO)}{dt} = -\frac{2\ell_2}{5}$ m/s.

106. Let $\sin^{-1}(2012)^x = t$
$\frac{1}{\sqrt{1-(2012)^{2x}}} \times (2012)^x \log_e 2012\, dx = dt$
$\therefore \int \frac{(2012)^{2x}}{\sqrt{1-(2012)^{2x}}} \cdot (2012)^{\sin^{-1}(2012)^x}\, dx$
$= (\log_{2012} e) \int (2012)^t\, dt$

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$= \log_{2012}e \times \frac{(2012)^t}{\log_e 2012} + c$

$= (\log_{2012}e)^2 \times (2012)^{\sin^{-1}(2012)^x} + c$

107. Let $y = f(x) = x^3 - 2x^{-3} + 10$

$\therefore y_1 - y_2 = (x_1^3 - x_2^3) + 2(x_2^{-3} - x_1^{-3})$

$= (x_1^3 - x_2^3) + 2\left(\frac{x_1^3 - x_2^3}{(x_1x_2)^3}\right)$

$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = (x_1^2 + x_2^2 + x_1x_2)\left(1 + \frac{2}{(x_1x_2)^3}\right)$

$= \frac{[(x_1 + x_2)^2 - x_1x_2][(x_1x_2)^3 + 2]}{(x_1x_2)^3}$

$= \frac{(4A^2 - G^2)(G^6 + 2)}{G^6} = 9$

$\Rightarrow 9G^6 - (4A^2 - G^2)(G^6 + 2) = 0$

108. $f'(x) = \cos\left(\frac{x^2 + 2x + 1}{5}\right)$

$= \cos\left(\frac{(x+1)^2}{5}\right)$

Since,

$0 \le x \le 2, \frac{1}{5} \le \frac{(x+1)^2}{5} \le \frac{9}{5}$

$\cos\frac{(x+1)^2}{5} = 0,$

only when $\frac{(x+1)^2}{5} = \frac{\pi}{2}$

$f'(x) > 0$, if $0 < x < \sqrt{\frac{5\pi}{2}} - 1$

$f'(x) > 0$, if $\sqrt{\frac{5\pi}{2}} - 1 < x < 2$

$\therefore x = \sqrt{\frac{5\pi}{2}} - 1$ is a point of local maximum.

109. $T_n = \frac{1}{n^2 + (n-2)}$

$= \frac{1}{(n+2)(n-1)}, n = 3, 4, 5, \ldots.$

$= \frac{1}{3}\left[\frac{1}{n-1} - \frac{1}{n+2}\right]$

$\therefore S = \sum_{n=3}^{\infty} T_n = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right) + \frac{1}{3}\left(\frac{1}{3} - \frac{1}{6}\right) + \frac{1}{3}\left(\frac{1}{4} - \frac{1}{7}\right) + \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right) \ldots\ldots\ldots\ldots$

$S = \frac{1}{3}\left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right] = \frac{1}{3}\left[\frac{6+4+3}{12}\right] = \frac{13}{36}$

110. $\int_0^{2/3} f(x)\,dx$

$\int_0^{2/3}\left([x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right]\right)dx$

$= \int_0^{2/3} 0\,dx + \int_0^{2/3} 0\,dx + \int_{1/3}^{2/3} dx$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

BIOLOGY

111. Allium type is bispore type but 7 called and 8 nucleate.

112. Sudden fall in level of progesterone, due to regression of corpus lutum results is breaking of uterine wall, causes menstrual flow.

115. Turner syndrome results from monosomy of sex chromosome which results from chromosomal non disjunction during gamete formation.

117. Neanderthal man had cranial capacity of about 1400 cc.

119. Murcury poisioning in marine water caused minamata disease in Japan.

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